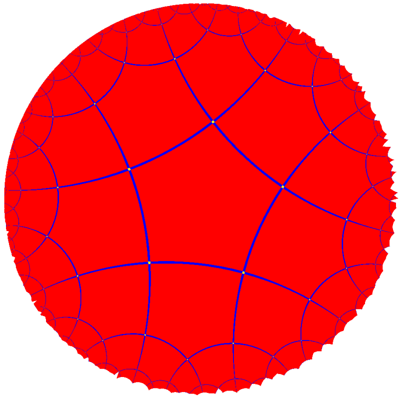
String theory, Holography and Quark-Gluon Plasma



Andrei Starinets

Rudolf Peierls Centre for Theoretical Physics
University of Oxford

21 May 2016

Outline

String theory and black holes: explaining black hole's entropy

Perturbative and non-perturbative results: a simple example

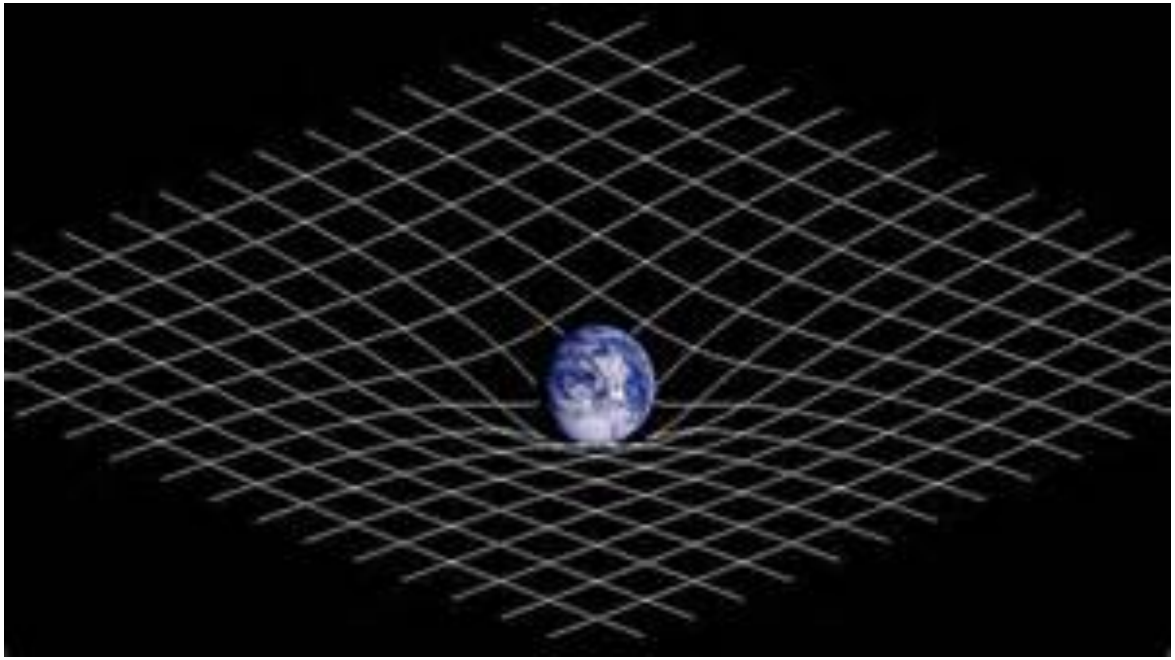
Simple example (continued): Duality as change of variables

AdS/CFT (holographic) duality

Physics of heavy ion collisions: quark-gluon plasma

How string theory helps to understand behavior
of hot and dense nuclear matter

Gravity and the metric



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Geometry (metric) = Energy (and/or mass)

Black holes

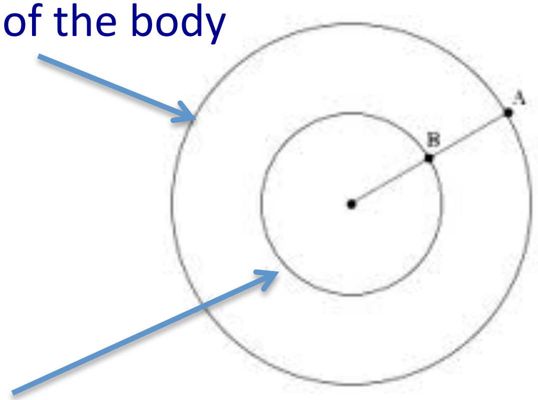
$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Describes the metric of space-time OUTSIDE of a body of mass M

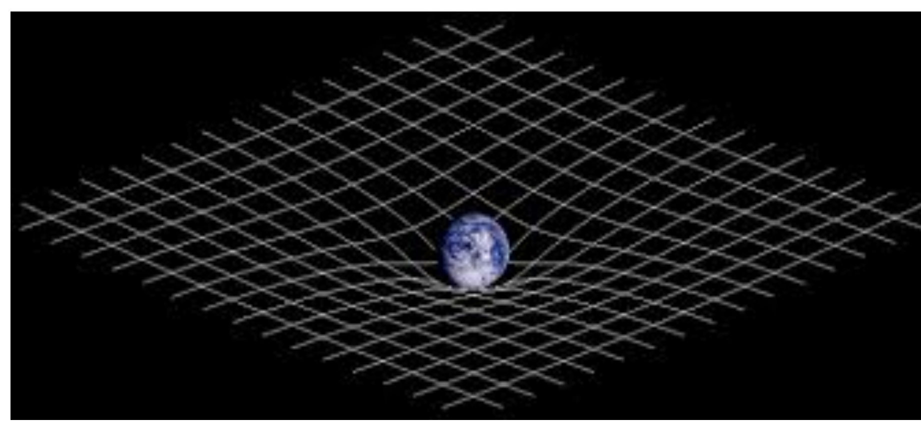
$$r = r_S = \frac{2GM}{c^2}$$

For most objects, the Schwarzschild radius is located deep inside the object and thus is not relevant since the solution is valid only outside (inside the metric is different). For example, for Earth the Schwarzschild radius is about 1 cm, for Sun – 3 km.

Surface of the body



Schwarzschild radius



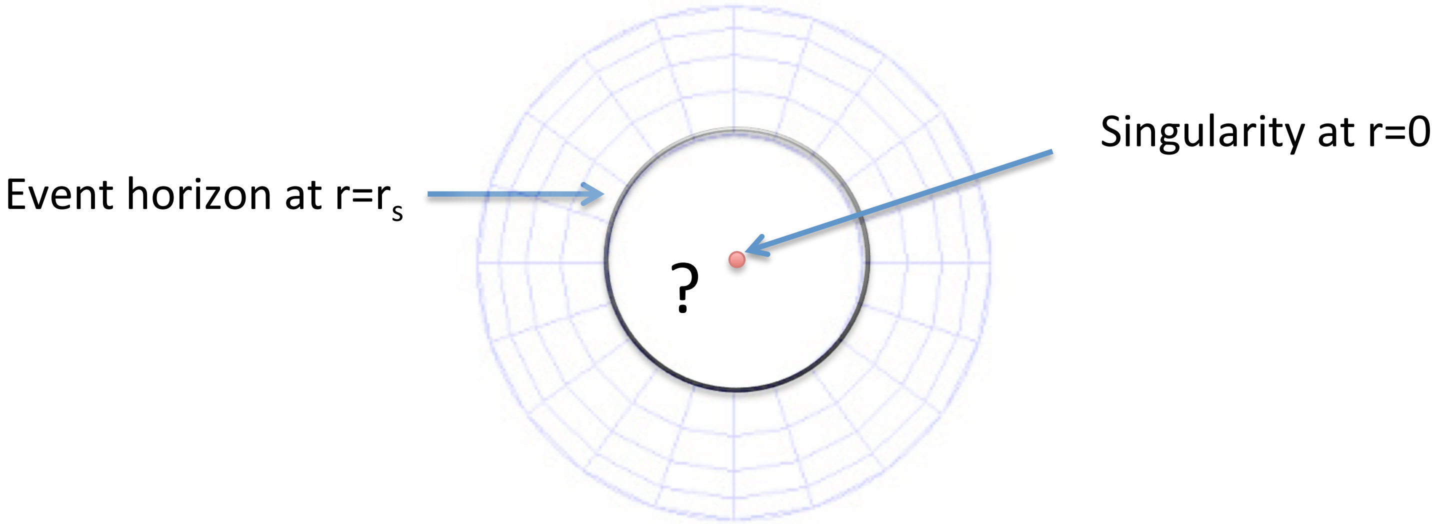
Black holes (continued)

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Describes the metric of space-time OUTSIDE of a body of mass M

