The vacuum comes alive

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Outline

- From action at a distance to a dynamic aether
- The fundamental symmetry of the aether: Lorentz invariance
- The language of field theory: Lagrangians
- How Lorentz invariance of the Lagrangian largely specifies the dynamics

Newton

1642 - 1727

- Gravity: action at a distance

$$F \propto \frac{Mm}{r^2}$$

Coulomb

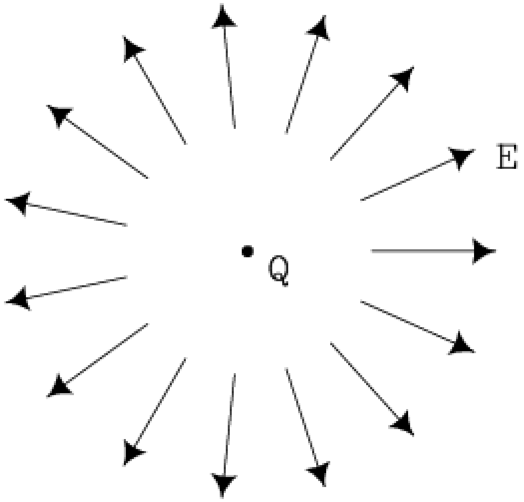
1736 - 1806

- Electrostatics

$$F \propto \frac{Qq}{r^2}$$

- Electric field as an abstraction

$$F = qE \qquad E(r) \propto \frac{Q}{r^2}$$



- Charges make a field; field pulls charges
- Same idea applies to gravity: F = mg

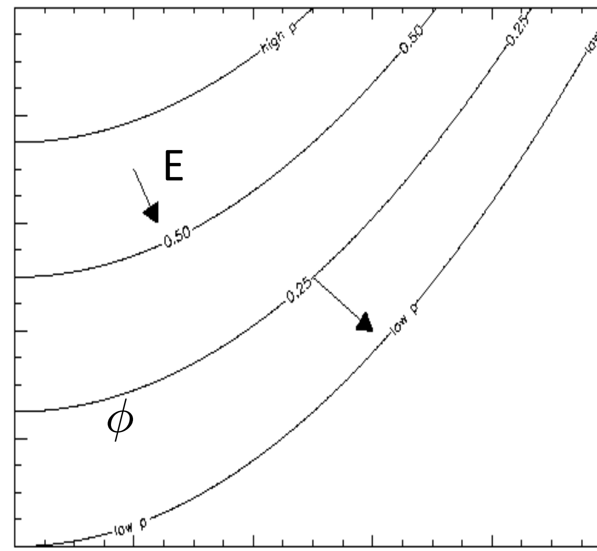
charge field

Potentials

- These fields are gradients (steepness) of a potential

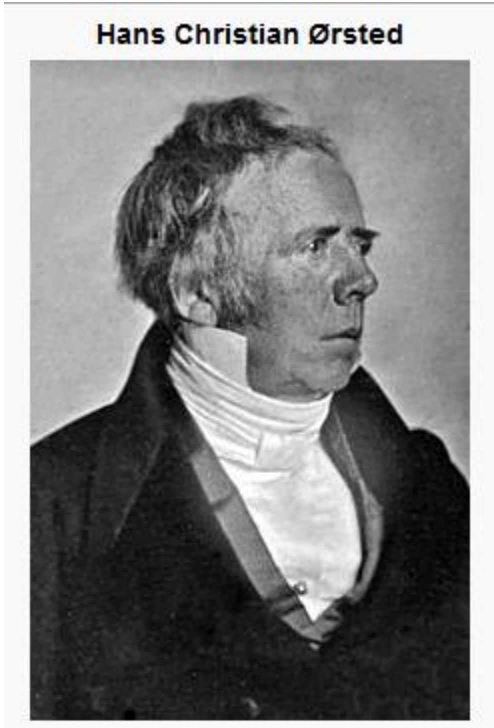
$$E = -\nabla \phi \qquad \phi = \frac{\kappa_E Q}{r}$$

