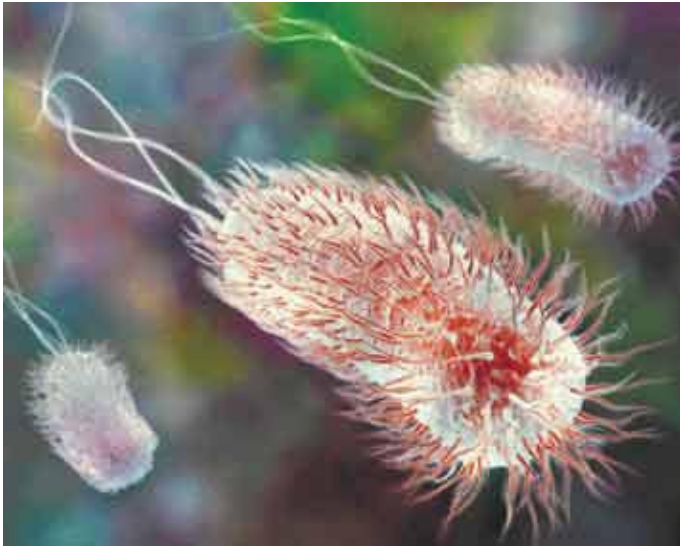


An aerial photograph of the University of Oxford, showing a dense cluster of historic stone buildings with red-tiled roofs. The central focus is the Christ Church spire, a tall, slender Gothic tower with a large, rounded dome. To the right, the long, narrow nave of Christ Church Cathedral is visible, characterized by its series of pointed Gothic arches. The surrounding area is filled with smaller, multi-story buildings, some with green roofs, and several green lawns are interspersed among the structures. The overall scene is a rich tapestry of medieval and early modern architecture.

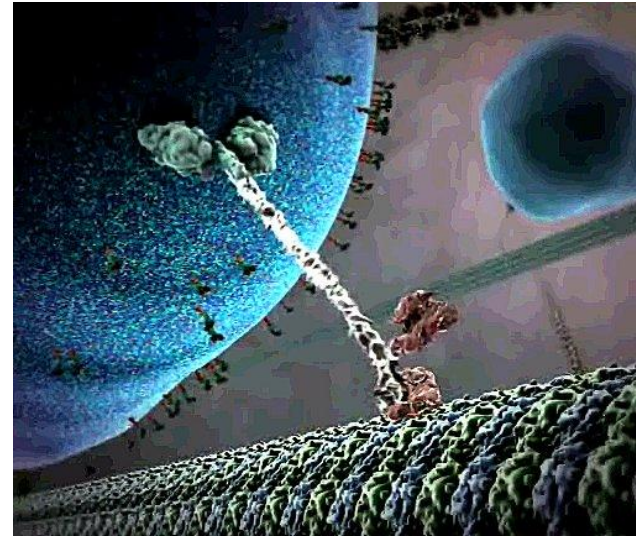
Motility in Living Matter

Julia Yeomans
University of Oxford

Active matter

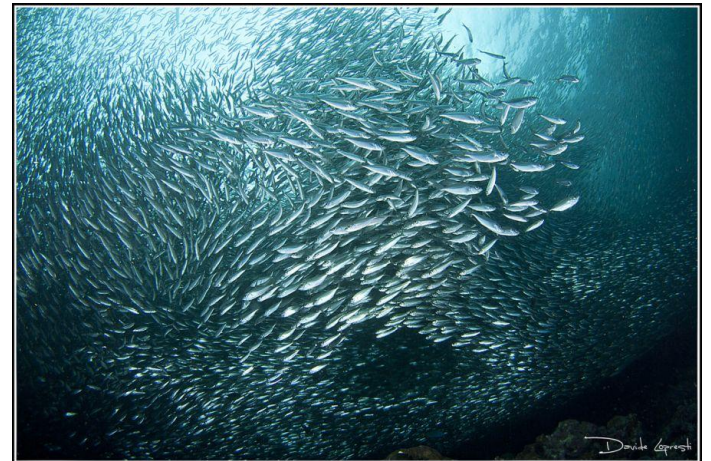


bacteria, self-propelled colloids



molecular motors, cells

active systems operate out of
thermodynamic equilibrium



birds, fish

Active matter

Why is active matter interesting?

1. Statistical physics beyond Boltzmann
2. Engineering applications
3. Medical applications



L. Angelani, R. Di Leonardo, Ruocco G.,
[*Phys. Rev. Lett.*, **102**, 048104, \(2009\)](#)

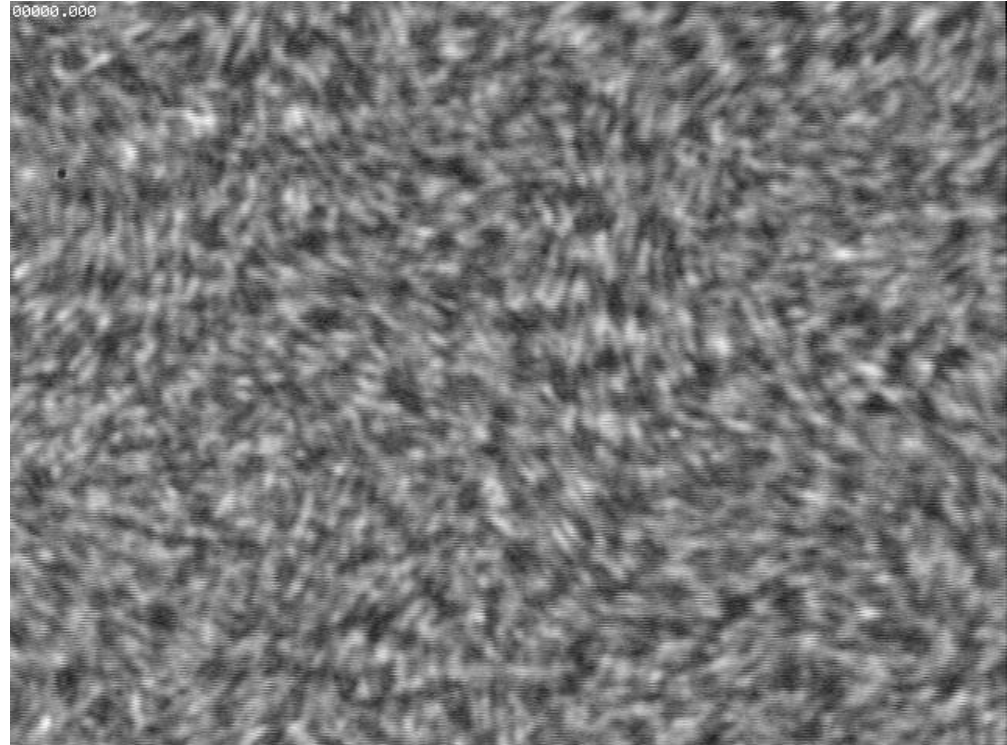
Active matter

Why is active matter interesting?

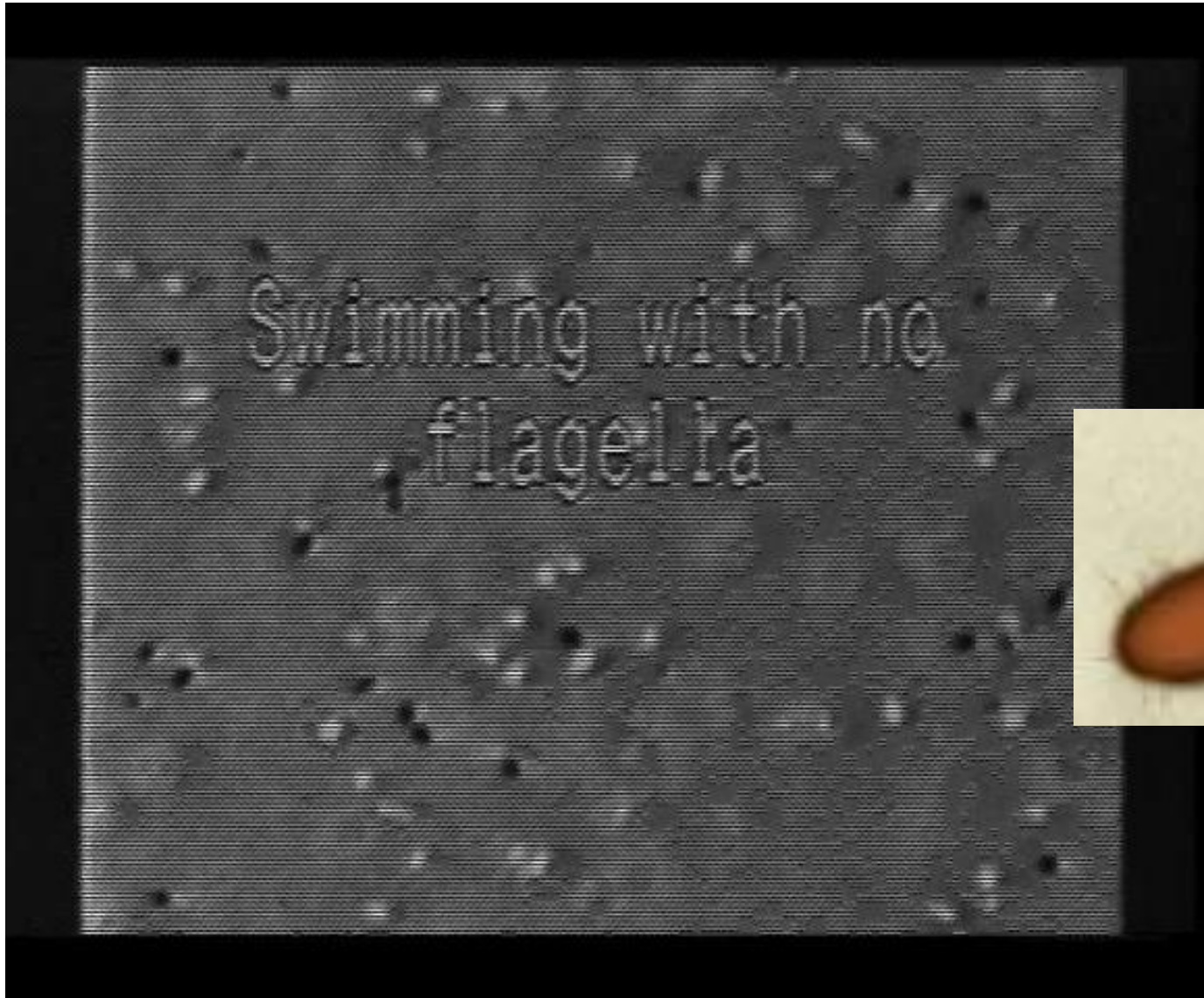
1. Statistical physics beyond Boltzmann
2. Engineering applications
3. Medical applications

Why is it interesting now?

1. Better microscopy
2. Nanotechnology
3. Faster computers



Swimmers



$$\rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

inertial terms

viscous terms

$$\text{Re} = \frac{\text{inertial response}}{\text{viscous response}} \sim \frac{\rho L_0 V_0}{\mu}$$

$$\rho \left\{ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right\} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

inertial terms

viscous terms

$$\text{Re} = \frac{\text{inertial response}}{\text{viscous response}} \sim \frac{\rho L_0 V_0}{\mu}$$

10^{-6}
 10^{-6}
 10^6

Stokes equations

$$\nabla p = \mu \nabla^2 \mathbf{v} + \mathbf{f}$$

$$\nabla \cdot \mathbf{v} = 0$$



Purcell's Scallop Theorem

$$\nabla p = \mu \nabla^2 \mathbf{v} + \mathbf{f} \qquad \nabla \cdot \mathbf{v} = 0$$

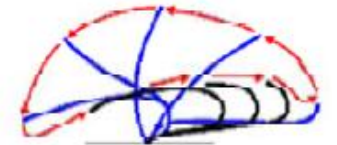
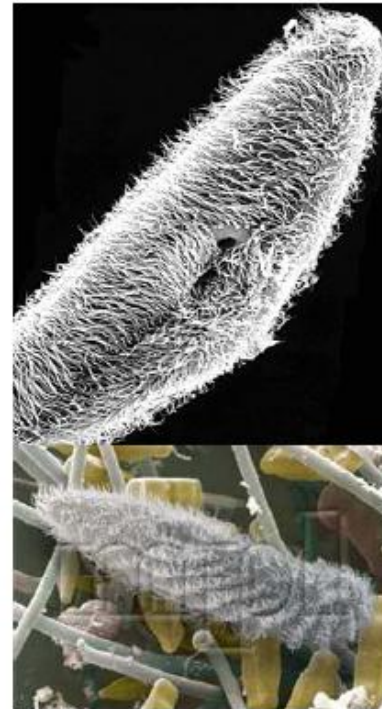
No time dependence implies the Scallop Theorem

A swimming stroke must not be invariant under time reversal



Purcell's Scallop Theorem

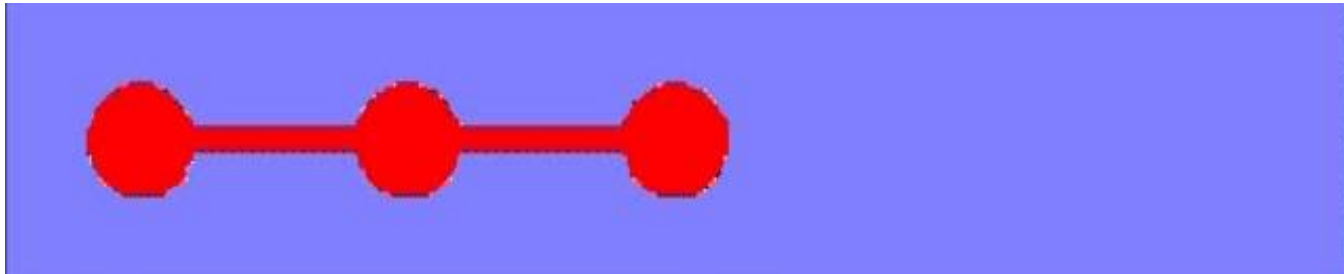
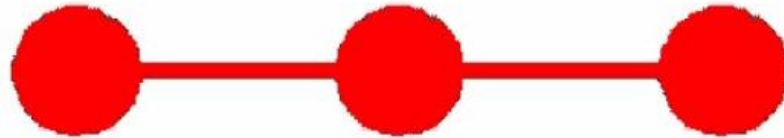
A swimmer strokes must be non-invariant under time reversal



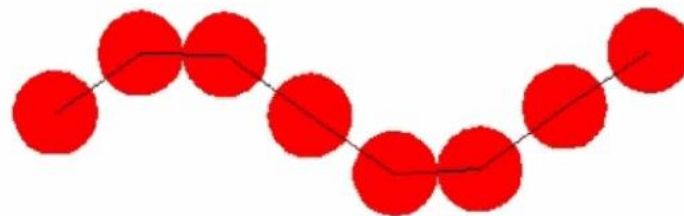
Model swimmers

three sphere swimmer

Najafi and Golestanian,



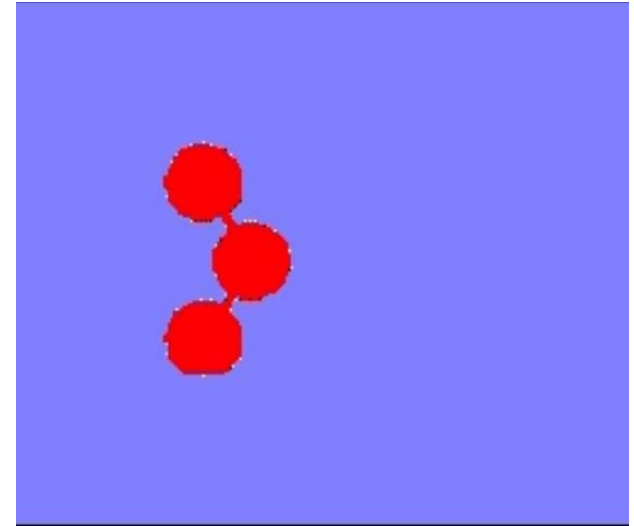
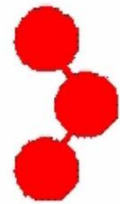
Snake swimmer