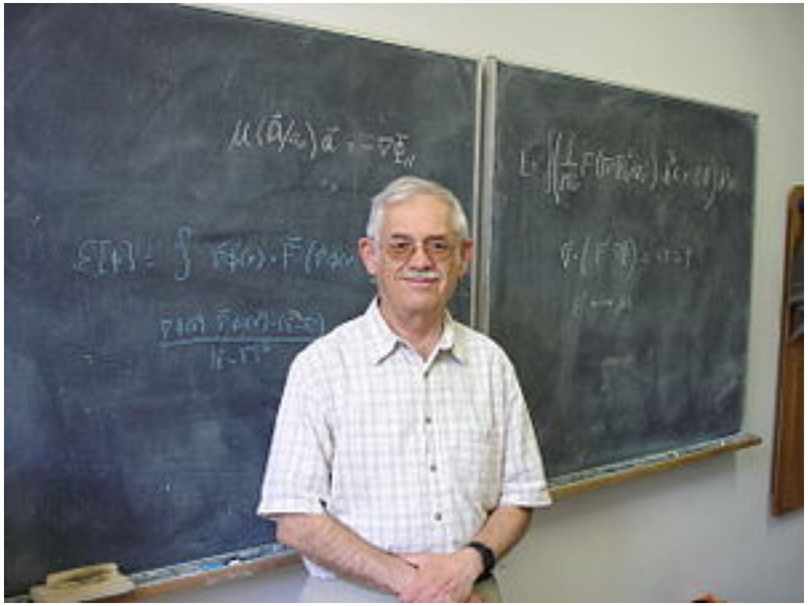
$$r = r_S = \frac{2GM}{c^2}$$

However, if the matter is squeezed inside its Schwarzschild radius (e.g. in the process of a gravitational collapse of a star), we get a black hole



Black hole thermodynamics

In 1972, Jacob Bekenstein suggested that a black hole should have a well defined entropy proportional to the horizon area. He was laughed at by some (many…) since black holes are BLACK (do not radiate) and thus cannot have a temperature associated with them



Jacob Bekenstein

Stephen Hawking

However, in 1974 Hawking demonstrated that black holes do emit radiation at a quantum level and so one can in fact associate a temperature with them

Hawking temperature and Bekenstein-Hawking entropy

Hawking showed that black holes emit radiation with a black-body spectrum at a temperature

$$T = \frac{\hbar c^3}{8\pi k_B G M} \approx \frac{1.2 \times 10^{23}\,\mathrm{kg}}{M}\, K$$

(a black hole of one solar mass has a Hawking temperature of about 50 nanoKelvin)

This fixes the coefficient of proportionality in Bekenstein's conjecture:

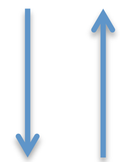
$$S_{BH} = \frac{c^3 A}{4 G \hbar}$$

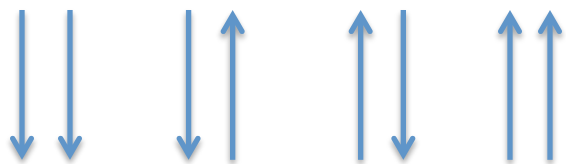
Immediate consequences and problems:

- Black holes "evaporate" with time
- Information loss paradox
- *What are the microscopic degrees of freedom underlying the BH thermodynamics?*

Entropy and microstates

In statistical mechanics, entropy is related to the number of microstates:

N=1 particle with spin:  2 microstates

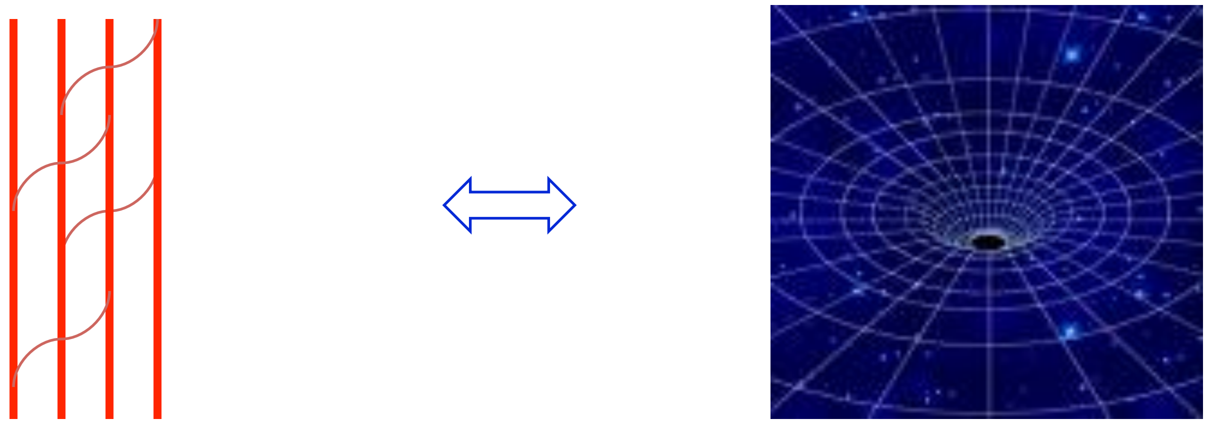
N=2 particles with spin:  4 microstates

N particles with spin: 2^N microstates

Boltzmann entropy: $S = k_B \ln W = k_B \ln 2^N = k_B N \ln 2$

Can we count the microstates of a black hole and recover the Bekenstein-Hawking result?

Counting microstates of a black hole



A.Strominger and C.Vafa (1996) were able to count the microstates of a very special (supersymmetric) black hole in 5 dimensions. The result coincides EXACTLY with the Bekenstein-Hawking thermodynamic entropy.

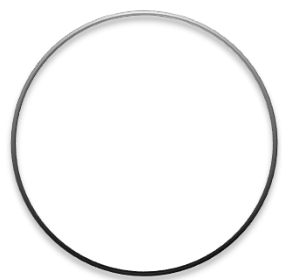
Strominger-Vafa result has been generalized in many ways since 1996.

However, we still do not know how to count the microstates of “normal” BHs, e.g. Schwarzschild BH in four dimensions…

Holographic principle

In thermodynamic systems without gravity, the entropy is extensive (proportional to volume)

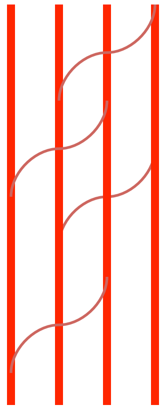
In gravitational systems, it is proportional to the AREA



$$S_{BH} = \frac{c^3 A}{4G\hbar}$$

It seems that gravitational degrees of freedom in D dimensions are effectively described by a theory in D-1 dimensions ('tHooft, Susskind, 1992)

Gauge-string duality (AdS-CFT correspondence)



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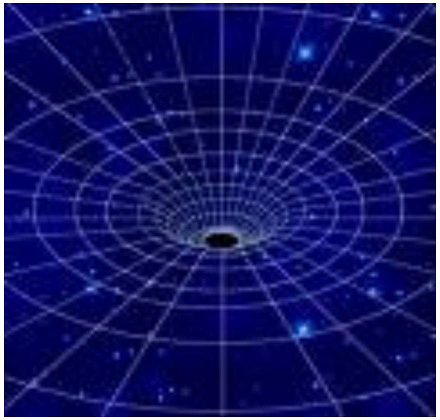


Open strings picture:
dynamics of strings & branes at low energy
is desribed by a
quantum field theory without gravity

Closed strings picture: dynamics of strings
& branes at low energy is described by
gravity and other fields
in higher dimensions

⟺

conjectured
exact equivalence

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

Quantum theory: the partition function

One way to define a quantum theory is to use Feynman path integral:

$$Z[\alpha] = \int [d\Phi] e^{-S[\Phi,\alpha]}$$

Here the partition function Z[α] depends on parameters of the theory α

The integration is over the fields of the theory Φ – e.g. scalar, vector, tensor or spinor fields, and S[Φ,α] is the action of the theory

Note that Φ is a "dummy variable": Z[α] depends only on α

The theory is "solved" completely, if we know how to calculate the path integral

Usually, this is a highly nontrivial task, and one resorts to approximations – for example, expanding for small α

Perturbative and non-perturbative approaches: a toy model

$$Z[\alpha] = \int_0^1 \frac{dx}{\sqrt{1-\alpha^2 x^2}} = ?$$

Note that x is a “dummy variable” – the result Z[α] depends ONLY on α!

Suppose we only know how to compute very simple integrals such as

$$\int dx = x + C \qquad \int_0^1 dx = 1$$

Prediction for very small α:

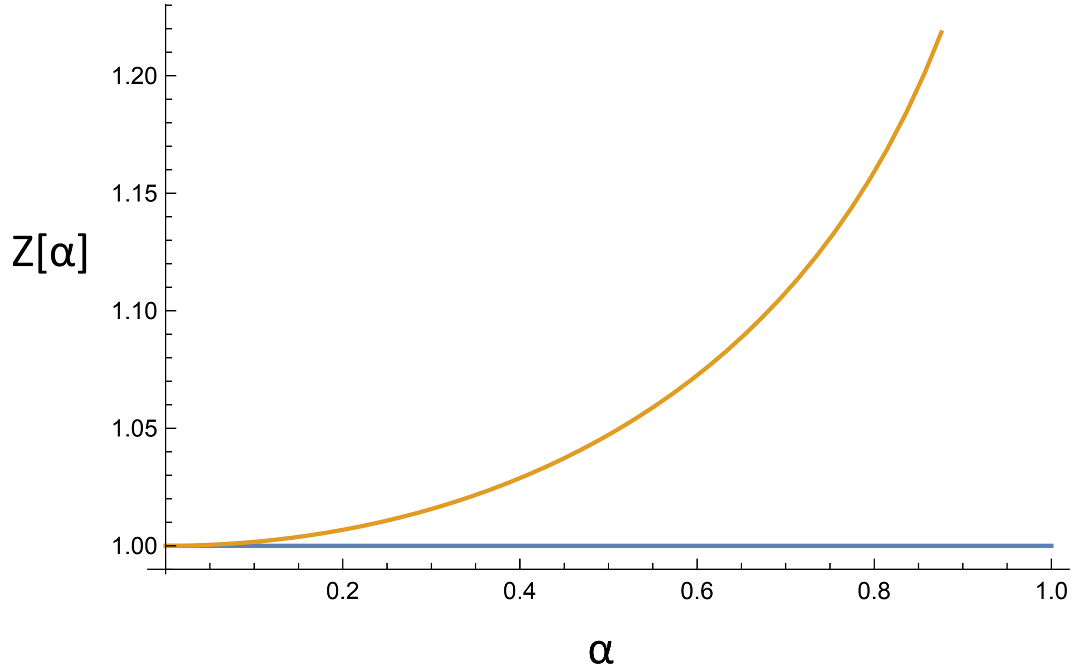
$$Z[\alpha] \approx \int_0^1 dx = 1\,, \quad \alpha \ll 1$$