$$g = -\nabla \Phi \qquad \Phi = -\frac{\kappa_G M}{r}$$

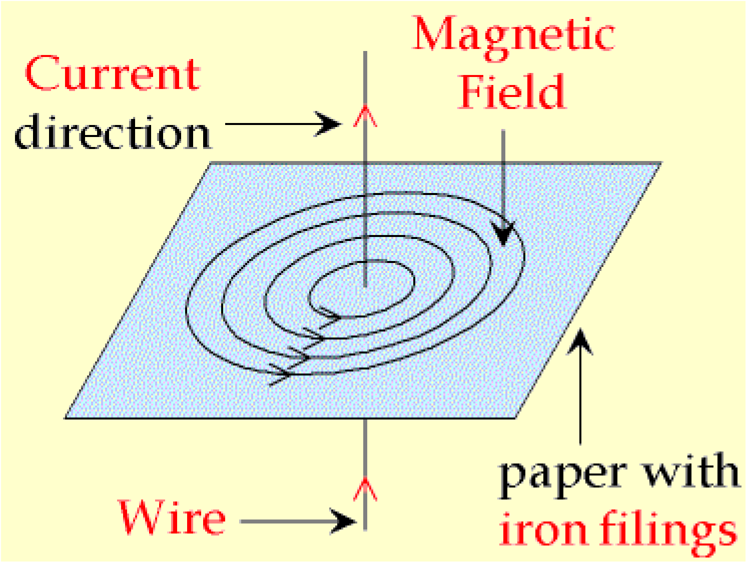


- Potential easier to work with because just one number ϕ versus E_x, E_y, E_z

Oersted
(1777-1851)



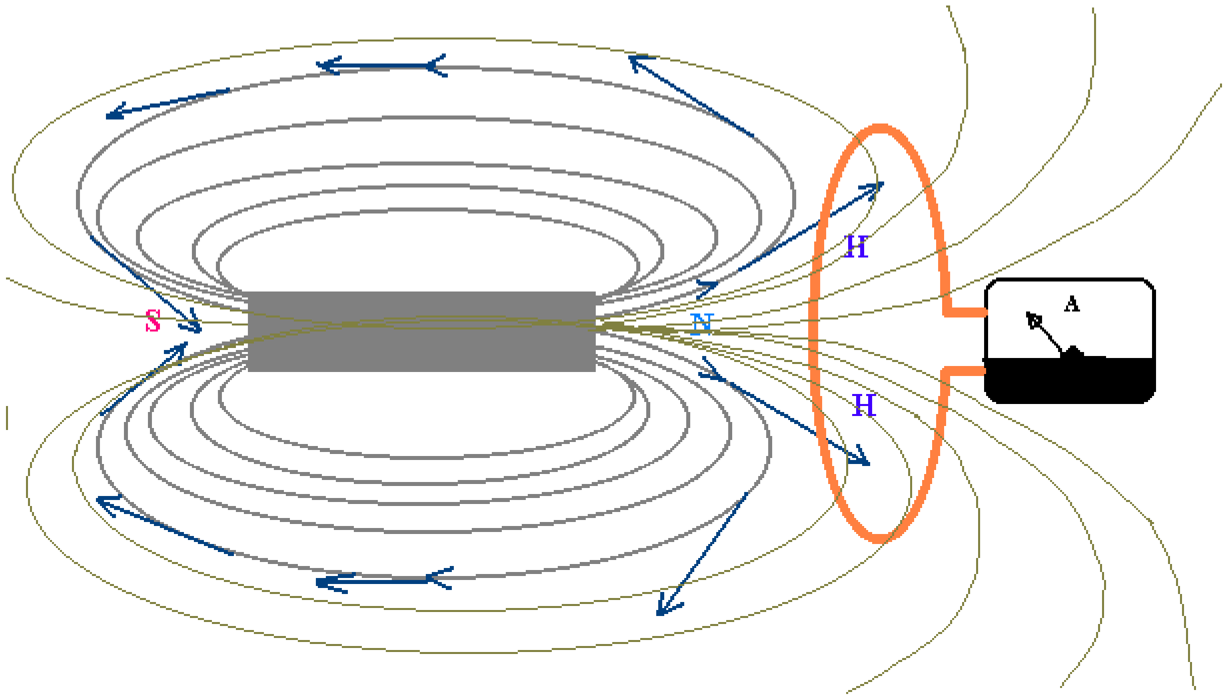
- Current flowing in a wire causes a B field to run around the wire so compass needle always points perpendicular to compass—wire line