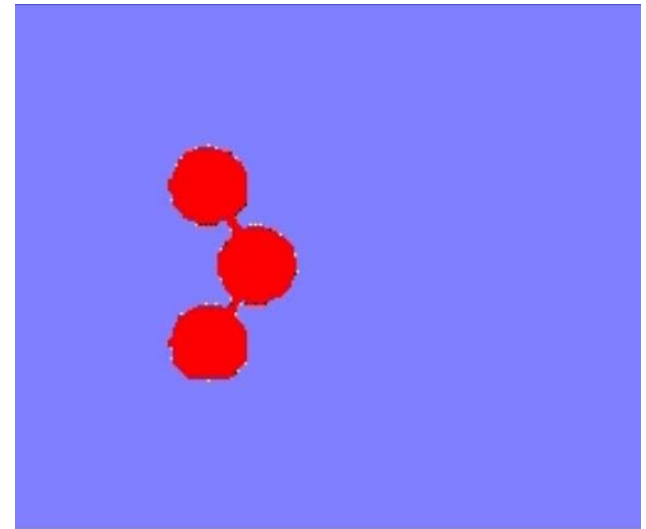
a generalised three sphere swimmer

Radial and tangential
motion

forward motion



turning motion



Green function of the Stokes equation

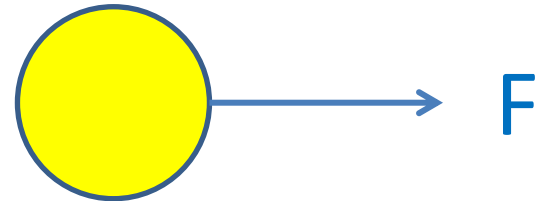
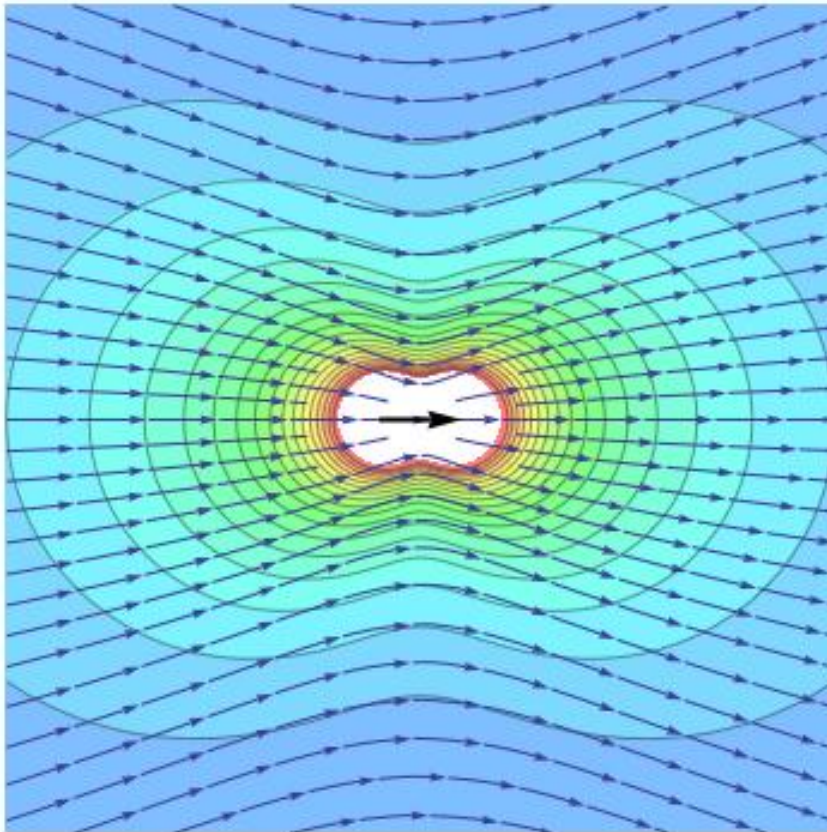
$$\nabla p = \mu \nabla^2 \mathbf{v} + \mathbf{f} \delta(\mathbf{r}) \qquad \nabla \cdot \mathbf{v} = 0$$

$$\mathbf{v} = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r} + \frac{\mathbf{r}\mathbf{r}}{r^3} \right) \qquad \text{Stokeslet}$$

$$p = p_0 + \frac{\mathbf{f} \cdot \mathbf{r}}{4\pi r^3}$$

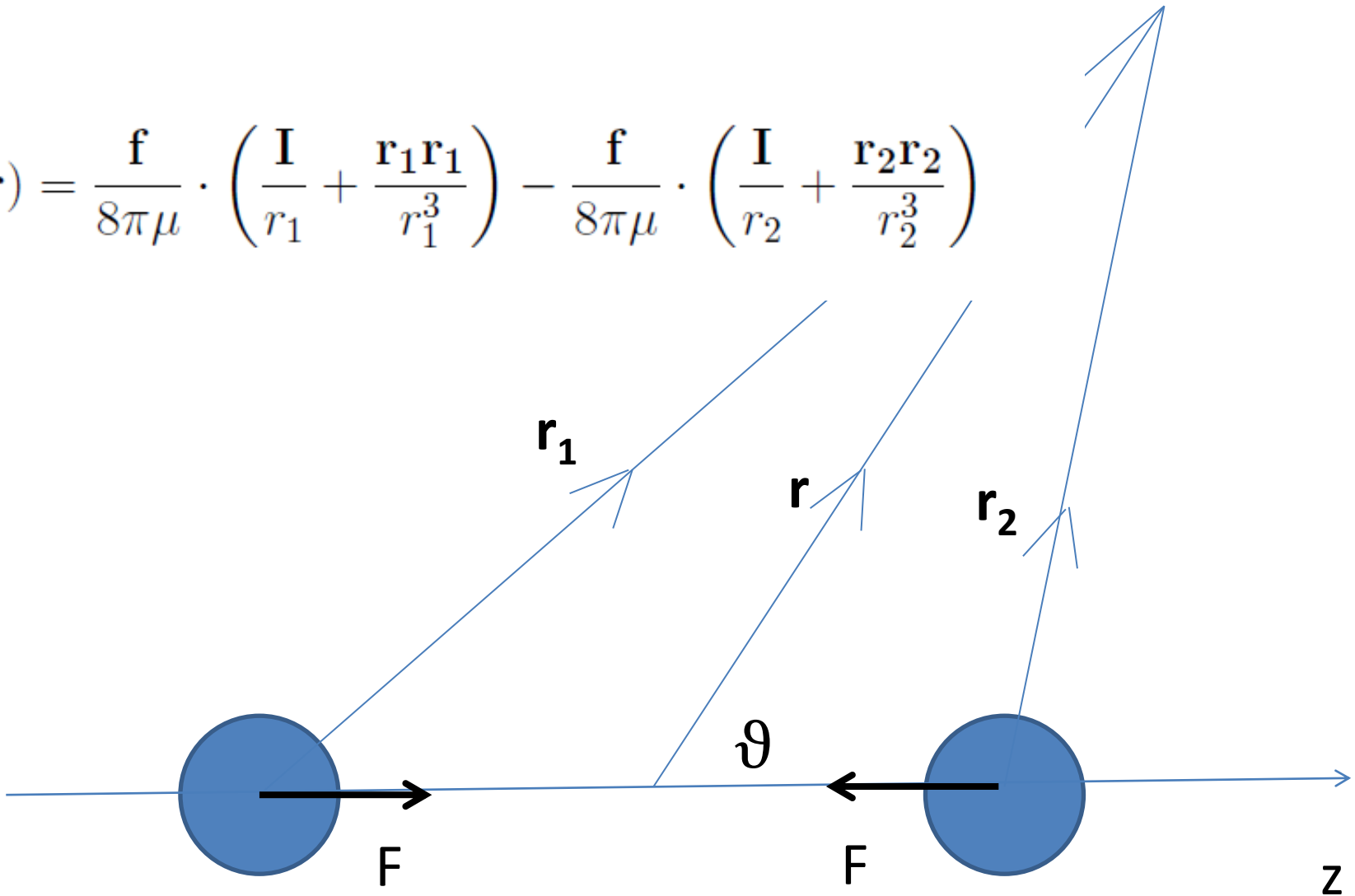
Green function of the Stokes equation

$$\mathbf{v} = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r} + \frac{\mathbf{r}\mathbf{r}}{r^3} \right)$$



Dipolar far flow field

$$\mathbf{v}(\mathbf{r}) = \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r_1} + \frac{\mathbf{r}_1\mathbf{r}_1}{r_1^3} \right) - \frac{\mathbf{f}}{8\pi\mu} \cdot \left(\frac{\mathbf{I}}{r_2} + \frac{\mathbf{r}_2\mathbf{r}_2}{r_2^3} \right)$$

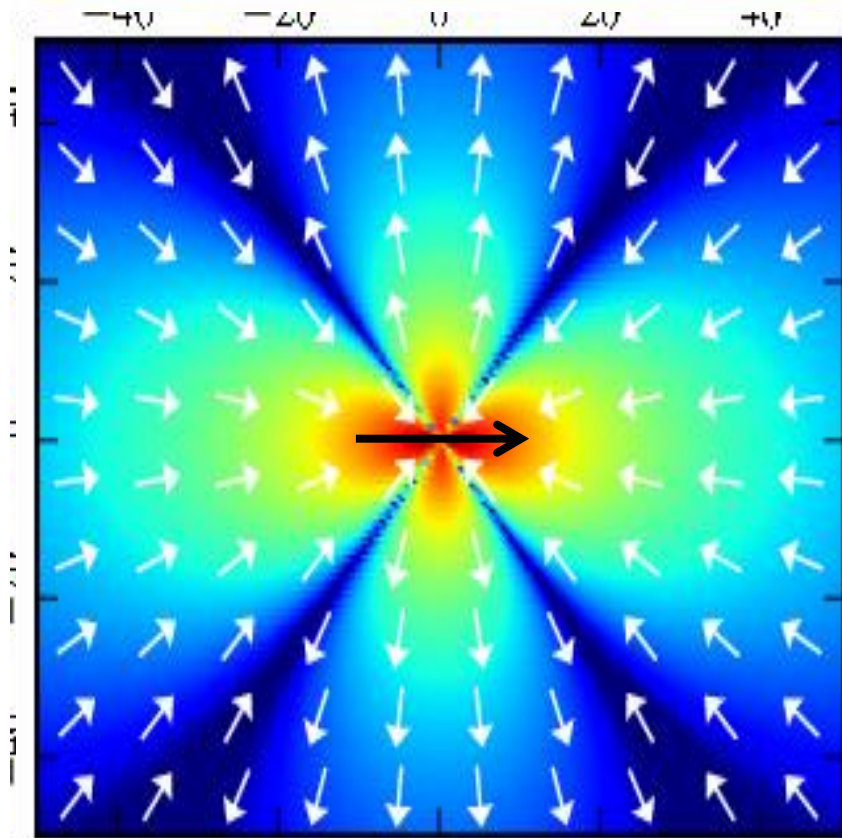


Far flow field of a swimmer

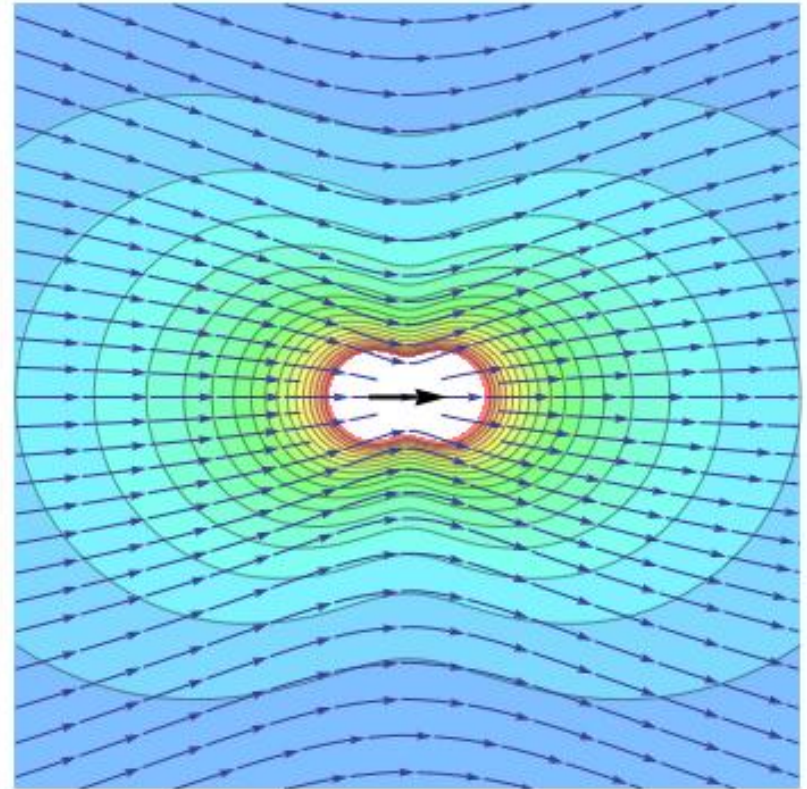
$$v_r = \frac{f}{4\pi\mu} \frac{L}{r^2} (3 \cos^2 \theta - 1)$$

Swimmers have dipolar far flow fields because they have no net force acting on them