Prediction for very small α:

$$Z[\alpha] = \int_0^1 \frac{dx}{\sqrt{1-\alpha^2 x^2}} = ?$$

$$Z[\alpha] \approx \int_0^1 dx = 1\,, \quad \alpha \ll 1$$

Compare this prediction to the exact plot Z[α]



Our prediction works well for small α but we want to know Z[α] for any α

$$Z[\alpha] = \int_0^1 \frac{dx}{\sqrt{1-\alpha^2 x^2}} = ?$$

We can do a clever change of the dummy variable x. Notice that:

$$1 - \sin^2 y = \cos^2 y$$

Change variables x -> y as:

$$\alpha\, x = \sin y$$

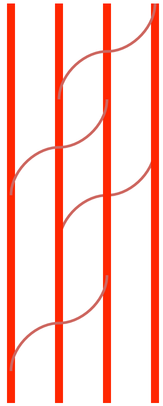
Then:

$$\alpha\, dx = \cos y\, dy$$

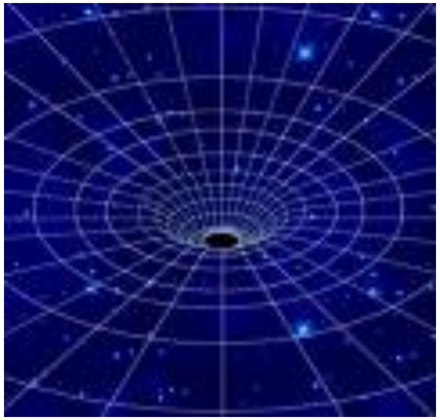
$$Z[\alpha] = \frac{1}{\alpha} \int_0^{\arcsin \alpha} \frac{\cos y\, dy}{\cos y} = \frac{1}{\alpha} \int_0^{\arcsin \alpha} dy = \frac{\arcsin \alpha}{\alpha}$$

This result is exact.

Gauge-string duality (AdS-CFT correspondence)



$\Longleftrightarrow$



Open strings picture:
dynamics of strings & branes at low energy is desribed by a quantum field theory without gravity

Closed strings picture: dynamics of strings & branes at low energy is described by gravity and other fields in higher dimensions

$$Z_{field\ theory}[\alpha] \Longleftrightarrow Z_{string\ theory}\left[\frac{1}{\alpha}\right]$$

Partition function of field theory in 3+1 dim

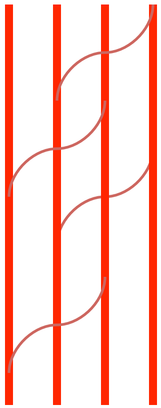
conjectured exact equivalence

Partition function of string theory in 10 dim

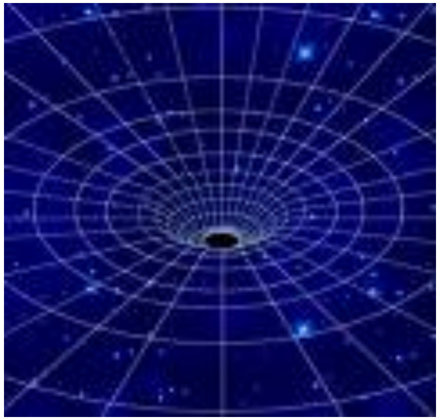
strong coupling

weak coupling

Gauge-String Duality, Gauge-Gravity Duality, AdS-CFT correspondence, Holography



$\Longleftrightarrow$



$$Z_{field\ theory}[\alpha] \Longleftrightarrow Z_{string\ theory}\left[\frac{1}{\alpha}\right]$$

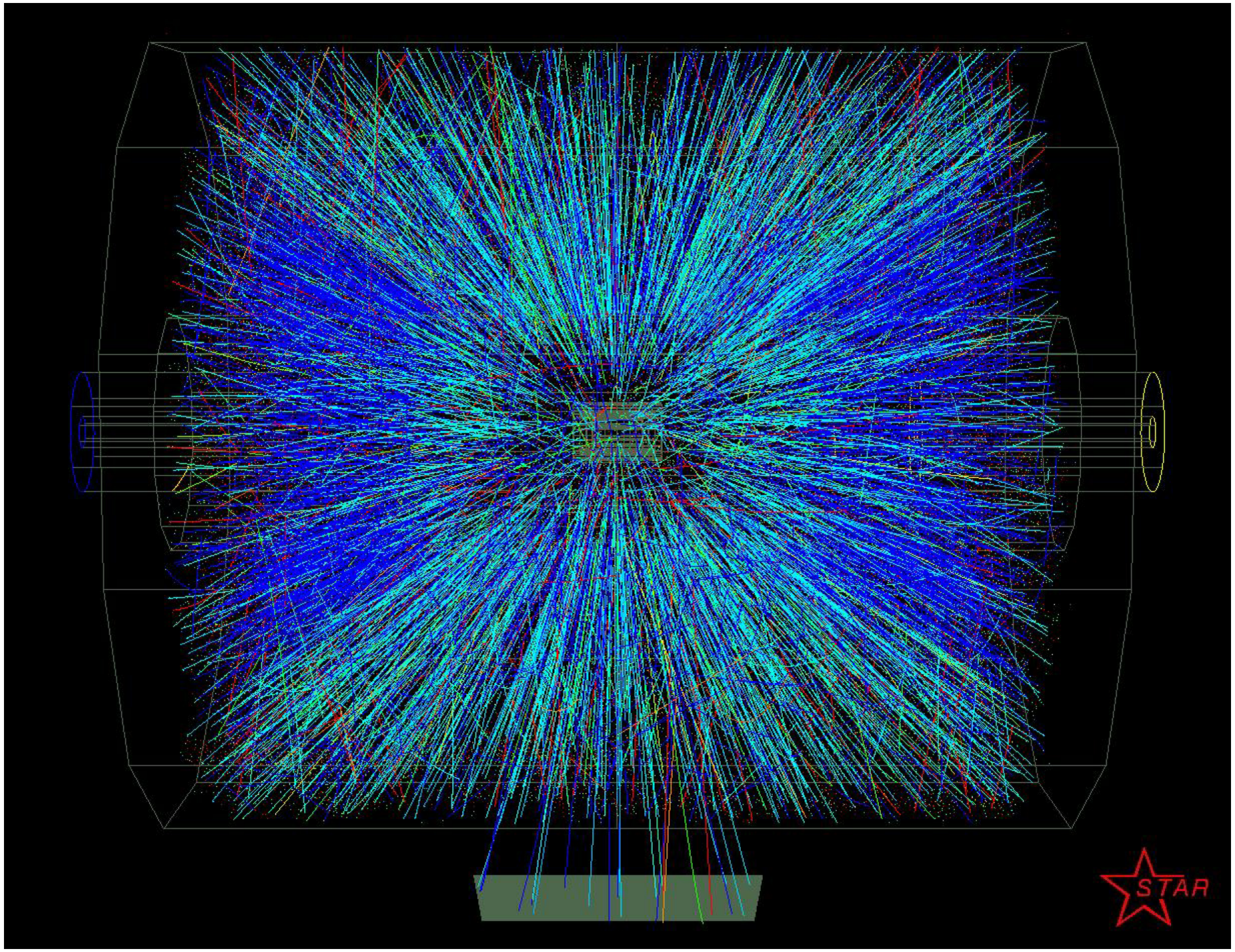
conjectured
exact equivalence

Using the duality, we can find various quantities in quantum field theory at strong coupling by doing the actual computation in the dual string theory at weak coupling