Faraday

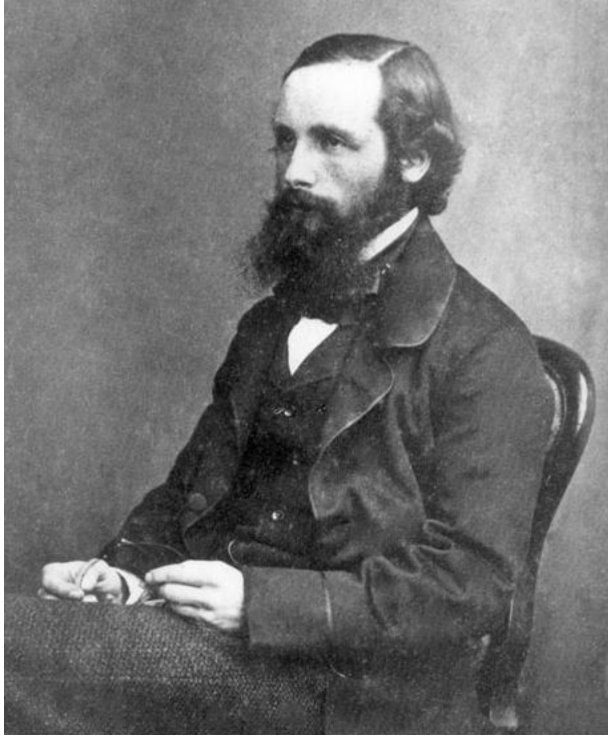
1791 - 1867

- A changing magnetic field generates a swirling electric field



$$\nabla \times E = -\frac{\partial B}{\partial t}$$

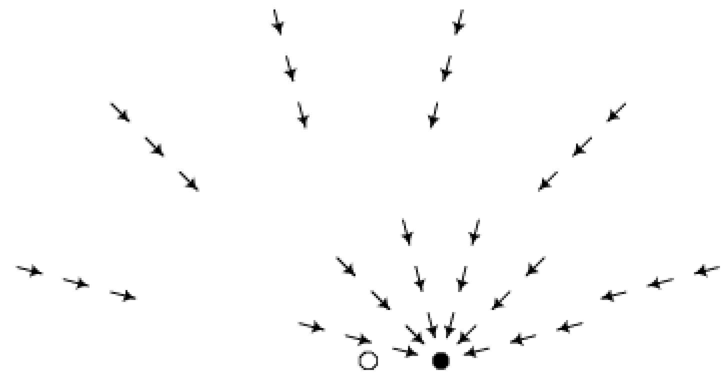
Maxwell
1831 - 1879



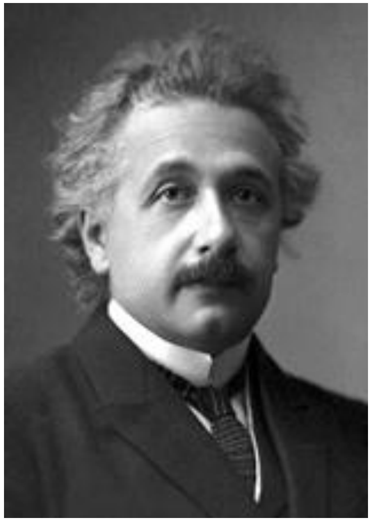
- A changing electric field generates a swirling magnetic field
- $\Rightarrow$ E, B form an independent dynamical system
- Don’t need charges to generate them!
- Ripples in E,B travel at speed of light

Maxwell (2)

- EB are manifestations of an “elastic” medium – the aether



- Action isn’t @ a distance!
- And it isn’t instantaneous
- A kink in E travels through the aether @ c
- An oscillating electric charge must radiate electromagnetic waves



Lorentz & Einstein

1853 – 1928 1879 - 1955

- Lorentz:
  - Maxwell's equations unchanged under

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \text{ where } \gamma^2 = \frac{1}{1-\beta^2}$$

  - What does this mean?
- Einstein:
  - equation relates times and positions measured by moving observers (β = v/c)
  - Invariance of equations implies physics is the same no matter how fast you move

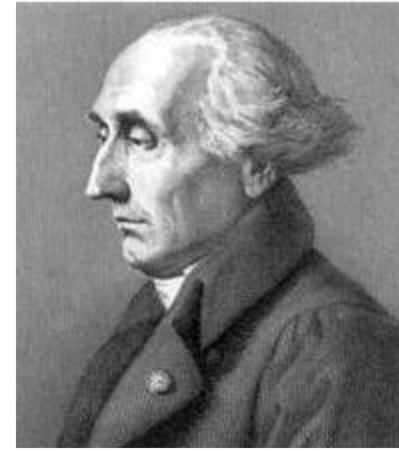
20^{th}c physics

- Aether = spacetime
- The material world is nothing but the excited aether
- Fundamental equations of physics must be invariant under Lorentz's transformation
- This requirement restricts the dynamics of the aether to just a handful of possibilities
- Lagrangian densities enable us to find these possibilities