Swimmer and colloidal flow fields

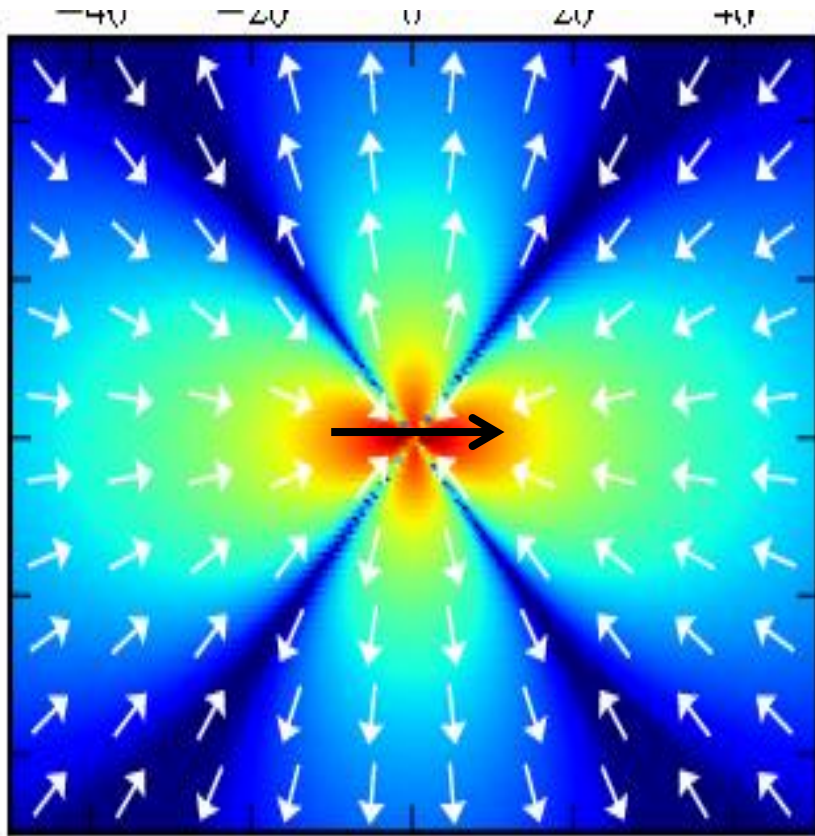


$$v \sim \frac{1}{r^2}$$

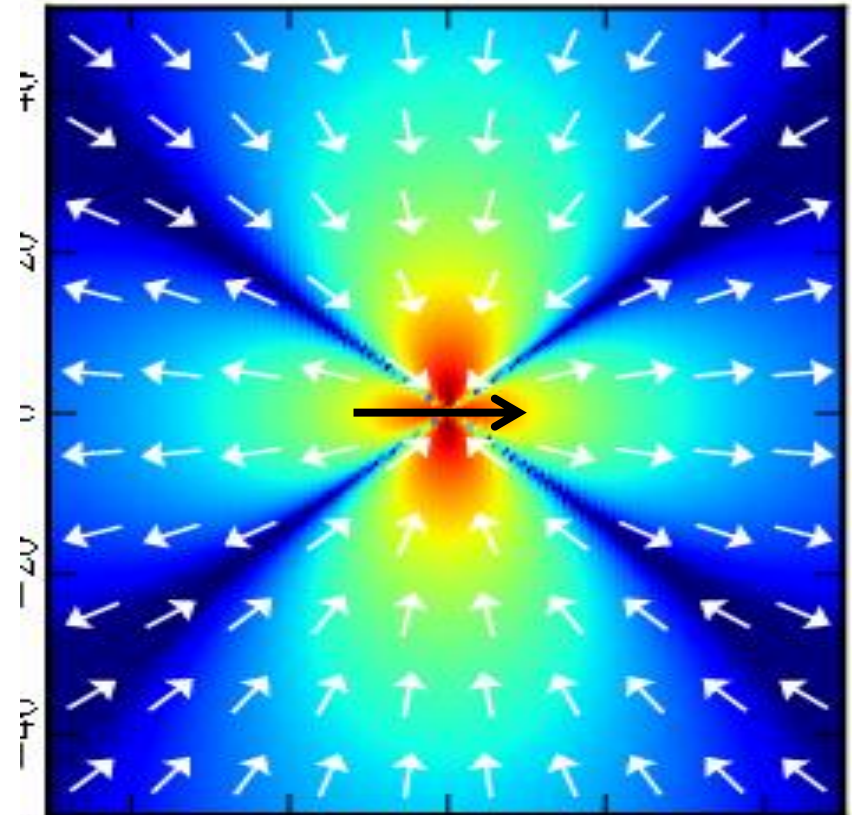


$$v \sim \frac{1}{r}$$

Dipolar flow field

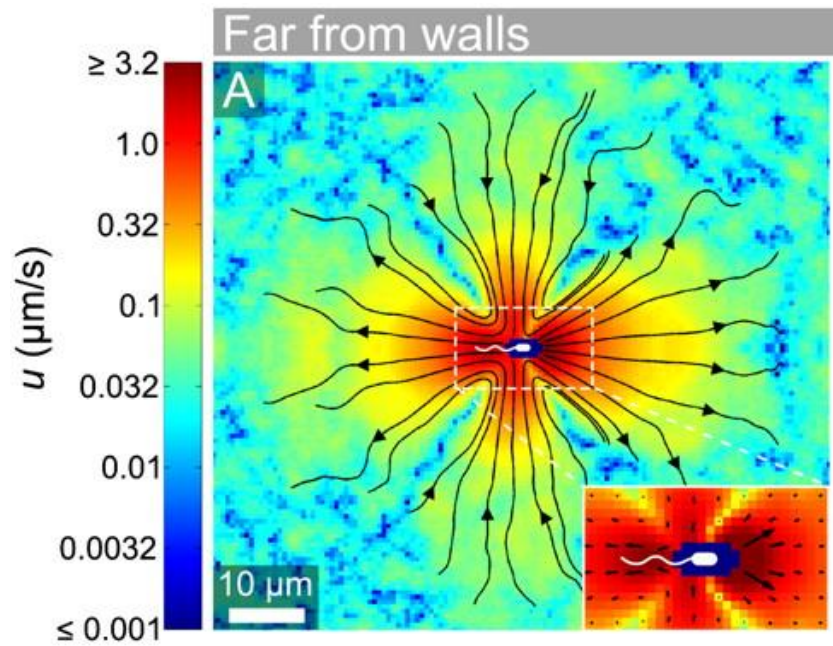


puller (contractile)

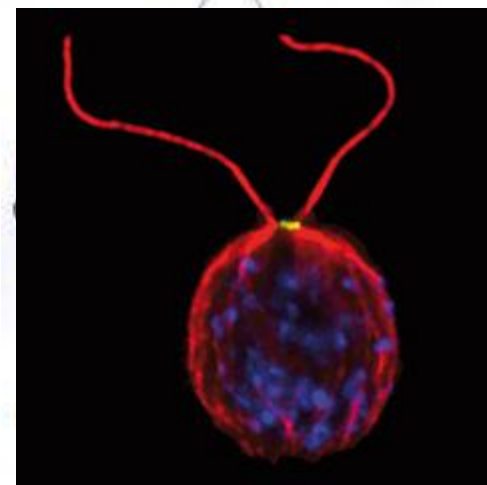
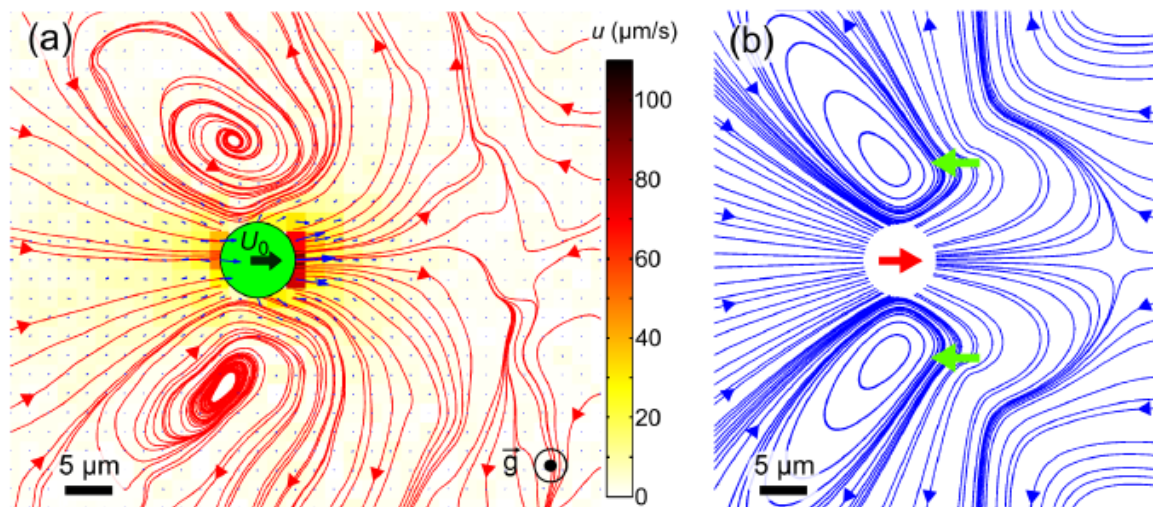


pusher (extensile)

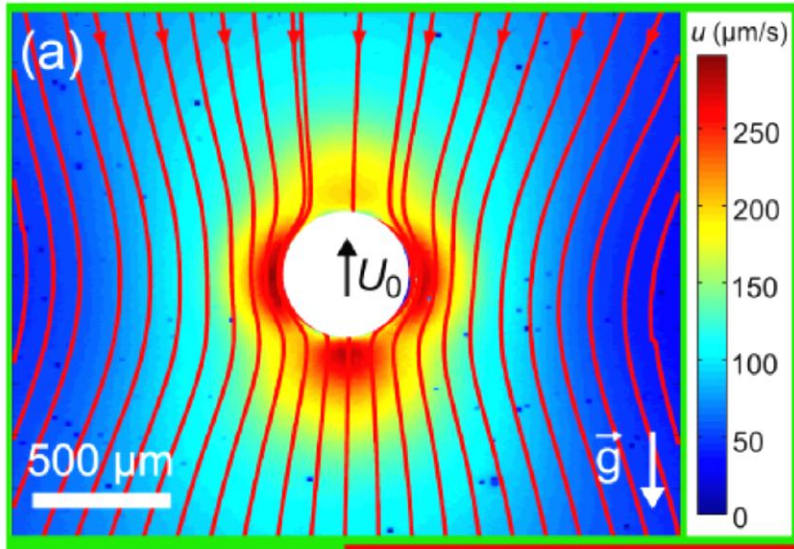
$$v_r = \frac{f}{4\pi\mu} \frac{L}{r^2} (3 \cos^2 \theta - 1)$$