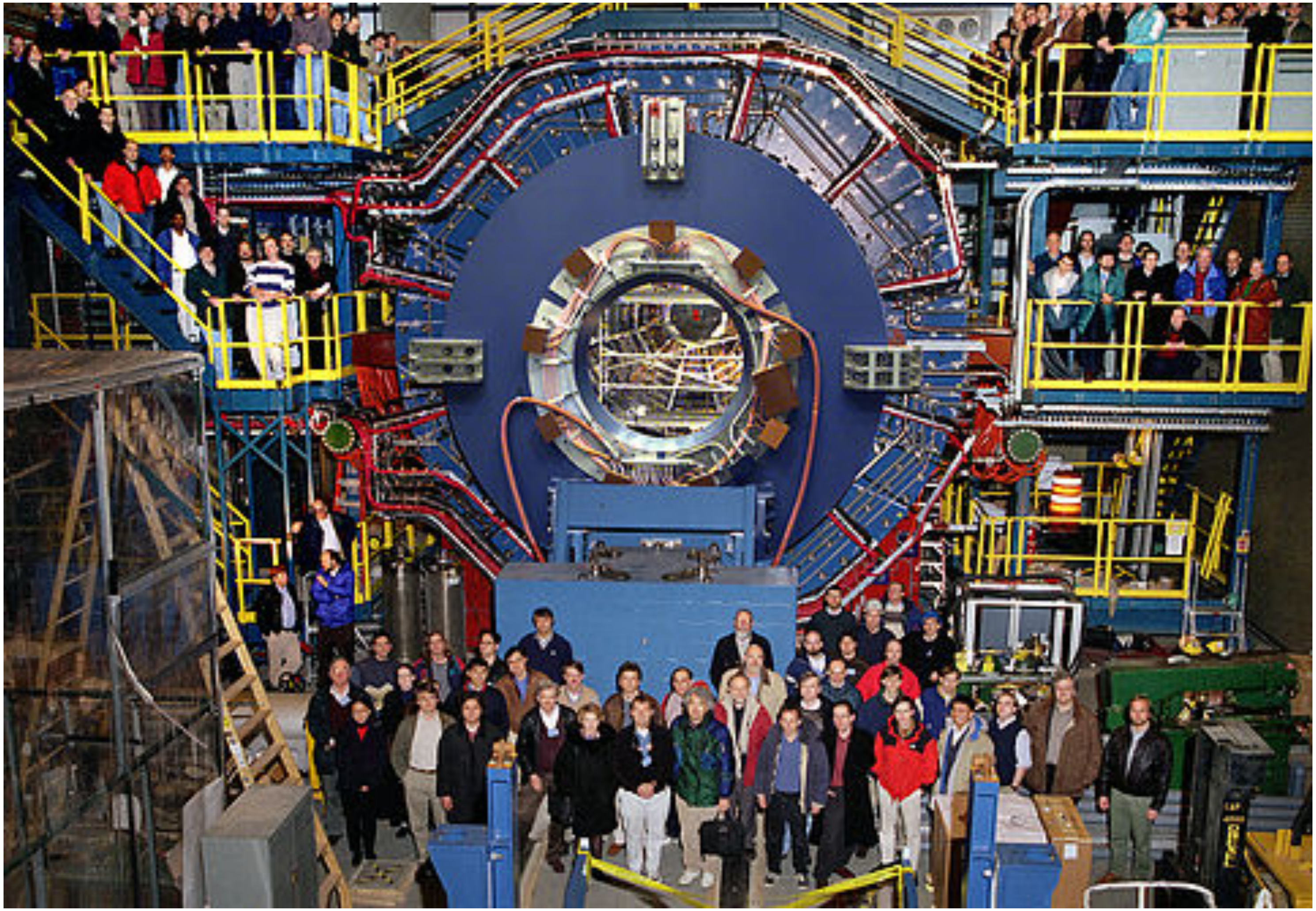
For some quantities, this is the only non-perturbative tool available

Note that the duality is independent of the status of String theory as THE Theory of Nature

Heavy ion collisions: RHIC/LHC

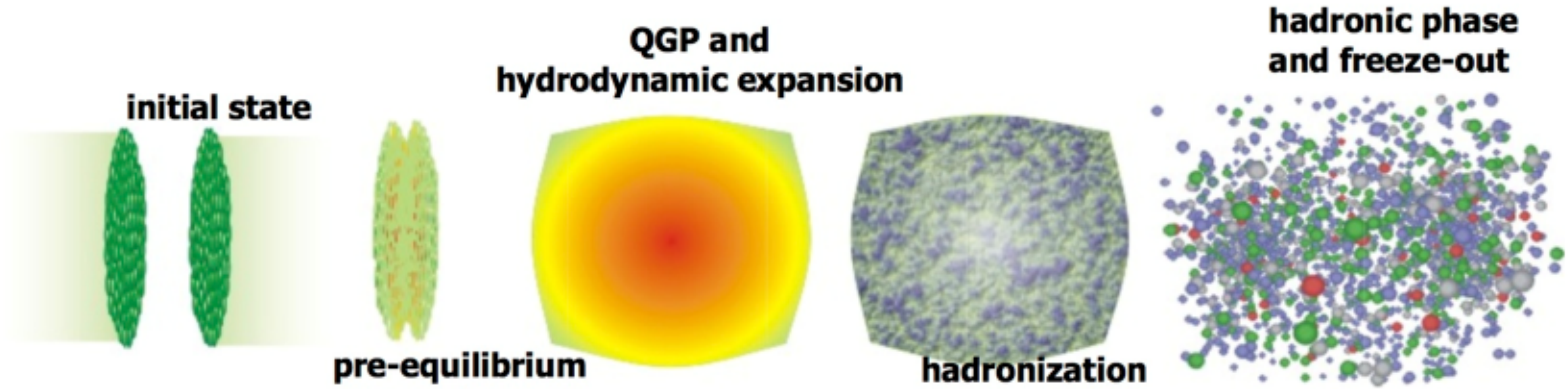


STAR: The Solenoidal Tracker at RHIC



Heavy ion collisions: evolution of the quark-gluon plasma

$$t_* \sim 10^{-24}\ sec$$



CGC	"Glasma"	Hydrodynamics		Hadronic gas
0	$1.2\,t_*$	$(2-3)\,t_*$	$(4-6)\,t_*$	$10\,t_*$

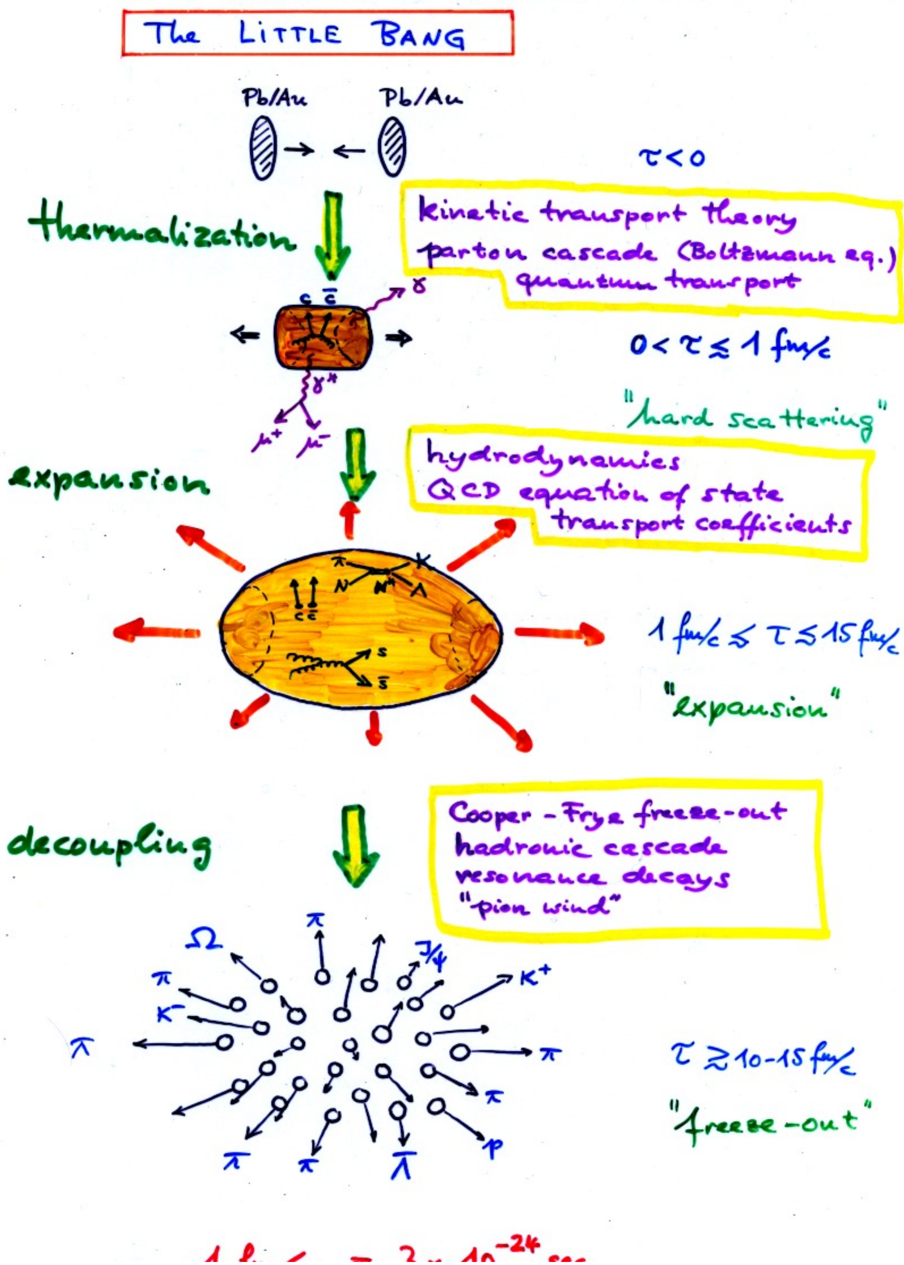


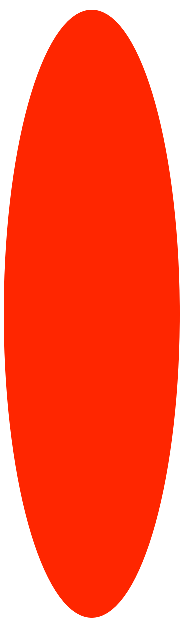
Figure from: U.Heinz, “Concepts of heavy-ion physics”, hep-ph/0407360