Lagrange

Giuseppe Luigi Lagrancia

1736 1813



- Projectiles etc follow path of "least action" $S = \int dt\, L(x,v)$
- L = KE – PE
- Given L(x,v) we can derive the eqns of motion from the "Euler-Lagrange" eqns
- A field evolves so as to minimise action $S = \int dt\, d^3x\, \mathcal{L}(\phi, \partial\phi/\partial x_i)$
- $\mathcal{L}$ is built up from the field and its gradients
- Given $\mathcal{L}$ we can recover the field eqns from the Euler-Lagrange eqns

Example: wave eqn

$$\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} = 0 \quad \leftrightarrow \quad \mathcal{L} = \frac{1}{c^2}\left(\frac{\partial \phi}{\partial t}\right)^2 - \left(\frac{\partial \phi}{\partial x}\right)^2$$

KE PE

- Wave eq linear in ϕ $\leftrightarrow$ L quadratic in ϕ
- Both d.e. and L unchanged by Lorentz transformation

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \text{ and } \phi(x', t') = \phi(x, t)$$

Example: emag

- Now field has 4 components A_x , A_y , A_z , $A_t = -\phi/c$

$$B = \nabla \times A \qquad \frac{E}{c} = \underset{\text{Electrostatics}}{\nabla A_0} - \underset{\text{Faraday}}{\frac{1}{c}\frac{\partial A}{\partial t}}$$

$$\frac{1}{c^2}\frac{\partial^2 A_j}{\partial t^2} - \sum_{i=1}^{3}\frac{\partial^2 A_j}{\partial x_i^2} = 0 \quad \leftrightarrow \quad \mathcal{L} = \frac{1}{2\mu_0 c}\left(\underset{\text{KE}}{\frac{E^2}{c^2}} - \underset{\text{PE}}{B^2}\right)$$

- $\mathcal{L}$ again quadratic in field gradients
- $\mathcal{L}$ again unchanged by Lorentz transformation given that A_i changes by same rule as x_i (is a vector)

$$\begin{pmatrix} A'_t \\ A'_x \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} A_t \\ A_x \end{pmatrix}$$

Dirac
1902-1984

- Dirac was seeking a relativistic generalisation of the Schroedinger eq
- He discovered a new aspect of the aether – a four-component, complex-valued field $\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$
- It transforms in a new way under Lorentz transformations – it's a `Dirac spinor' $\psi \to \psi` = M(\beta)\psi$
- Electrons & positrons are excitations of ψ just as photons are excitations of A

Dirac equation

$$\mathrm{i}\sum_j \gamma_j \frac{\partial\psi}{\partial x_j} - \frac{2\pi}{\lambda_C}\psi = 0 \quad \leftrightarrow \quad \mathcal{L} = \mathrm{i}\sum_j \overline{\psi}\gamma_j \frac{\partial\psi}{\partial x_j} - \frac{2\pi}{\lambda_C}\overline{\psi}\psi$$

4×4 matrix

Compton λ

Dirac eqn

$$\gamma_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

- Linear coupled wave eqns now 1st order (hence i)
- $\mathscr{L}$ unchanged by Lorentz transformation given that ψ changes as a spinor

Coupling fields

$$\frac{1}{c^2}\frac{\partial^2 A_j}{\partial t^2} - \sum_{i=1}^{3}\frac{\partial^2 A_j}{\partial x_i^2} = -\underset{\substack{\uparrow \\ \text{antenna}}}{\mu_0 J_j} \quad \leftrightarrow \quad \mathcal{L} = \frac{1}{2\mu_0 c}\left(\frac{E^2}{c^2} - B^2\right) + \frac{1}{c}\underset{\substack{\uparrow \\ \text{Current density}}}{J} \cdot A$$

- But current density from electrons is

$$J_i = e\overline{\psi}\gamma_i\psi$$

- So $\mathcal{L}$ is

$$\mathcal{L} = \underbrace{\frac{1}{2\mu_0 c}\left(\frac{E^2}{c^2} - B^2\right)}_{\text{Emag waves}} + \underbrace{\frac{e}{c}A_i\overline{\psi}\gamma_i\psi}_{\text{coupling}} + \underbrace{\mathrm{i}\overline{\psi}\gamma_i\frac{\partial\psi}{\partial x_i} - \frac{2\pi}{\lambda_C}\overline{\psi}\psi}_{\text{Free electrons}}$$

- Coupling term is here cubic in the fields

Summary

- The spacetime (aether) is a complex dynamical system that carries several (many-component) fields
- We can derive all the wave eqns that describe this system from a single Lagrangian density
- Lorentz invariance of L strongly restricts the possible field eqns!
- It comprises a block for each mode (free particles) plus coupling terms between blocks