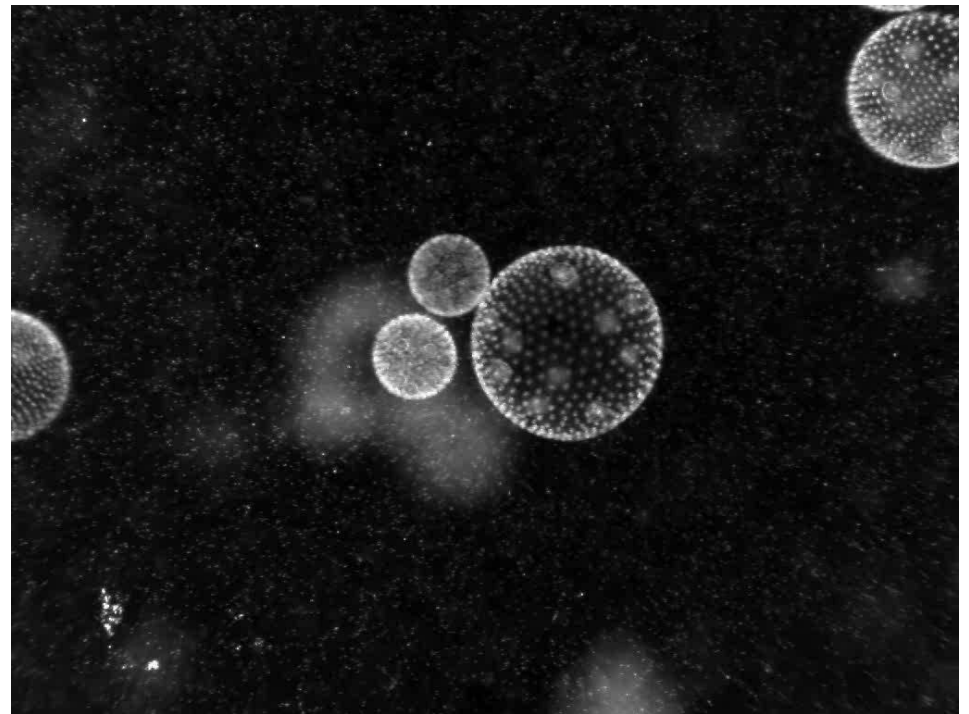
E-coli



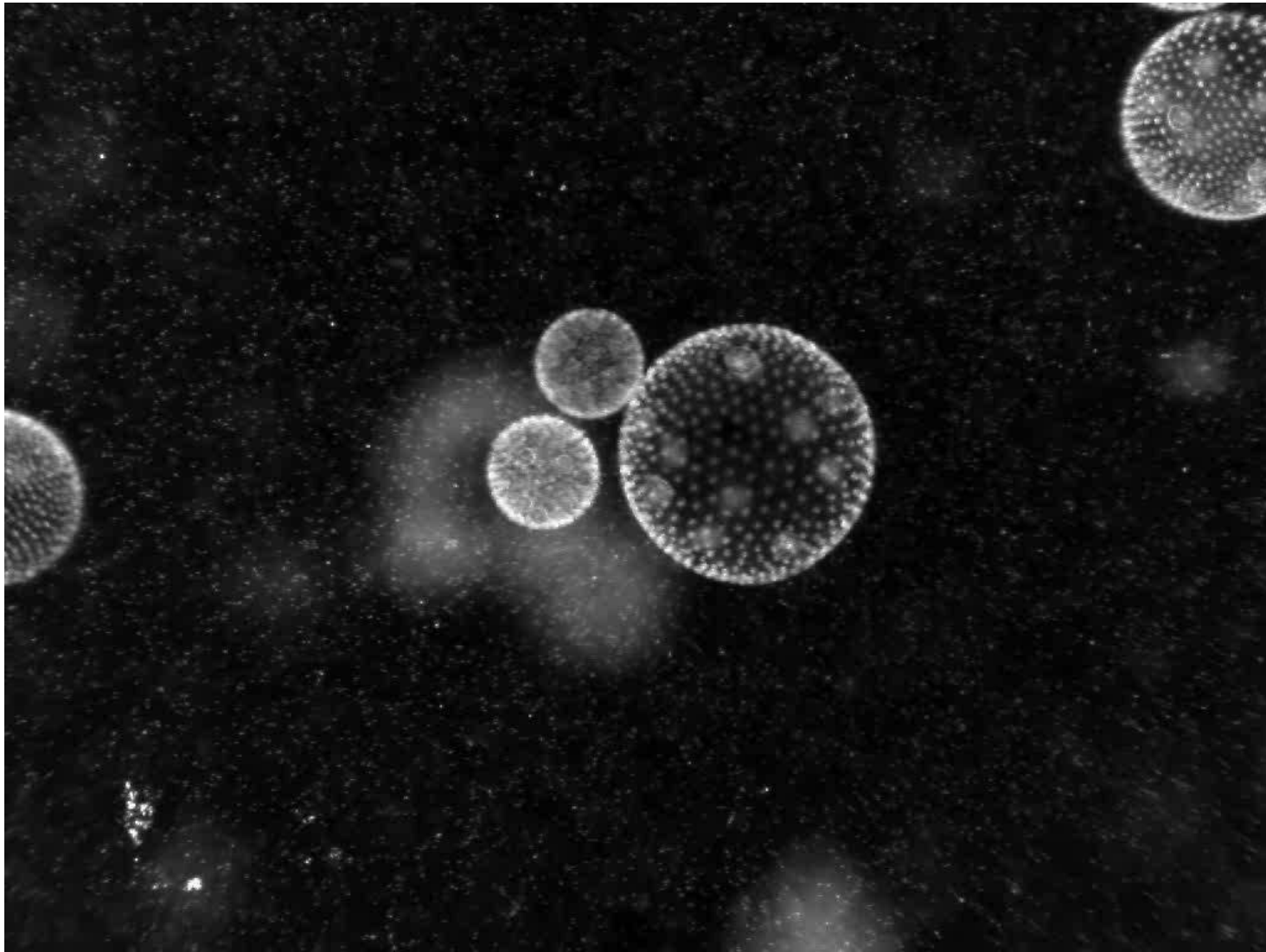
Chlamydomonas



Volvox



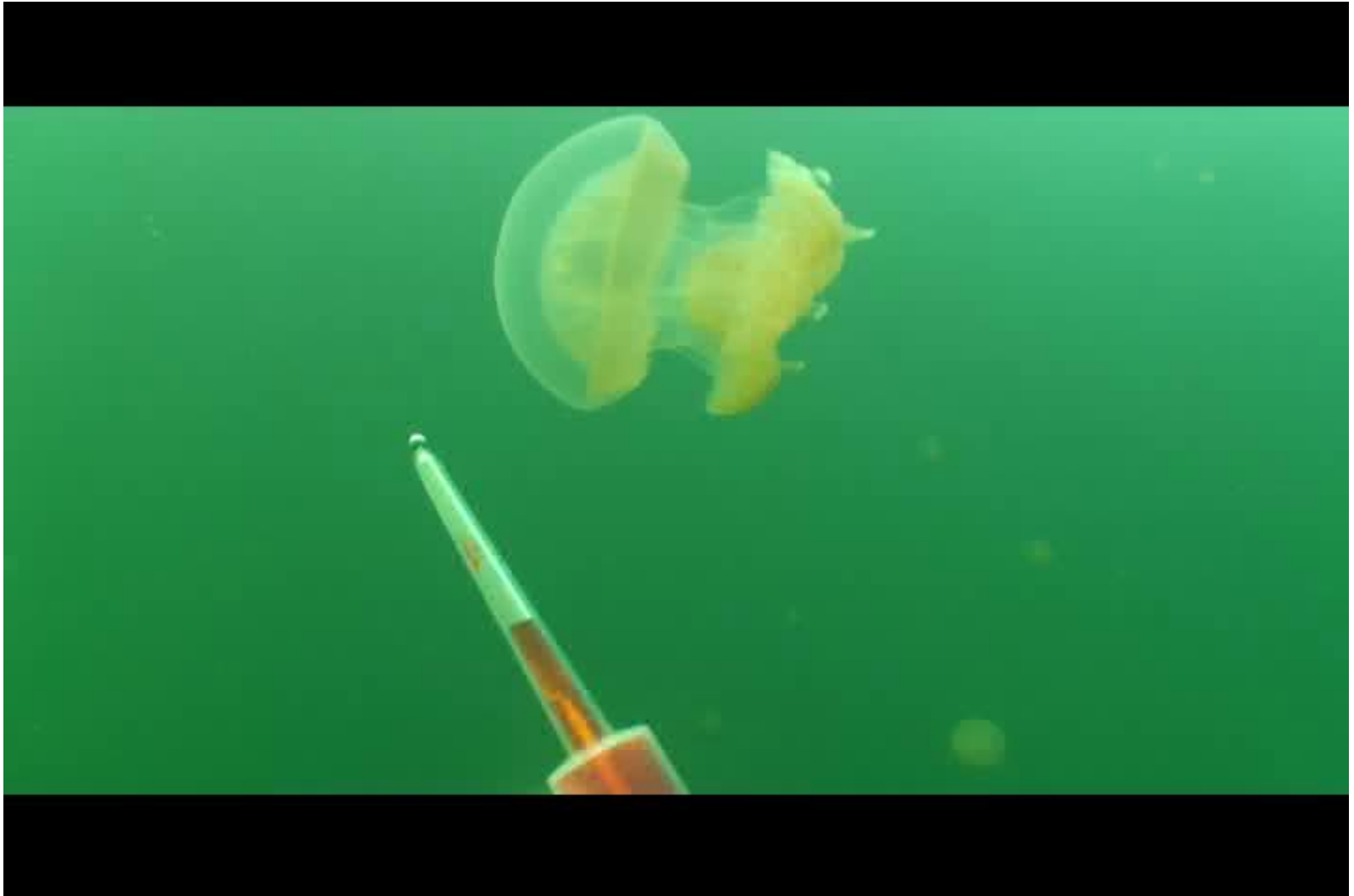
Dresher et al, PRL 105 (2010)
PNAS 108 (2011)

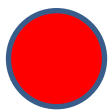
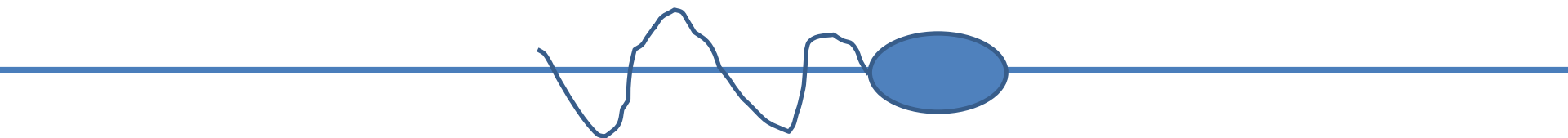


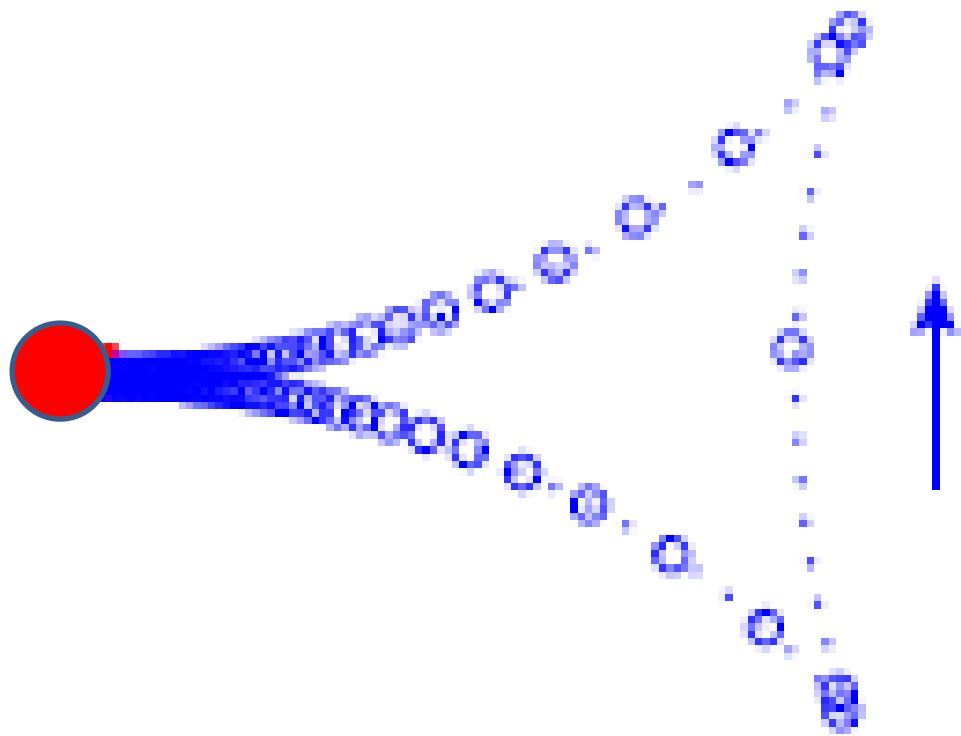
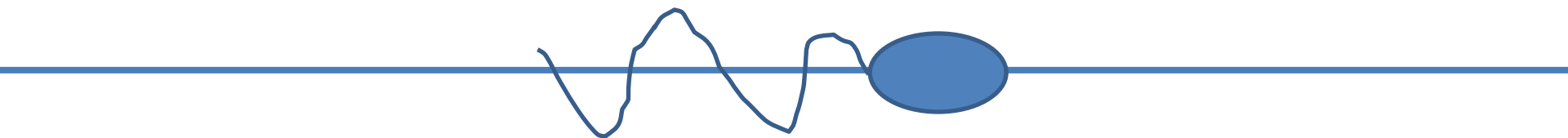
Volvox

Goldstein group, Cambridge

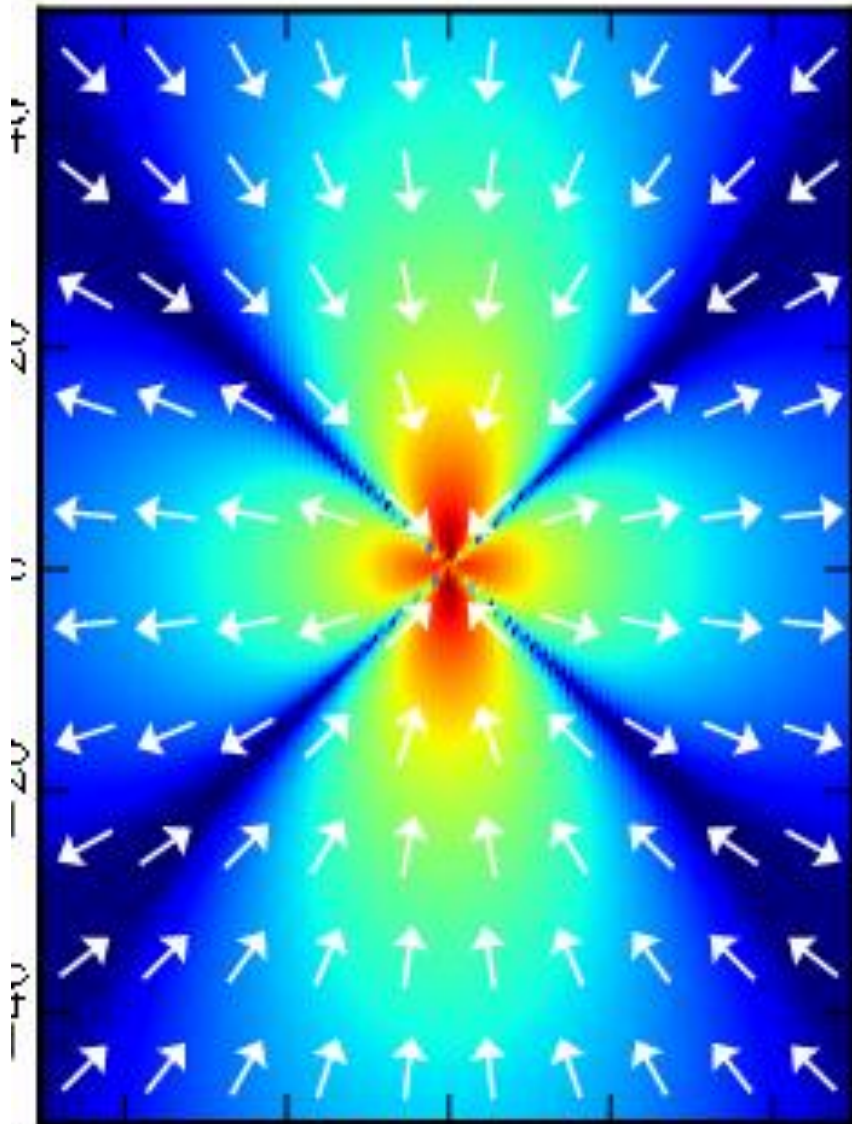
Do small swimmers mix the ocean? K. Katija, J.O. Dabiri, G. Subramanian,
A.M. Leshansky, L.M. Pismen, A.W. Visser