Heavy ion collision experiments at RHIC (2000-current) and LHC (2010-current) create hot and dense nuclear matter known as the "quark-gluon plasma"

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma "fireball" is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know



thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

$$\alpha_s(T_{\mathrm{RHIC}}) \sim 1$$

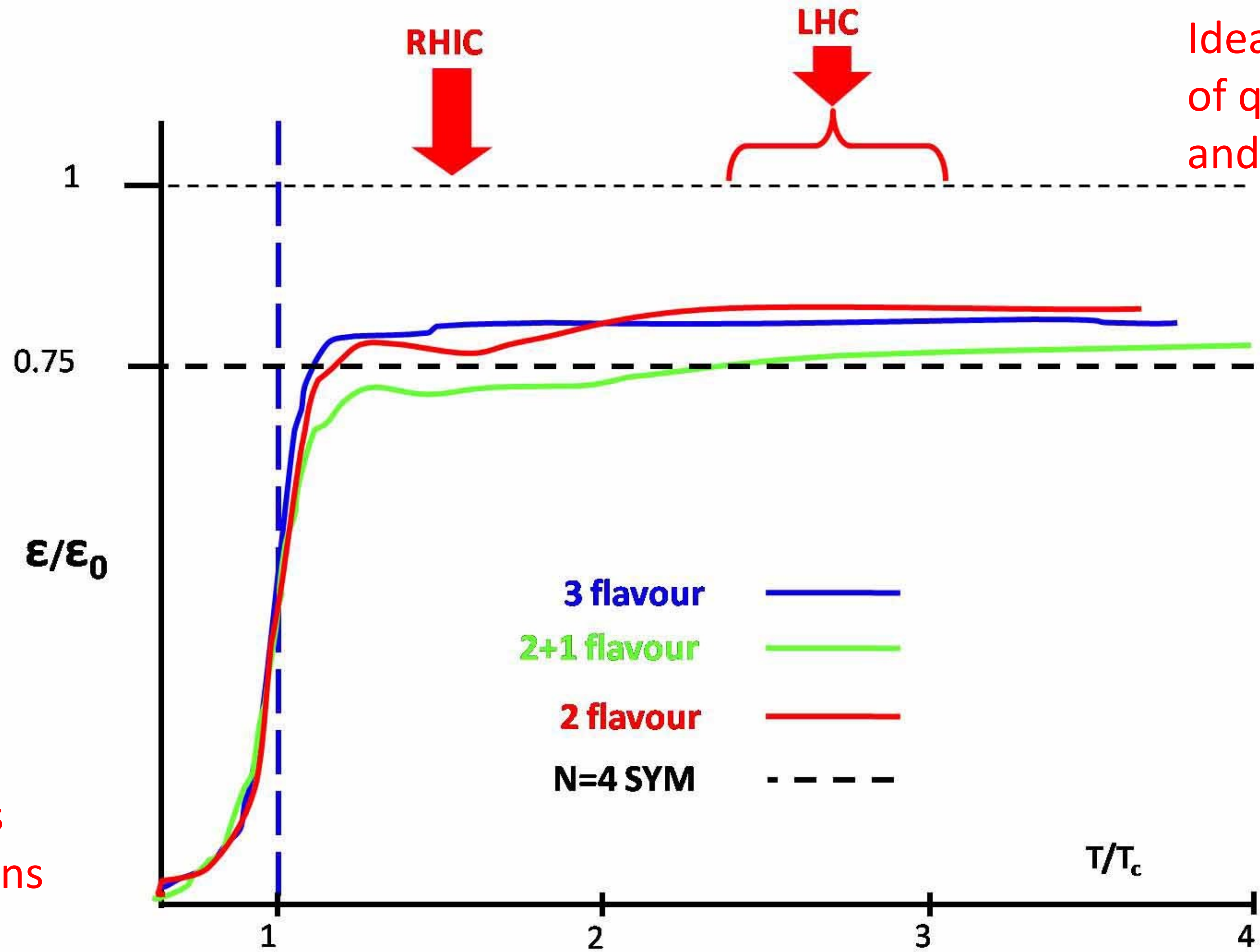
initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Energy density vs temperature for various gauge theories

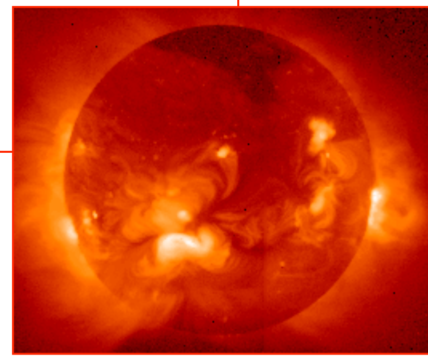
Ideal gas
of quarks
and gluons



Ideal gas
of hadrons

Figure: an artist's impression from Myers and Vazquez, 0804.2423 [hep-th]

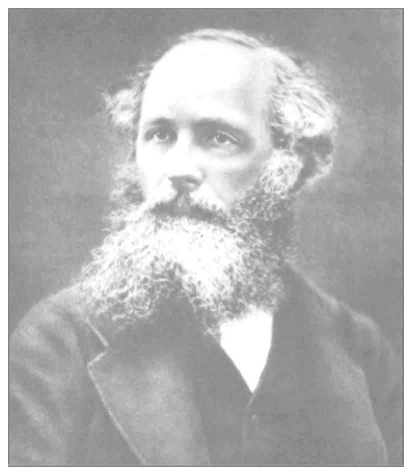
Quantum field theories at finite temperature/density



Equilibrium

Near-equilibrium

entropy
equation of state
....



transport coefficients
emission rates
....

perturbative
pQCD

non-perturbative
Lattice

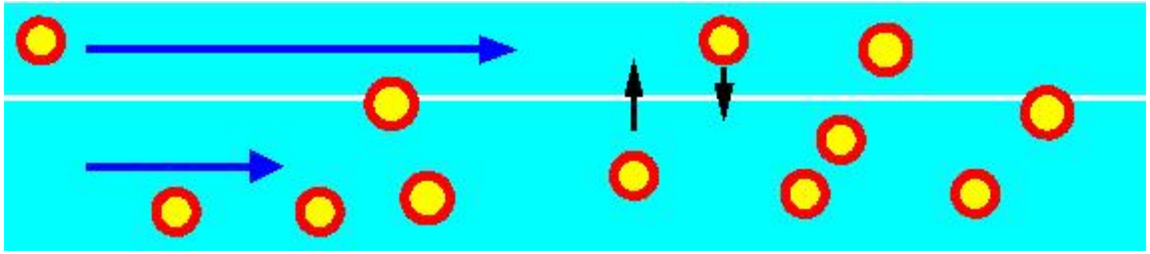
perturbative
kinetic theory

non-perturbative
????

Shear viscosity

One of the most important characteristics of quark-gluon plasma is shear viscosity

Shear viscosity can be understood as the measure of internal friction in liquid or gas



Momentum transfer between two layers of liquid or gas moving with different velocities leads to gradual equilibration of the velocities

Viscosity of gases and liquids

Gases (Maxwell, 1867): $$\eta \sim \rho \, \bar{v} \, l_{mfp} \sim \frac{m_o \bar{v}}{\sigma} \sim \frac{m_o^{1/2}}{\sigma} \sqrt{T}$$

Viscosity of a gas is

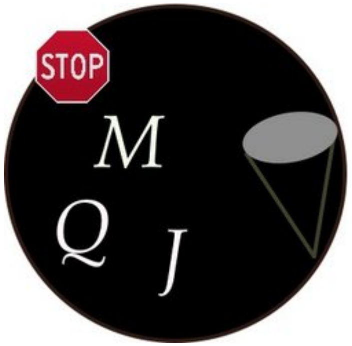
- independent of pressure
- scales as square of temperature
- inversely proportional to cross-section

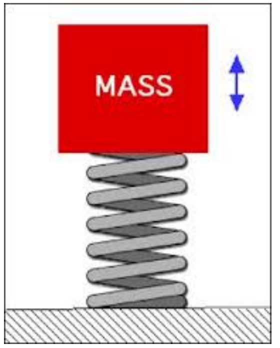
Liquids (Frenkel, 1926): $$\eta \sim A(P,T) \, \exp \frac{W}{T}$$

- In practice, A and W are chosen to fit data
- No good theory to determine viscosity of liquids from first principles
- Can try AdS/CFT correspondence to compute it at strong coupling

Black holes beyond equilibrium

Undisturbed black holes are characterized by global charges: M, Q, J…





A thermodynamic system in equilibrium is characterized by conserved charges: E, Q, J…

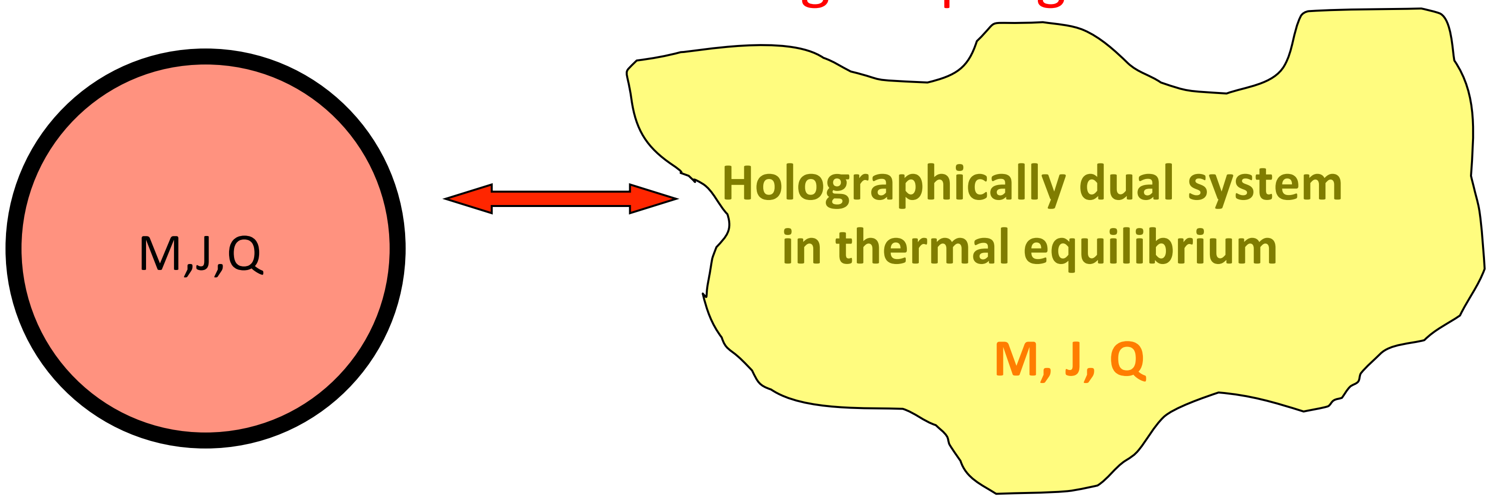
If one perturbs a non-gravitational system (e.g. a spring pendulum), it will oscillate with eigenfrequencies (normal modes) characterizing the system

$$\omega = \sqrt{\frac{k}{M}}$$

What happens if one perturbs a black hole?

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box"\ h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

Black hole's quasinormal spectrum encodes properties of a dual microscopic system

Comparing eigenfrequencies of a black hole

$$\omega = \pm \frac{c}{\sqrt{3}} k - \frac{i}{6\pi T} k^2 + \frac{3 - 2\ln 2}{24\pi^2 \sqrt{3} T^2} k^3 + \cdots$$

with eigenfrequencies of a dual microscopic system (described by fluid mechanics)

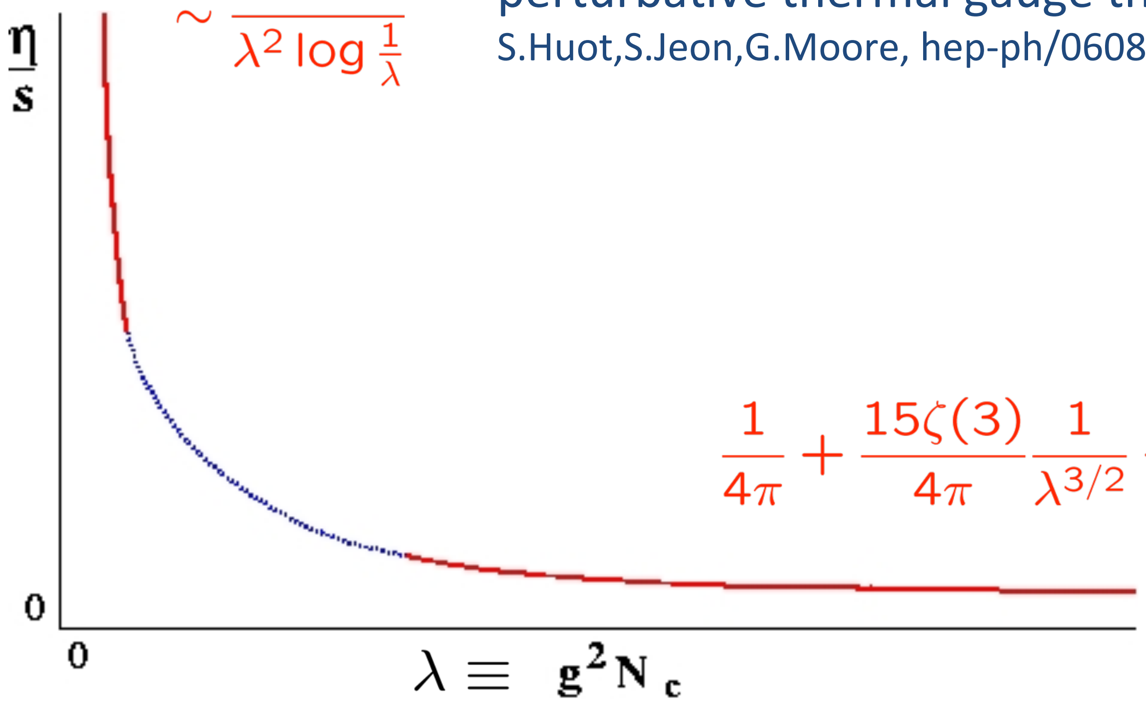
$$\omega = \pm v_s k - i \frac{2\eta}{3sT} k^2 + \cdots$$

one can compute viscosity-entropy ratio and other quantities of a microscopic system

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} + \cdots$$

Moreover, one can relate Navier-Stokes equations and Einstein's equations…

Shear viscosity in $\mathcal{N} = 4$ SYM



$\sim \frac{1}{\lambda^2 \log \frac{1}{\lambda}}$

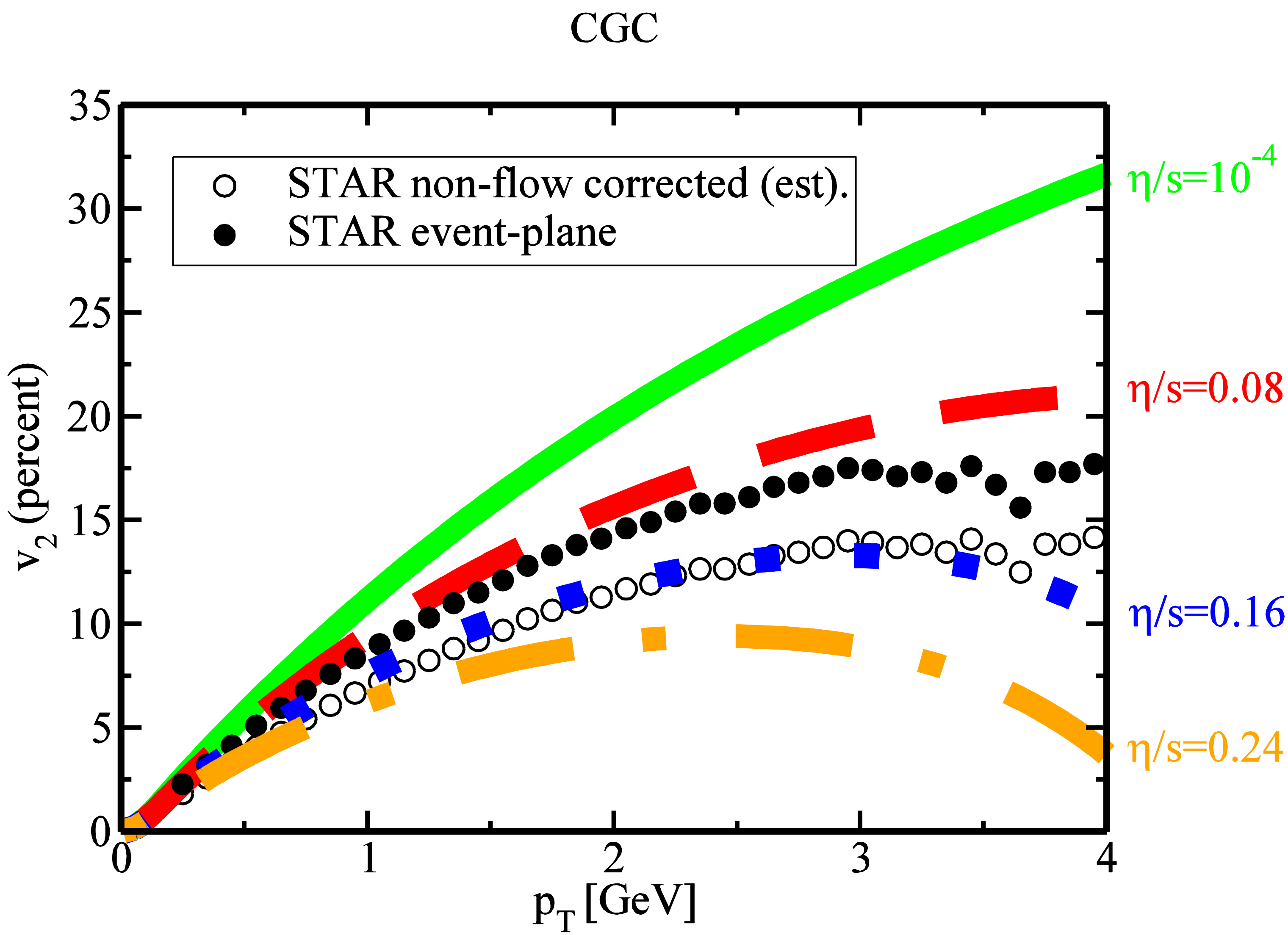
perturbative thermal gauge theory
S.Huot,S.Jeon,G.Moore, hep-ph/0608062