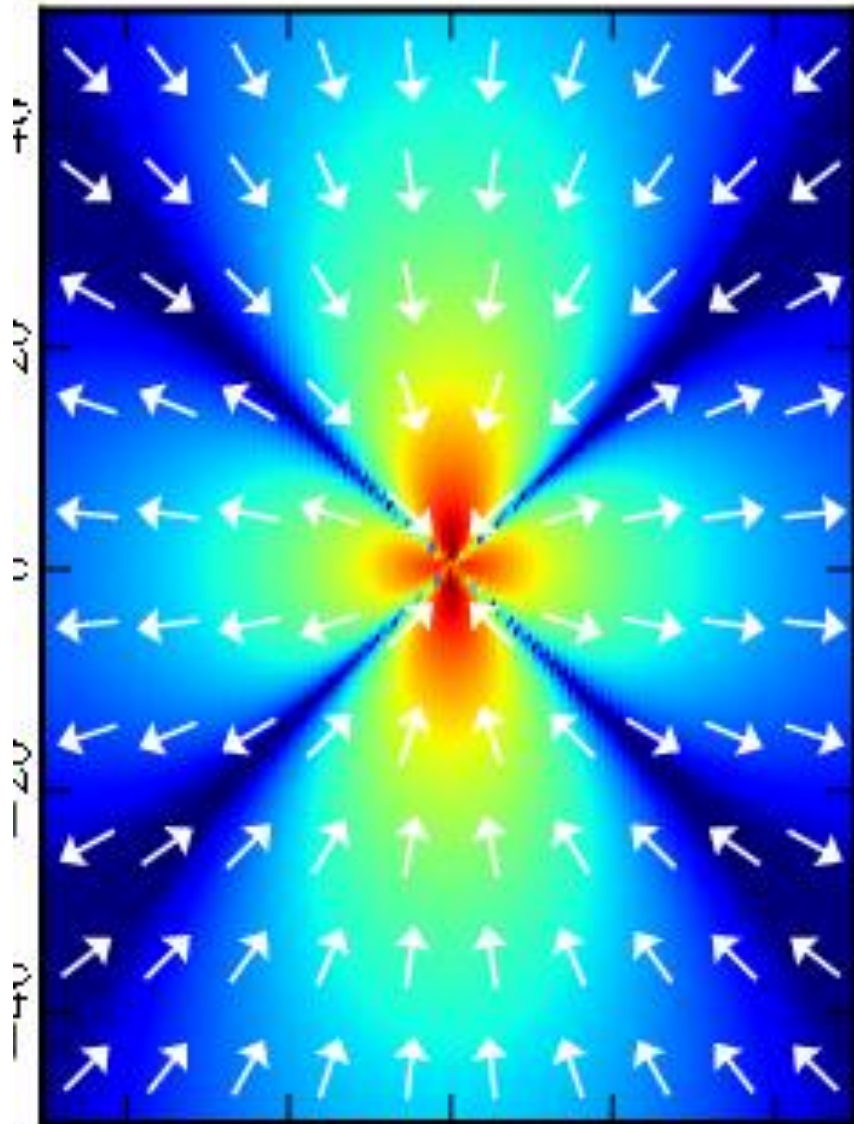


Multipole flow fields

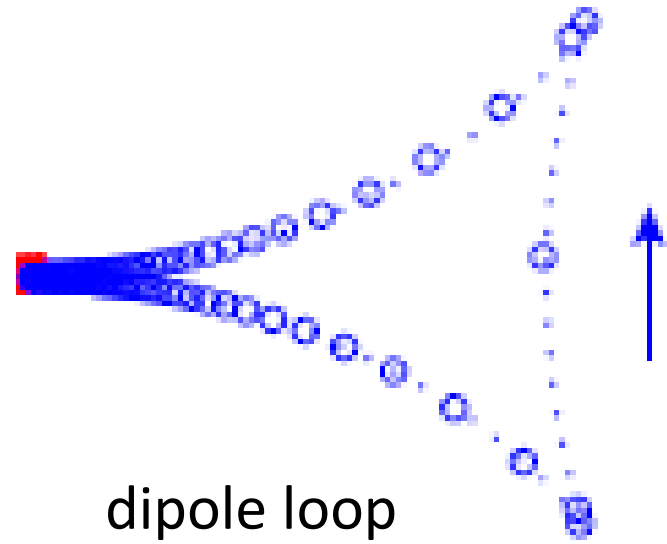


Dipole flow field

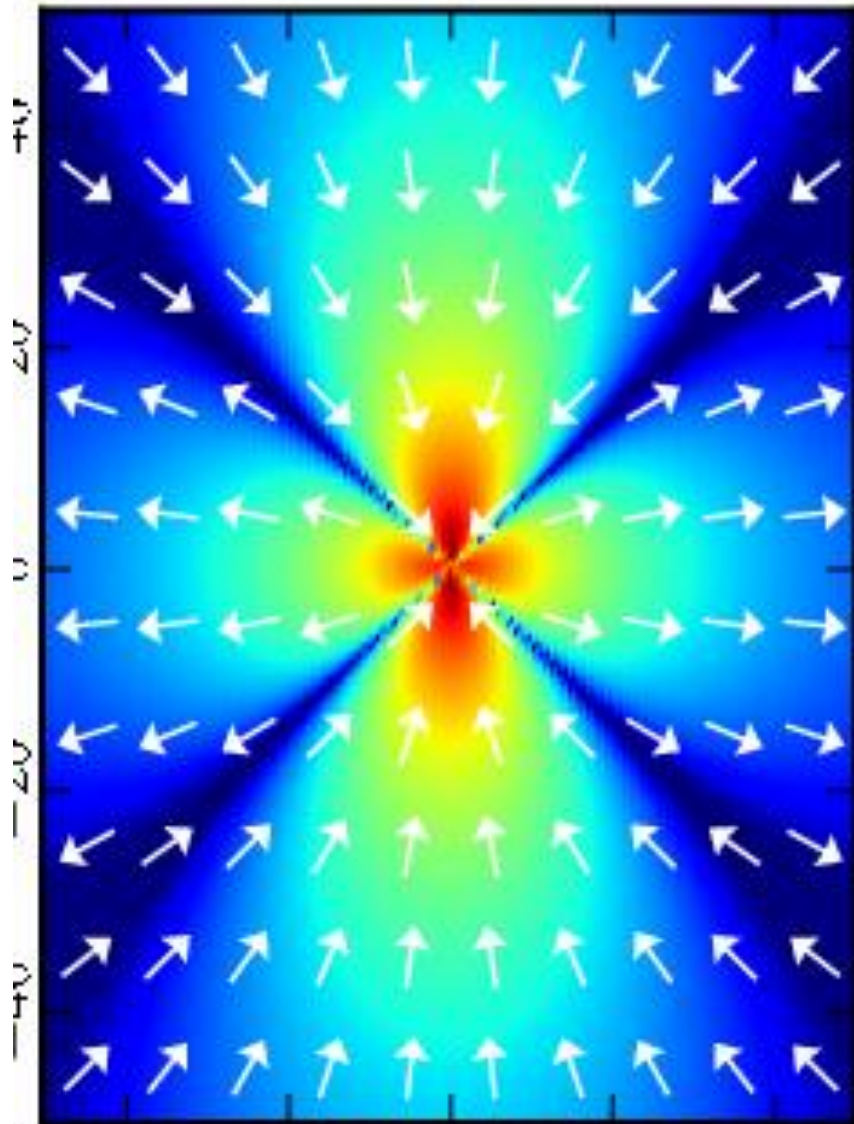
Multipole flow fields



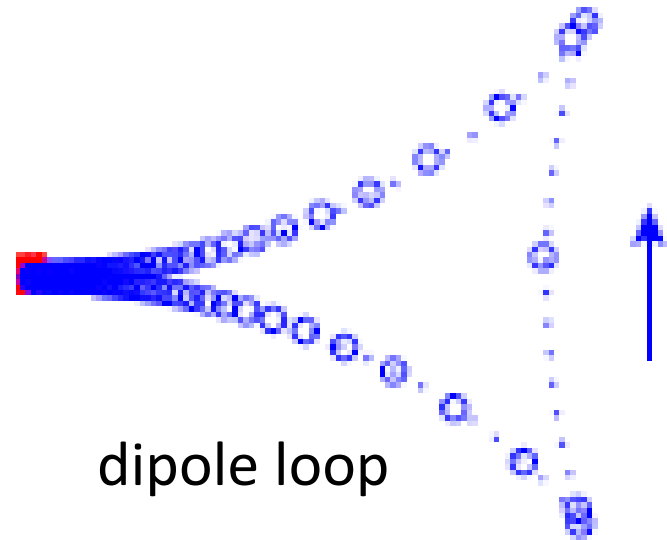
Dipole flow field



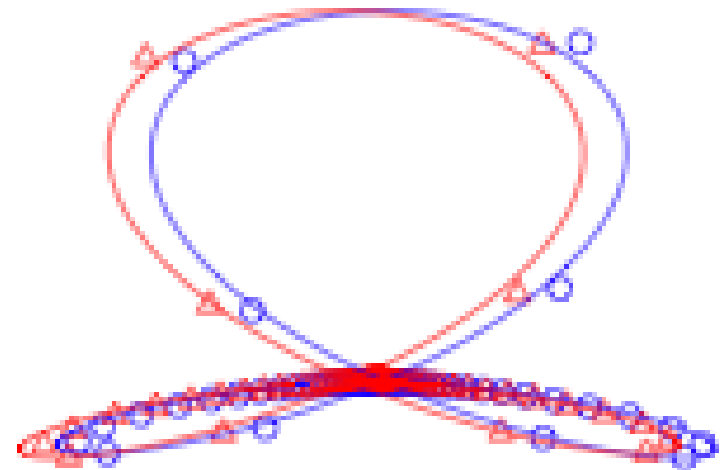
Multipole flow fields



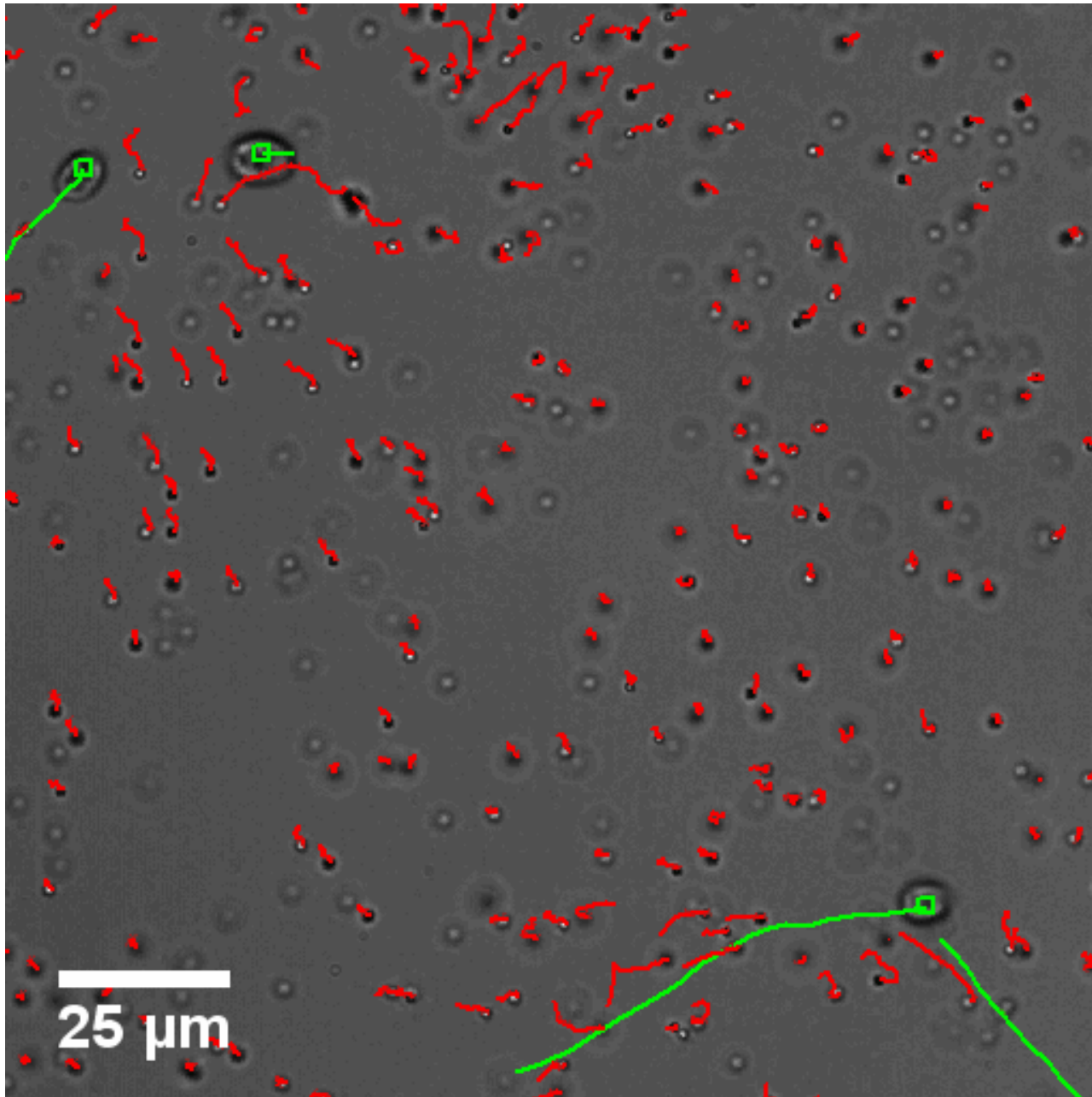
Dipole flow field



dipole loop



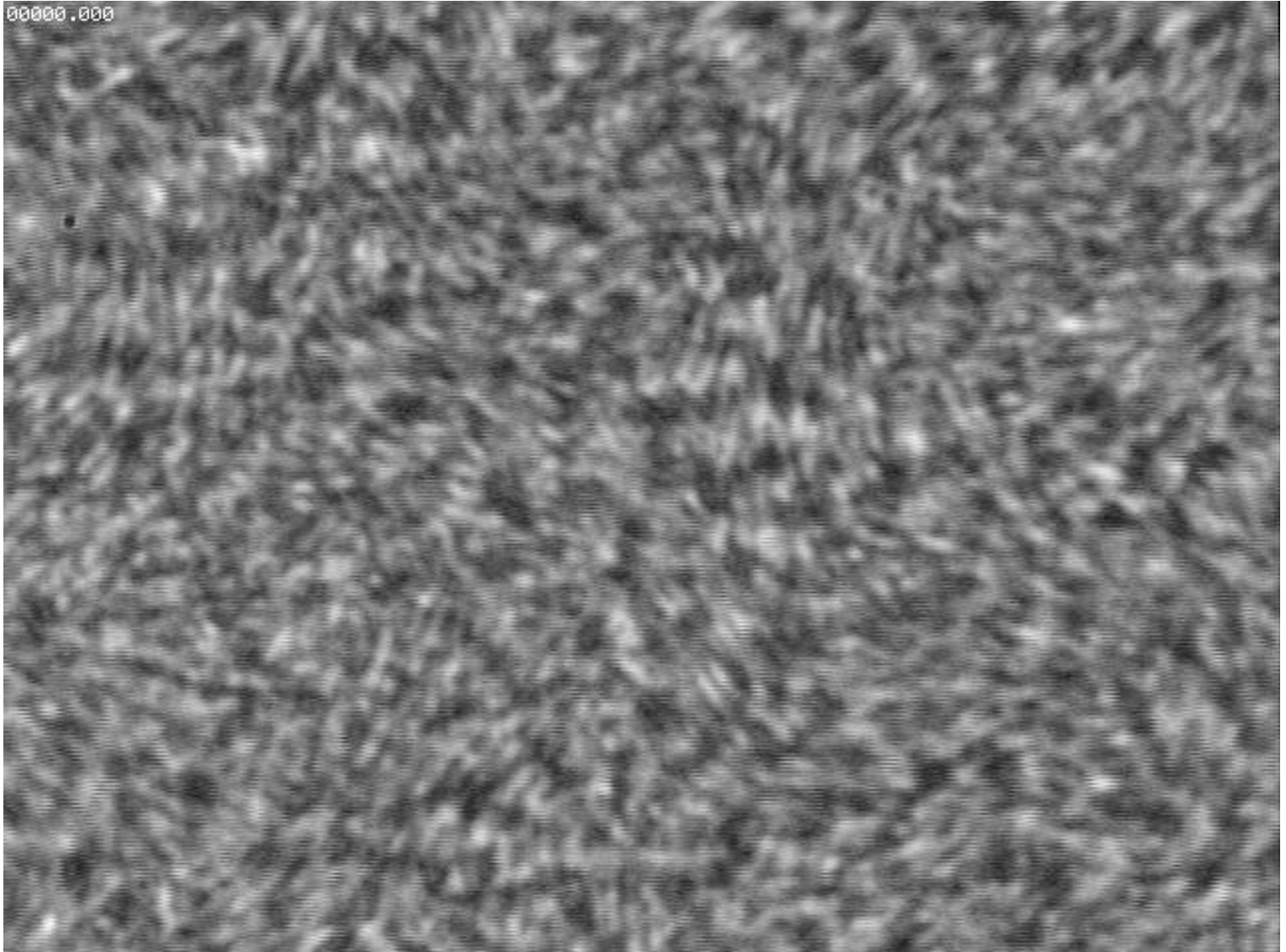
quadrupole loop



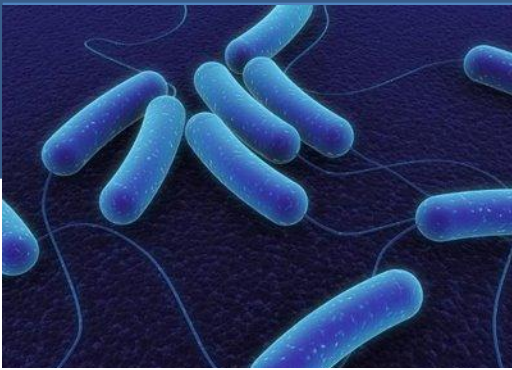
Guasto
website

Bacteria

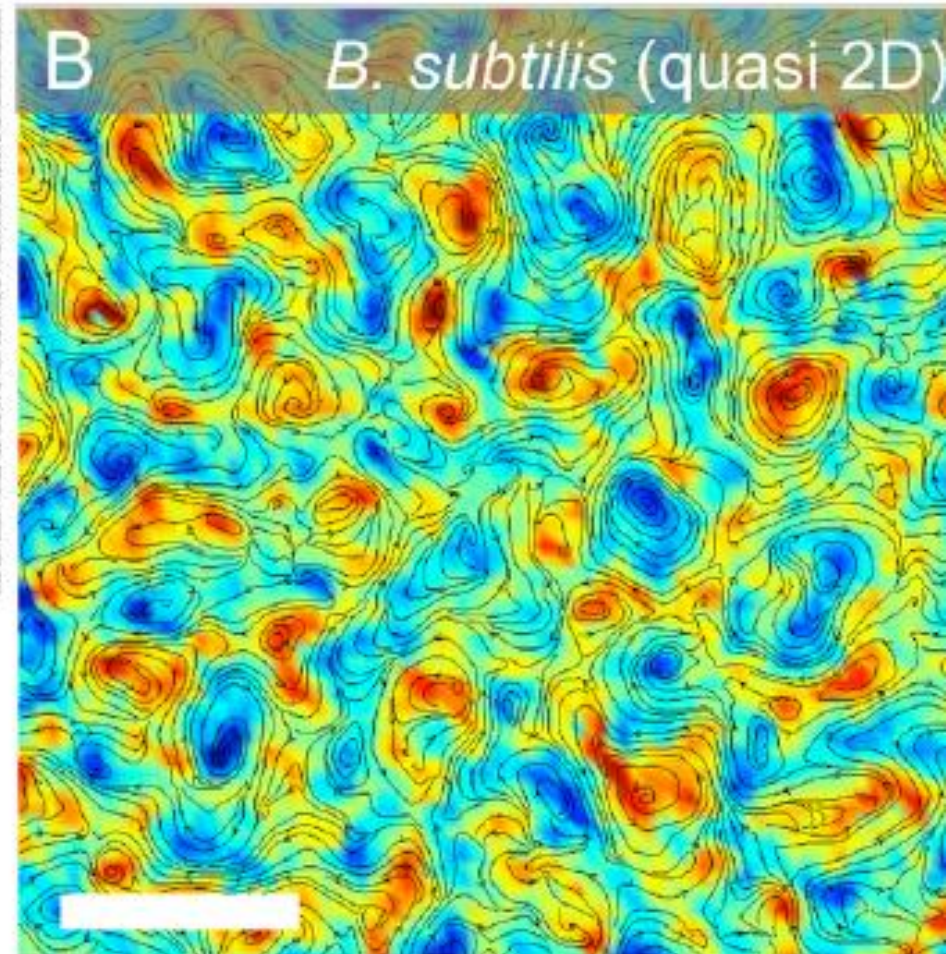
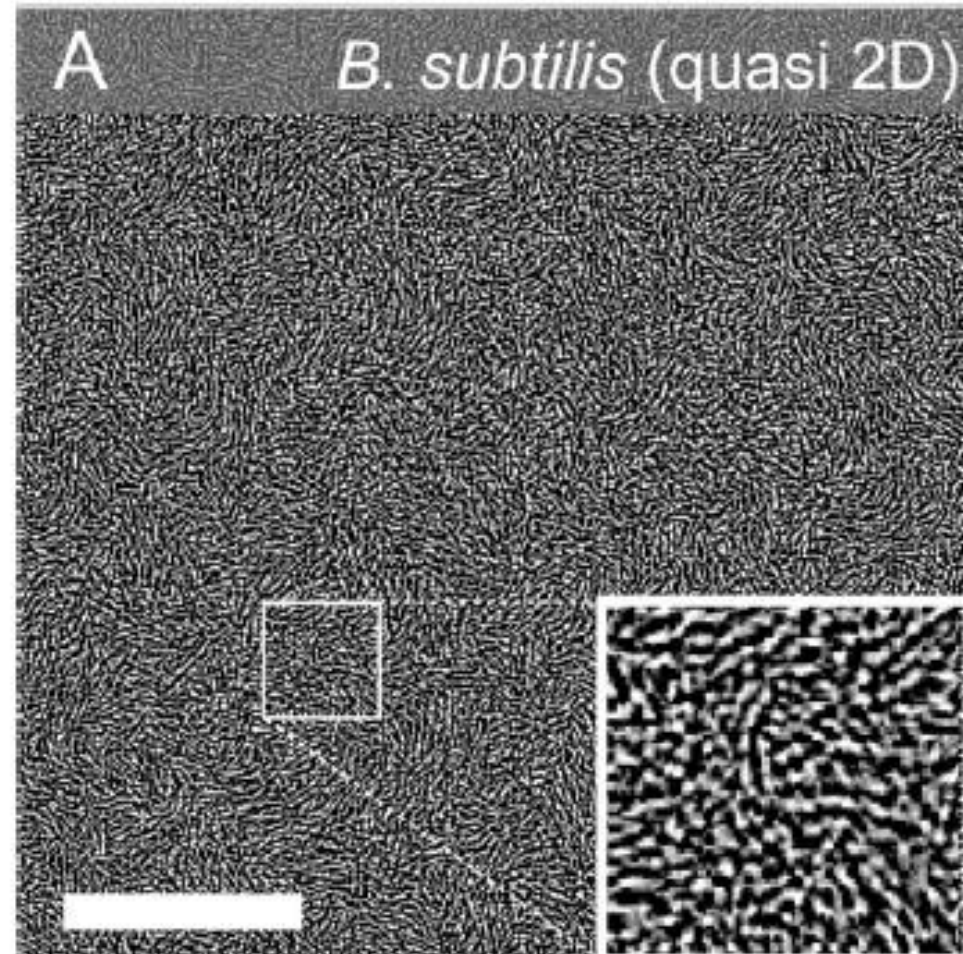
00000.000