$$\frac{1}{4\pi} + \frac{15\zeta(3)}{4\pi} \frac{1}{\lambda^{3/2}} + \cdots$$

Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

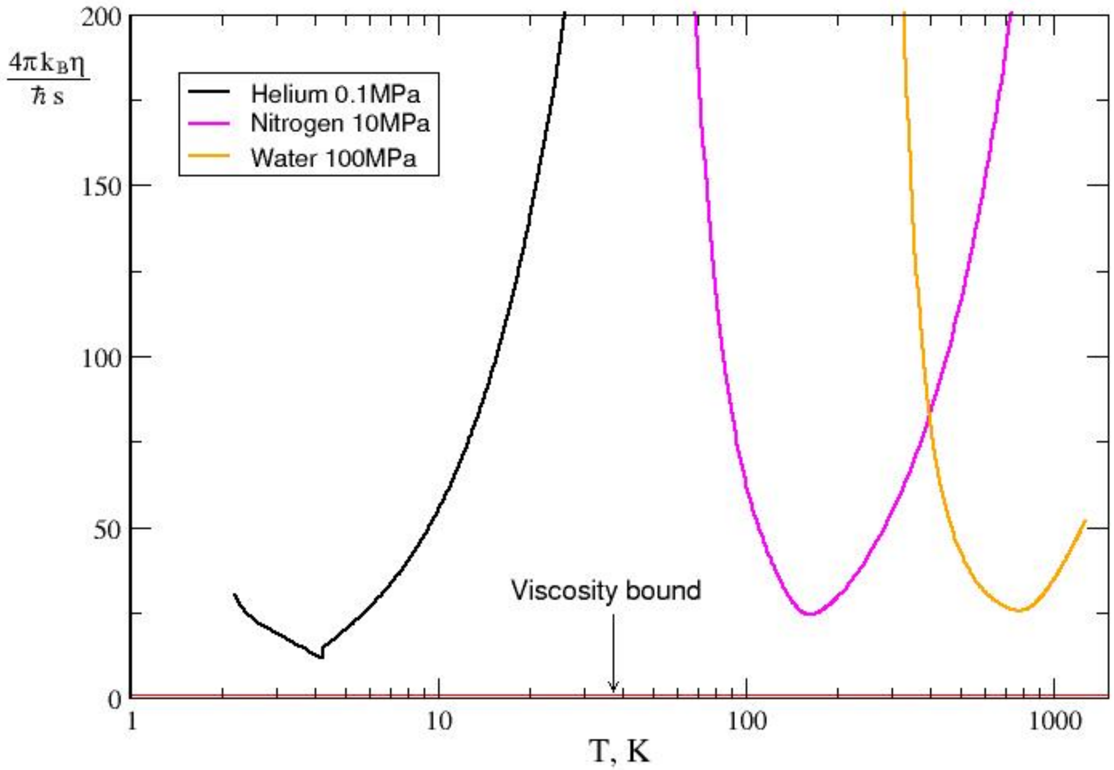
Shear viscosity "measurements" at RHIC and LHC



Luzum and Romatschke, 0804.4015 [nuc-th]

A viscosity bound conjecture

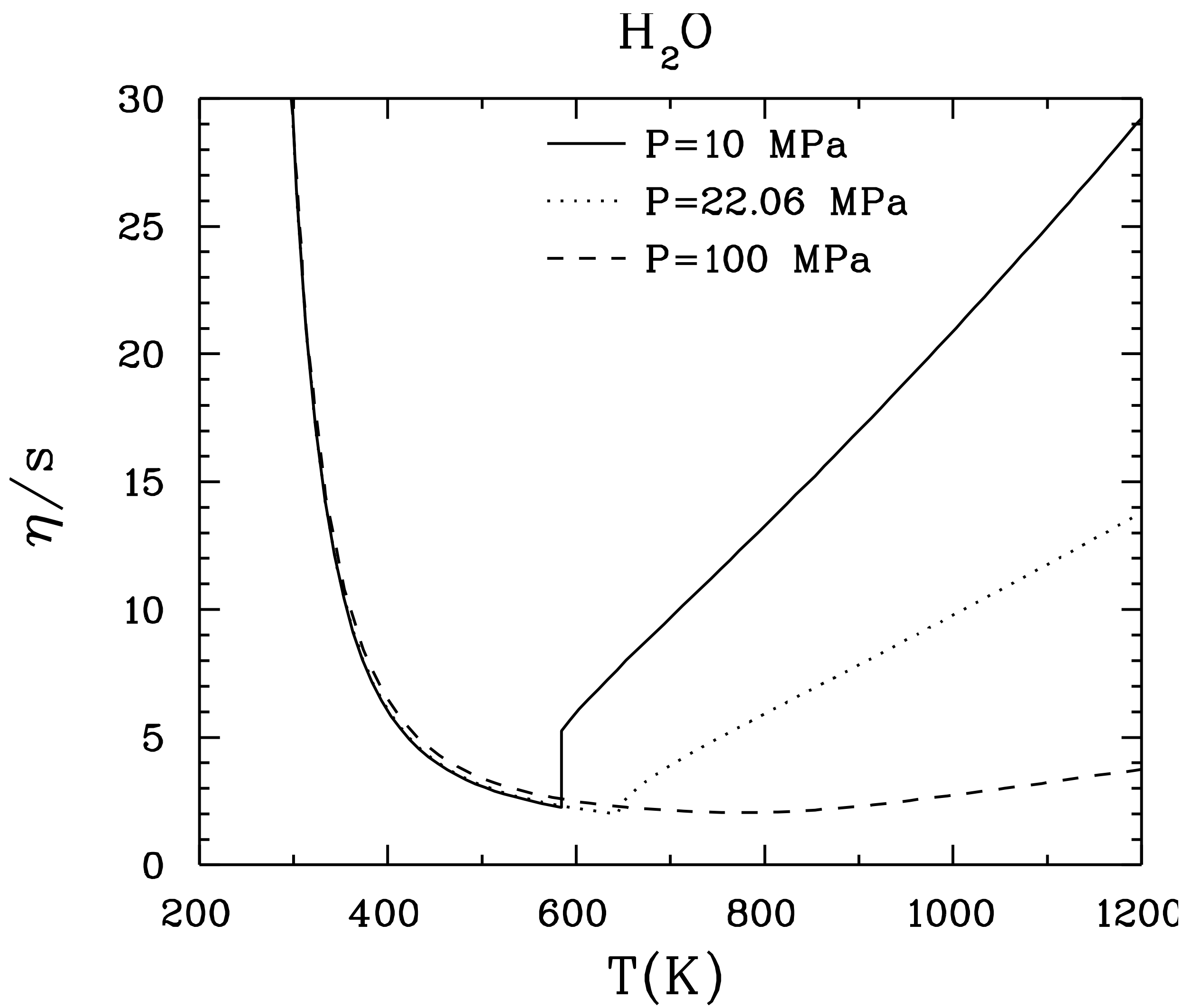
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

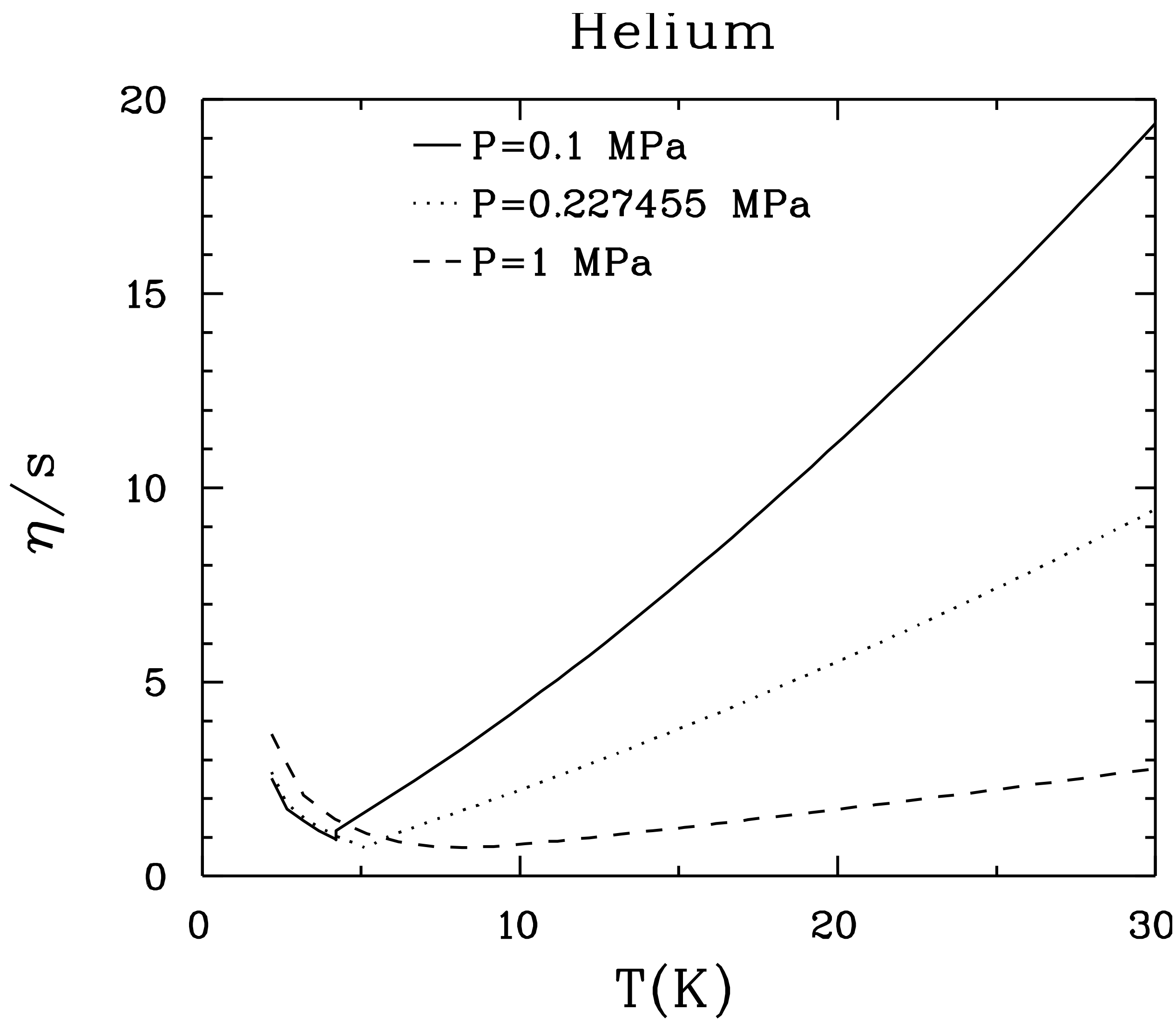
Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