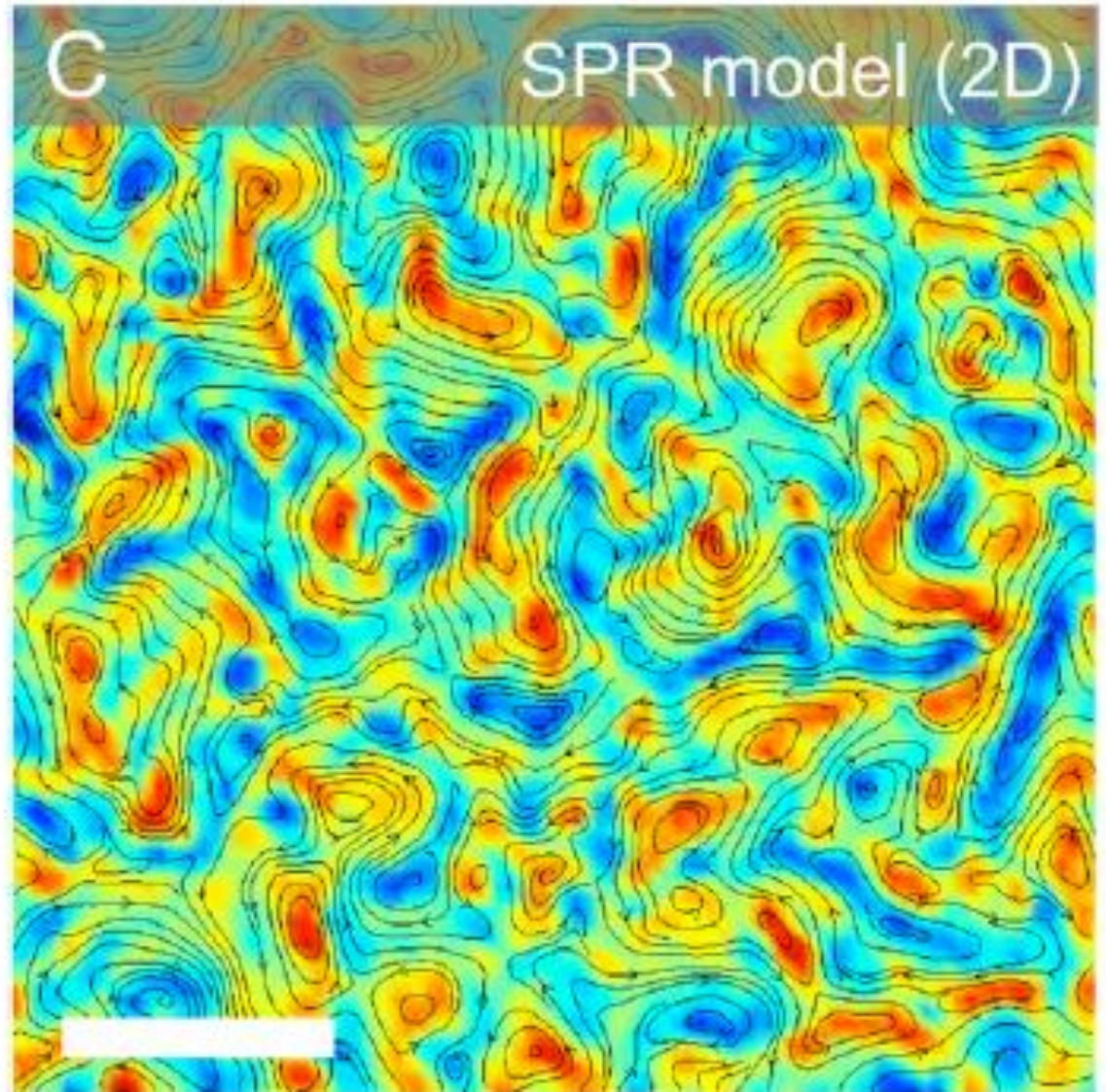
Bacteria



vorticity

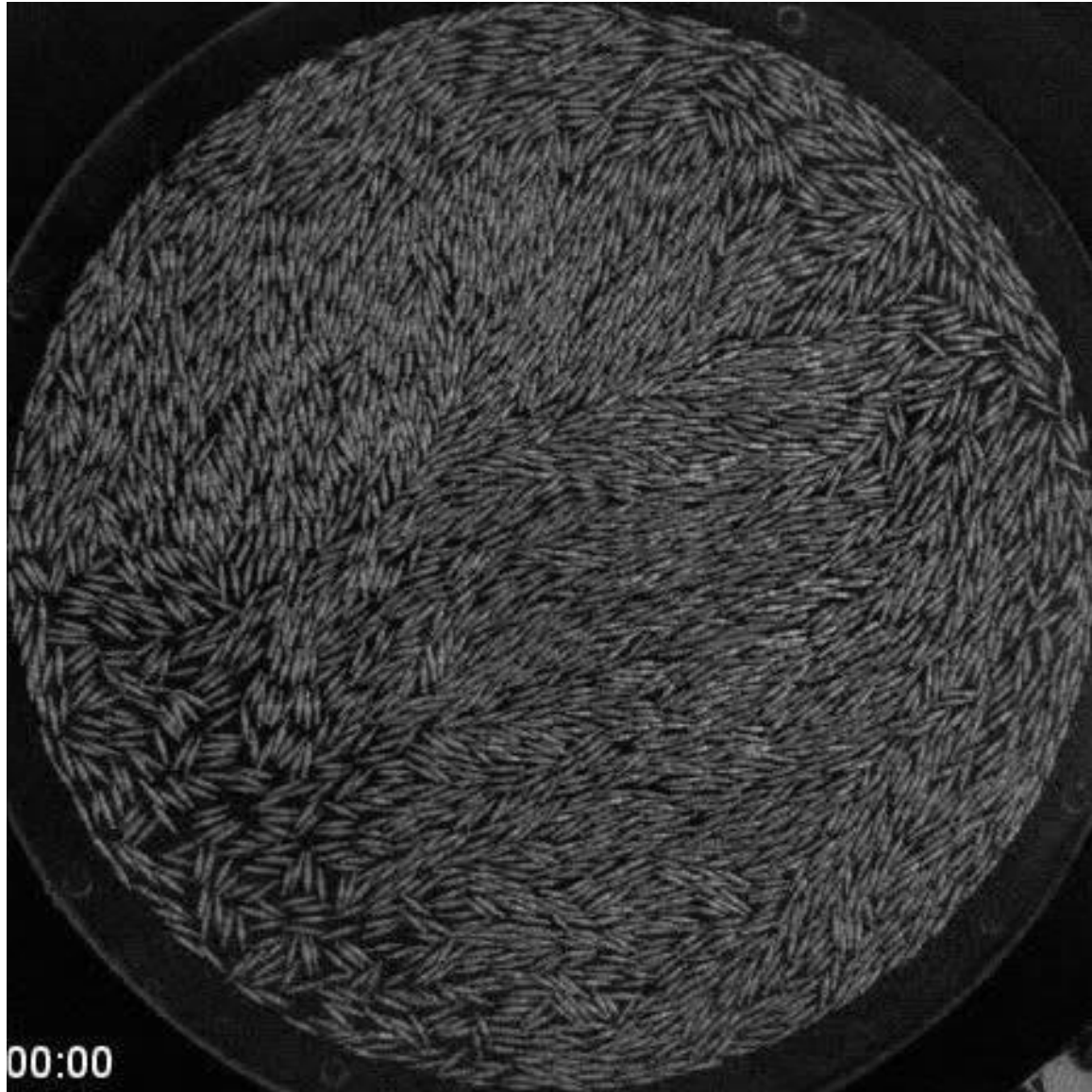


Discrete simulations



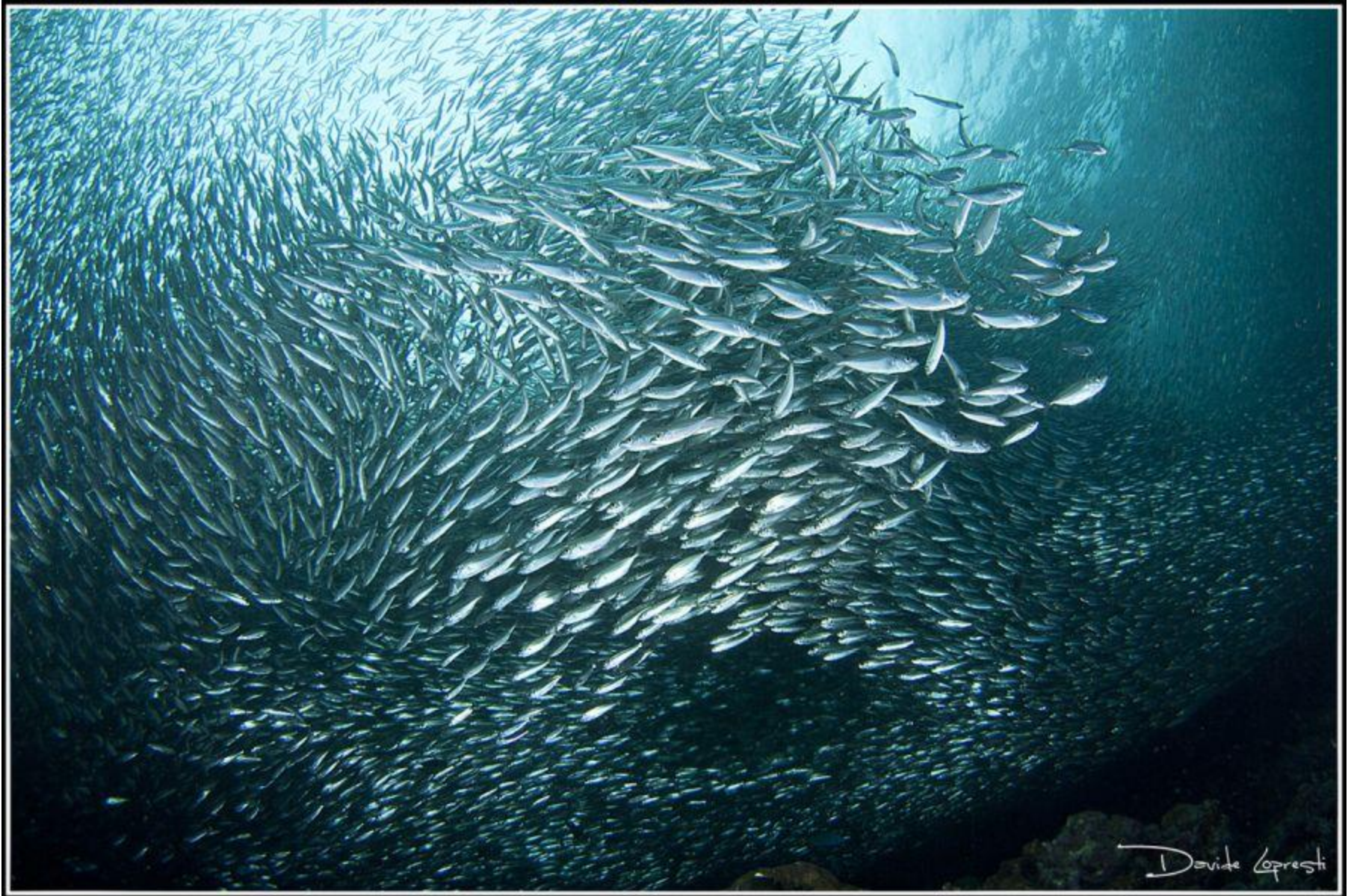
Wensink, Dunkel, Heidenreich,
Drescher, Goldstein, Lowen,
Yeomans, PNAS 2012

Driven grains



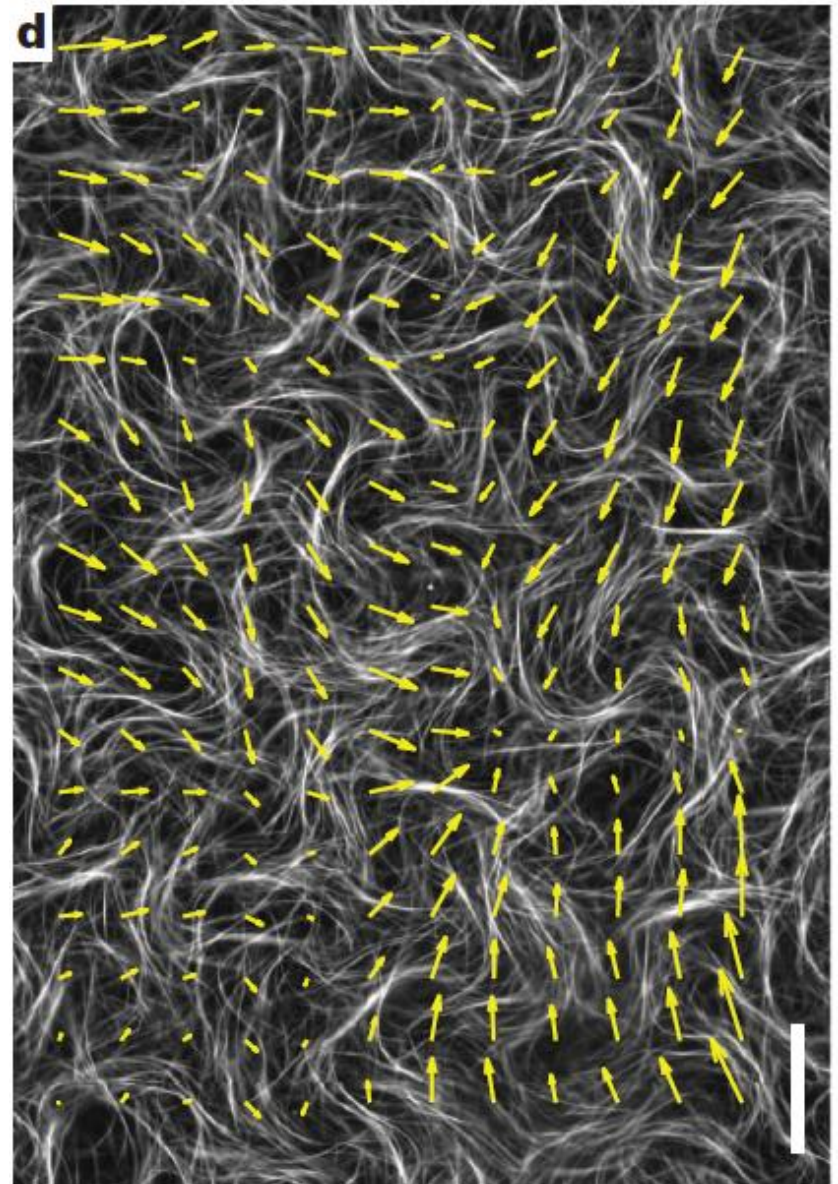
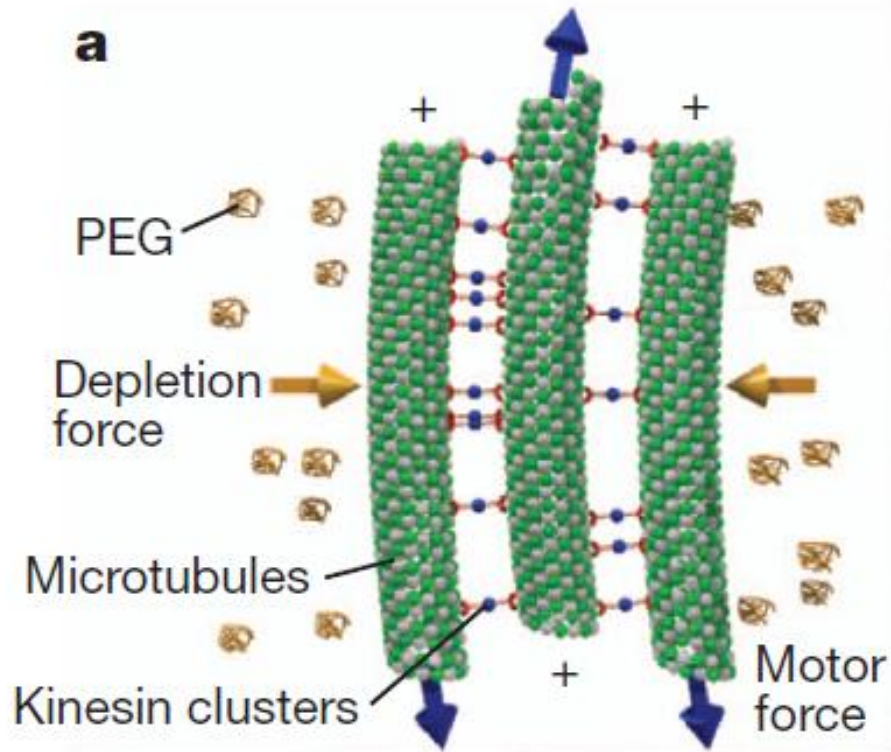
V Narayan, S Ramaswamy, N Menon - Science, 2007

Fish?



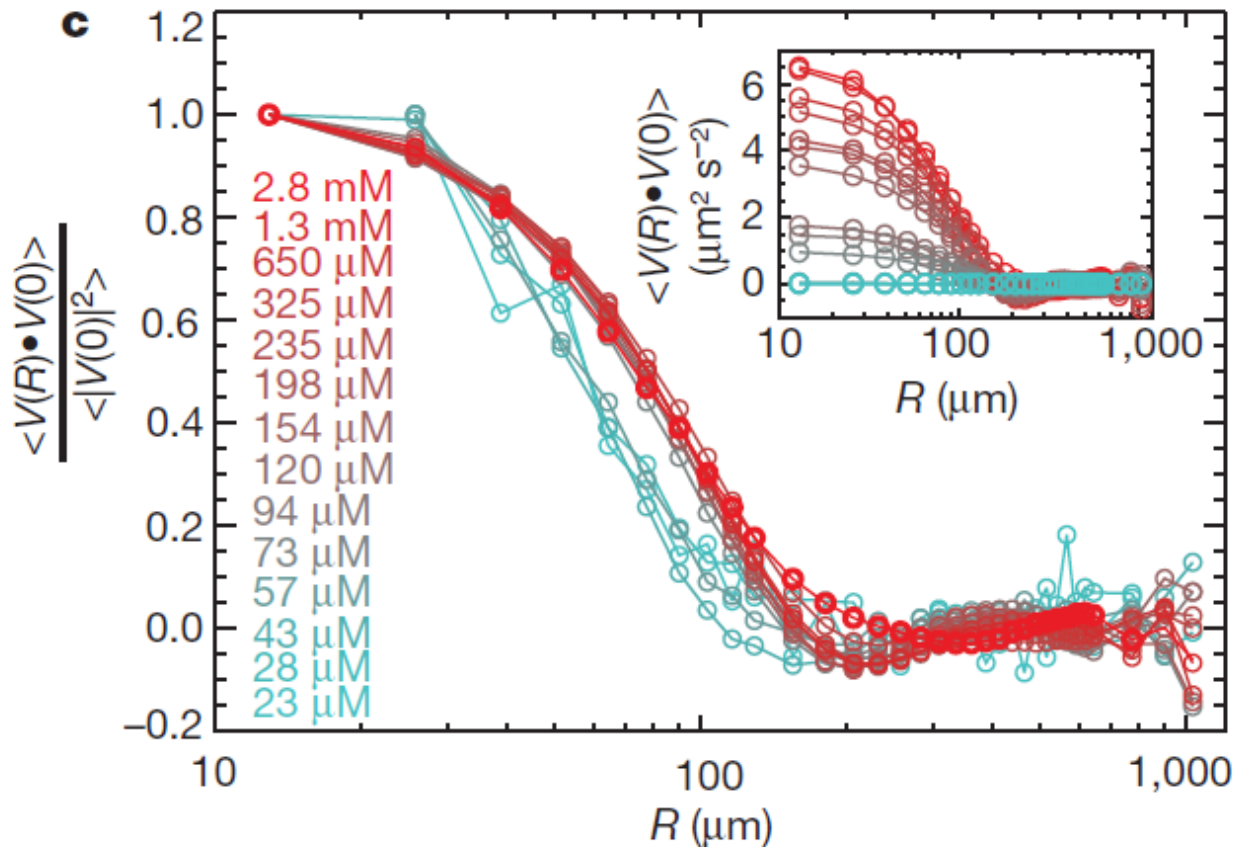
Davide Copresti

Molecular motors



Sanchez, Chen, DeCamp, Heymann, Dogic,
Nature 2012

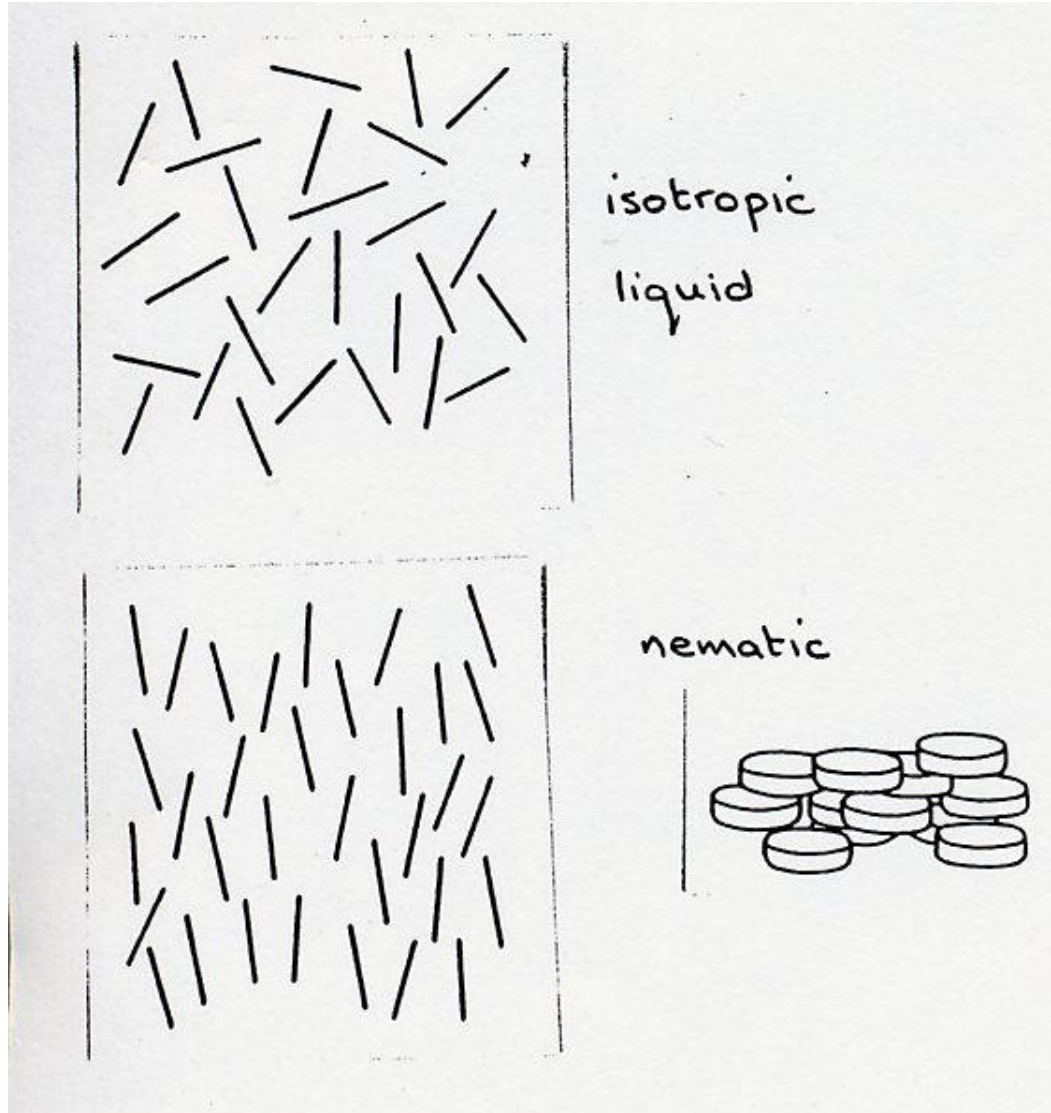
Molecular motors



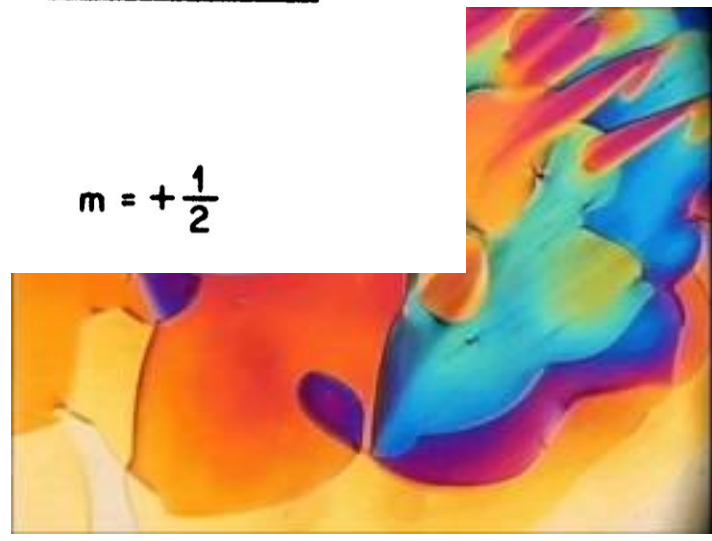
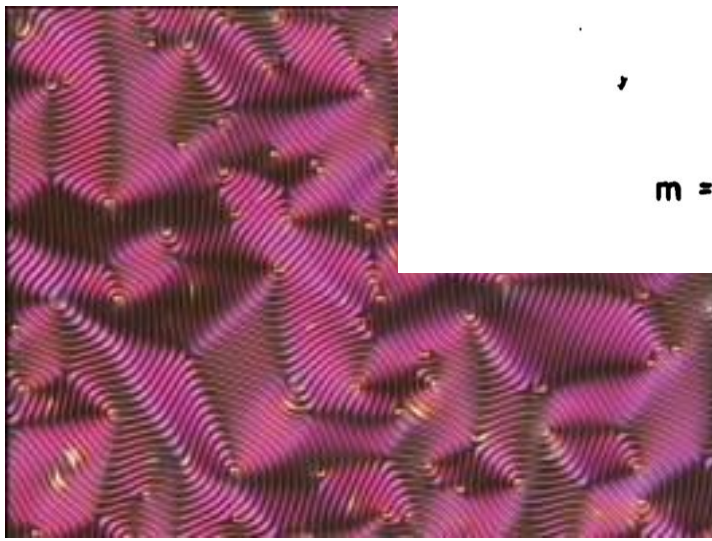
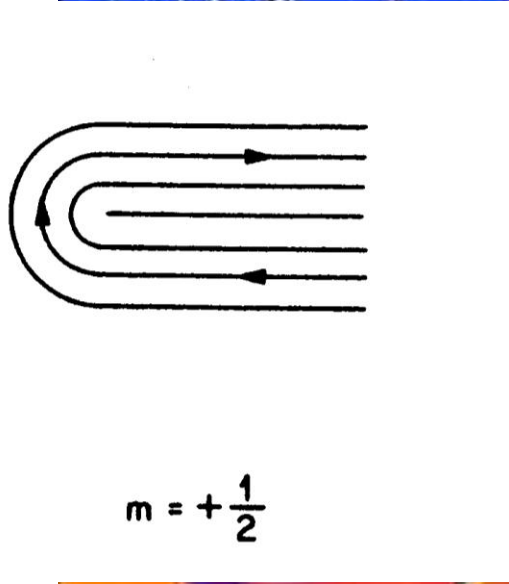
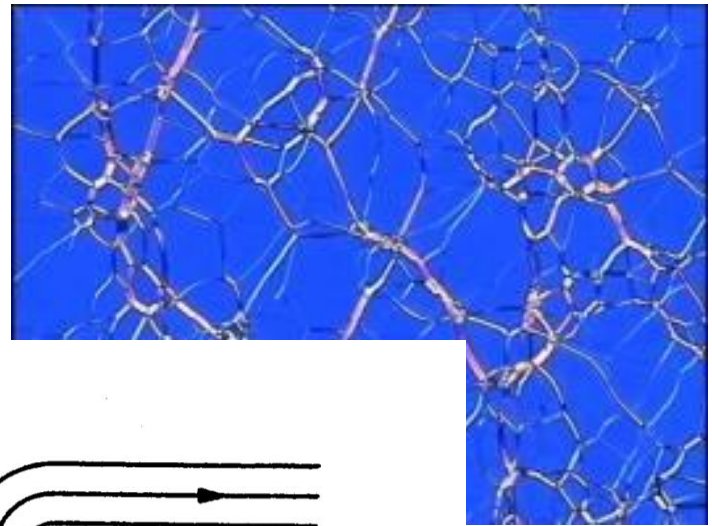
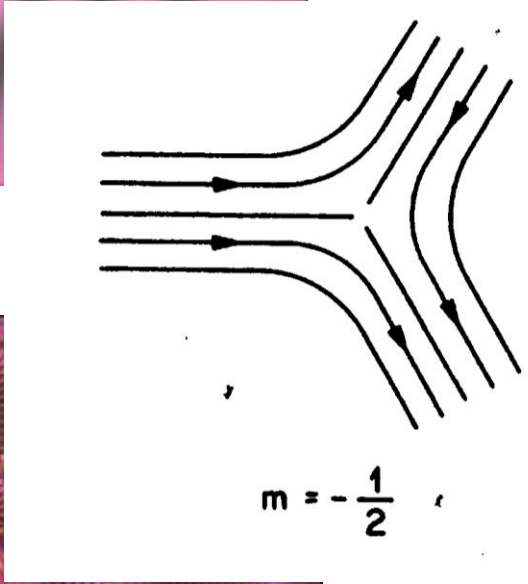
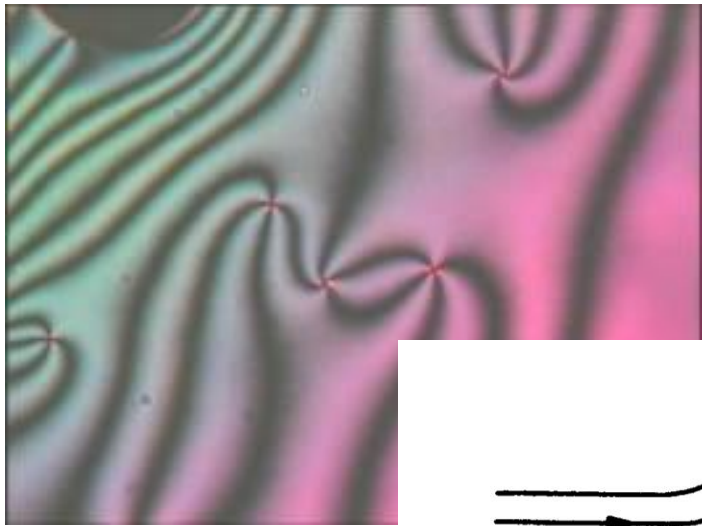
Velocity increases with activity