$(\eta/s)_{\text{min}} \sim 25$ in units of $\frac{\hbar}{4\pi k_B}$

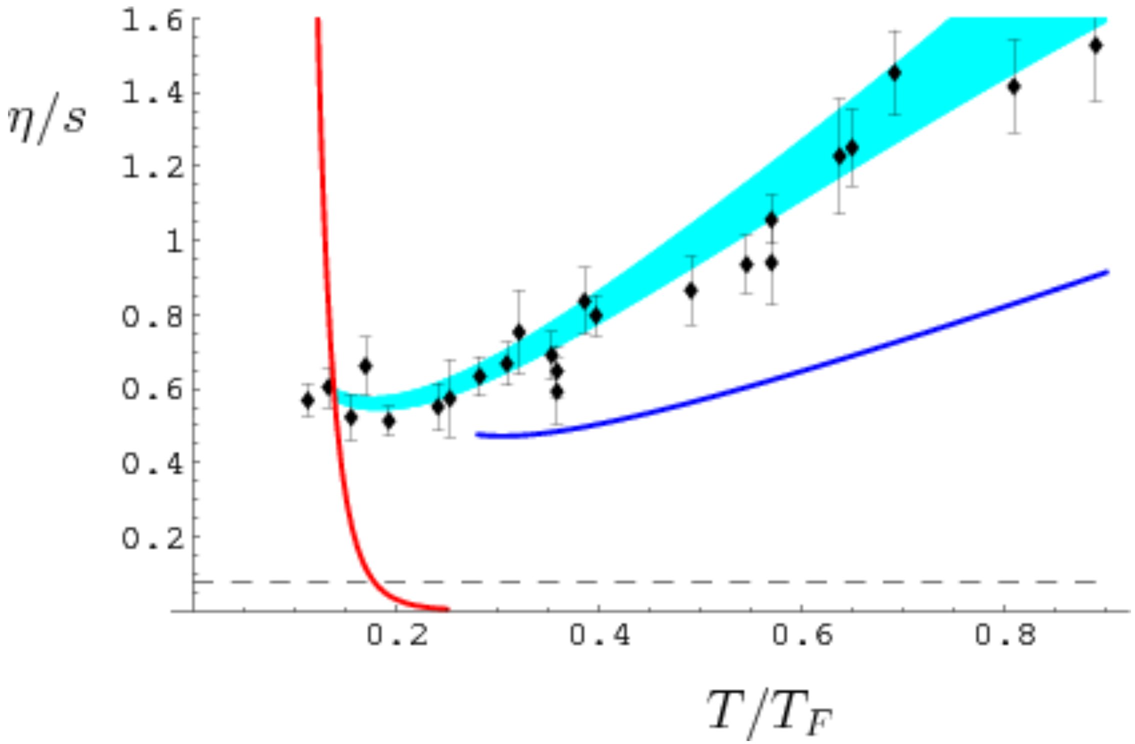
Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\text{min}} \sim 8.8$ in units of $\frac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity-entropy ratio of a cold Fermi gas

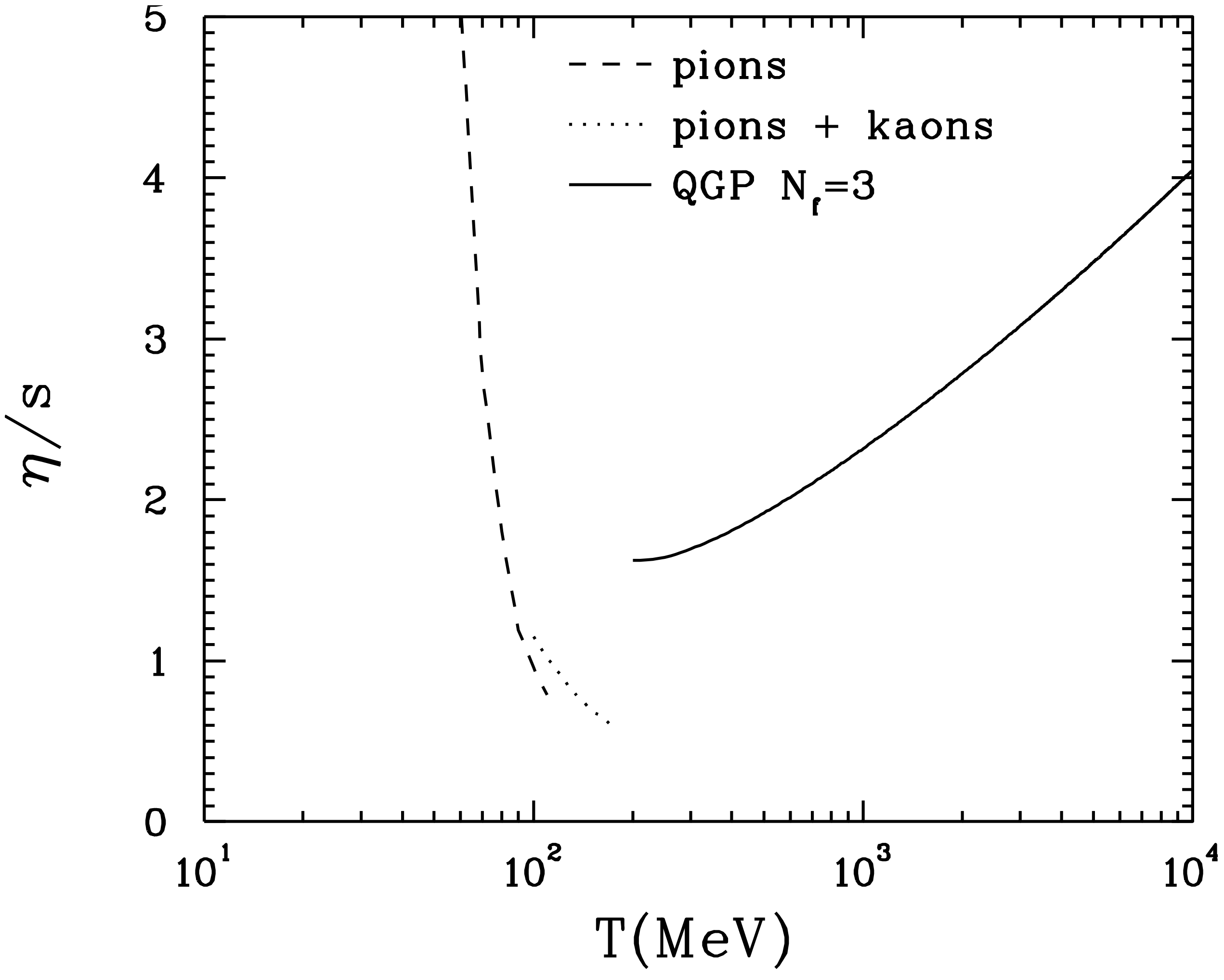


$$\eta/s \sim 5.5 \text{ in units of } \frac{\hbar}{4\pi k_B}$$

T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

QCD



Chernai, Kapusta, McLerran, nucl-th/0604032

Conclusions

String theory was (partially) successful in explaining entropy of black holes

Black holes have entropy and temperature and behave like thermodynamic systems, and we think we know why (holographic principle)

String theorists' work on black hole physics has led to the (accidental) discovery of AdS/CFT (or gauge-string) duality

The duality can be viewed as a non-trivial change of variables in path integral
We know it works (it has been tested in numerous examples) but rigorous proof is still lacking

Gauge-string duality can compute field theory quantities at strong coupling

These computations helped to explain properties of quark-gluon plasma (e.g. low shear viscosity-entropy density ratio) and are of great interest for other strongly interacting systems

The duality is a two-way street – can it be used to understand strongly coupled gravity?

THANK YOU!