Length scale controlling decay of $\langle vv \rangle$ independent of activity

Liquid crystals



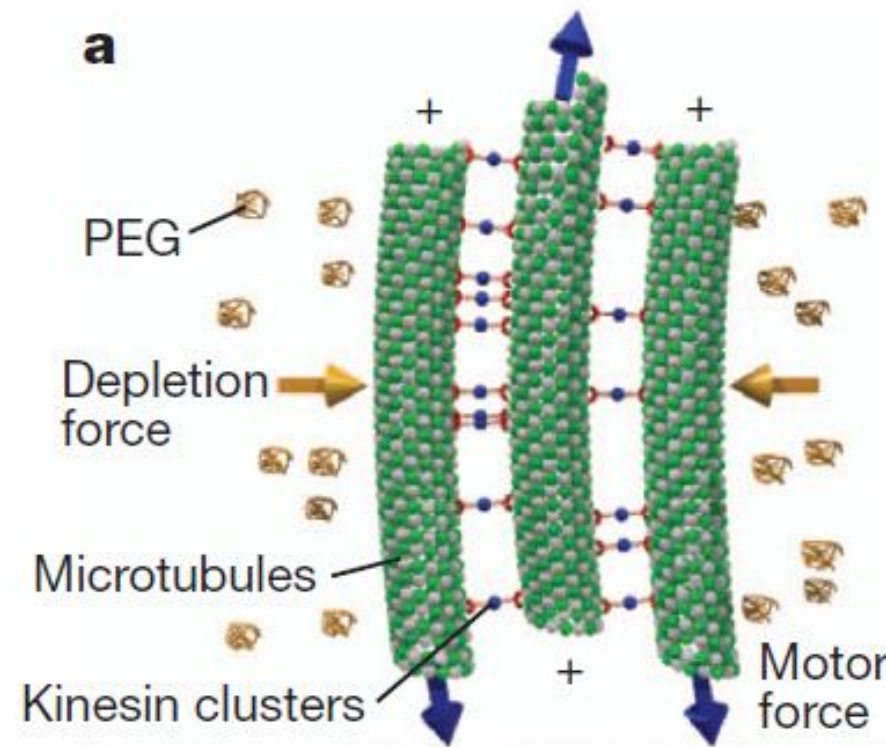
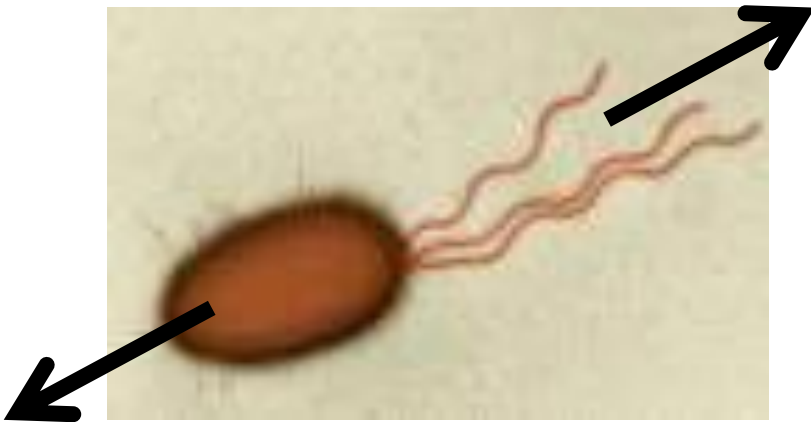
Topological defects



Continuum equations of active nematics

Liquid crystal equations of motion +
additional term in the stress tensor

Consequence of a dipolar source term



Hatwalne, Ramaswamy, Rao, Simha, PRL 2003

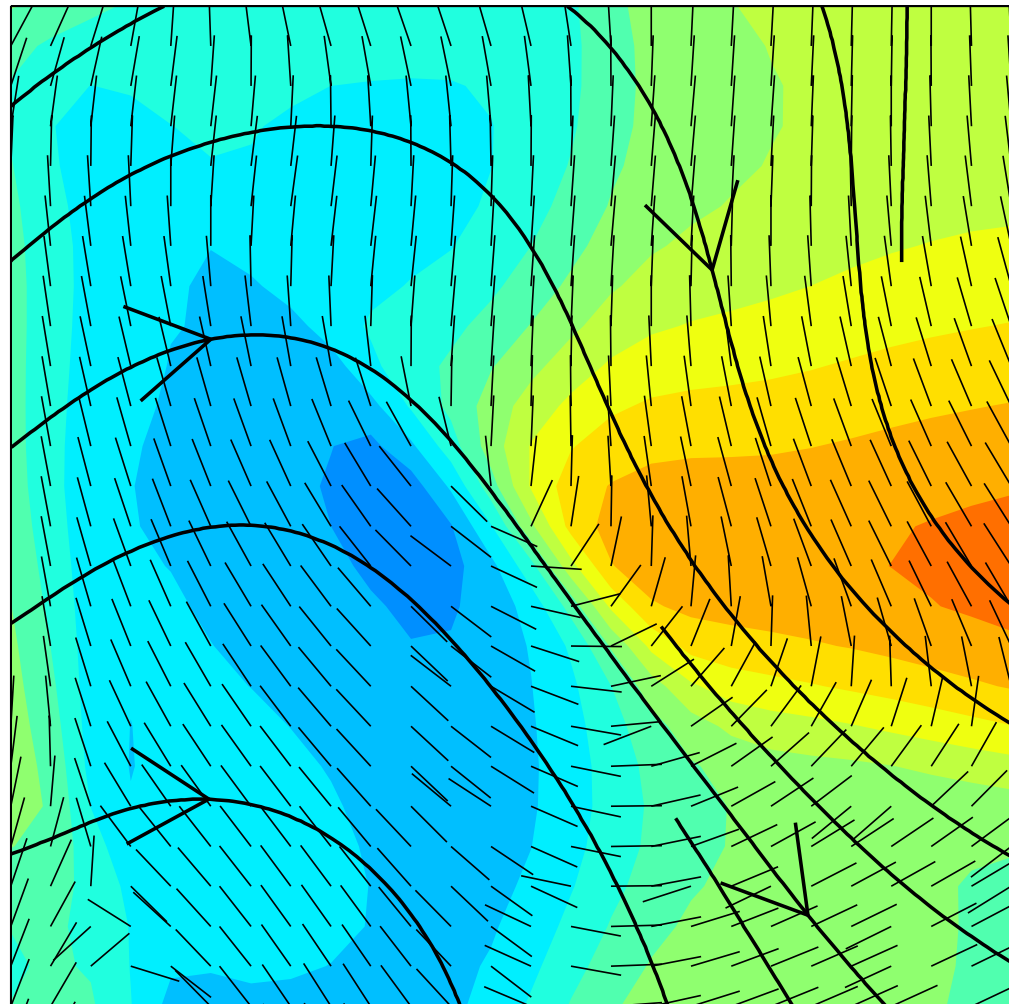
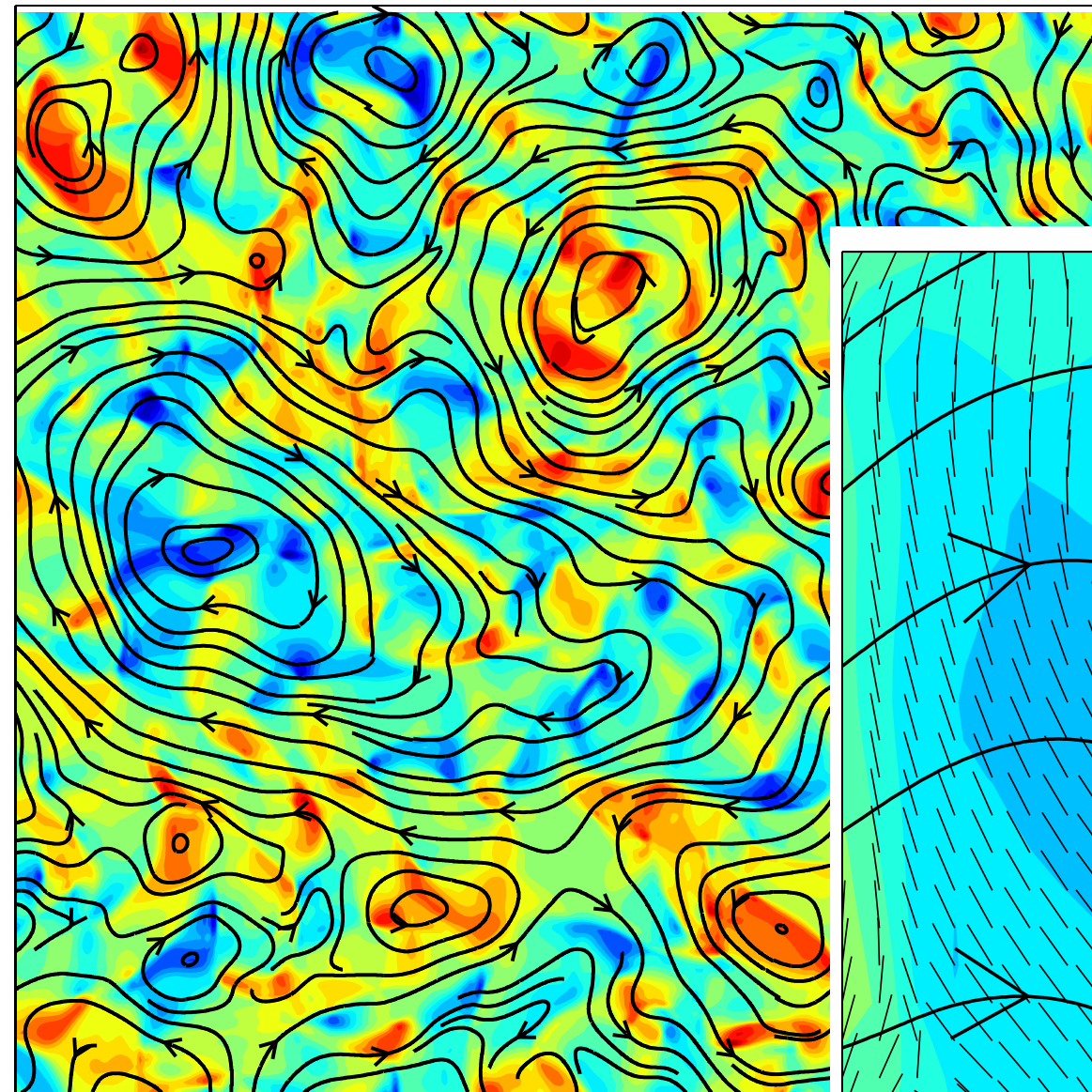
Continuum equations of active nematics

Liquid crystal equations of motion +
additional term in the stress tensor

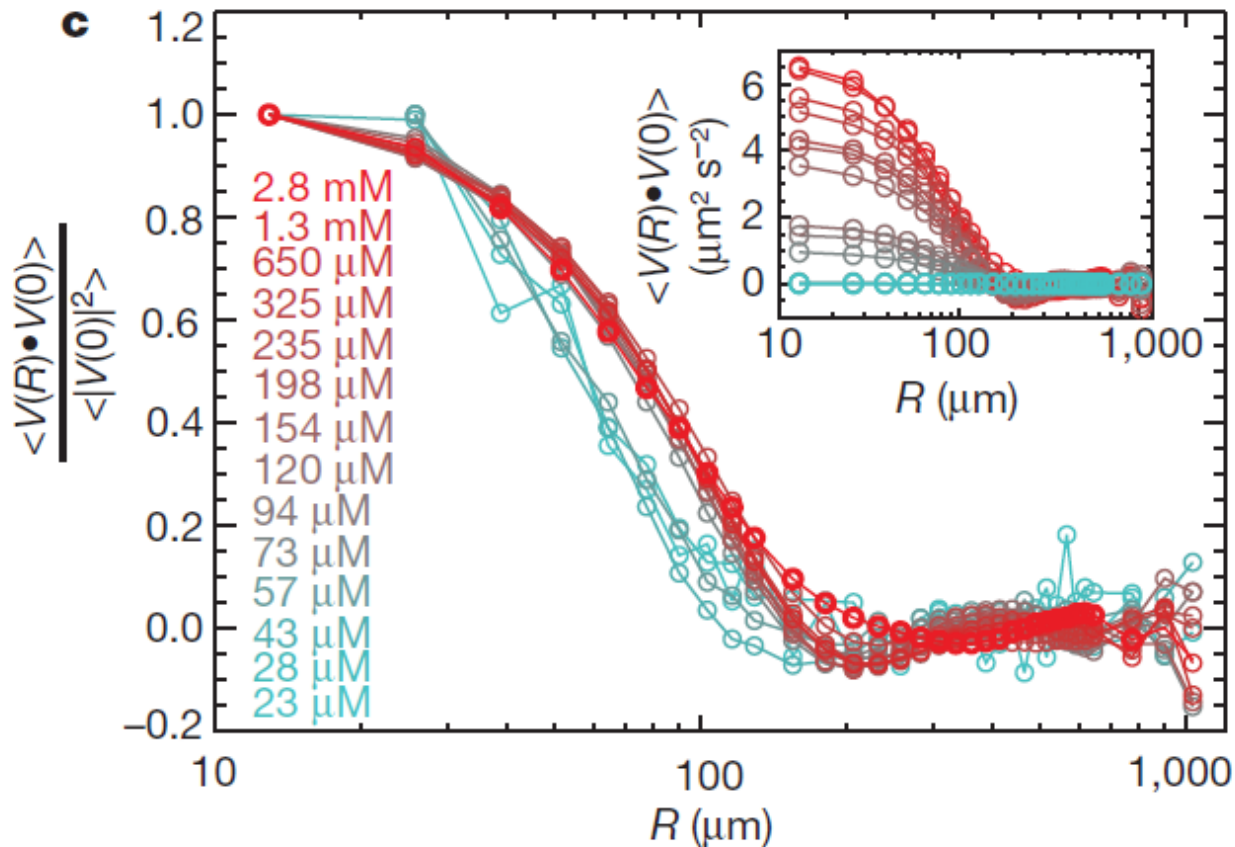
Gradients in the order parameter field induce a flow

Nematic state is unstable

Active turbulence in extensile suspensions



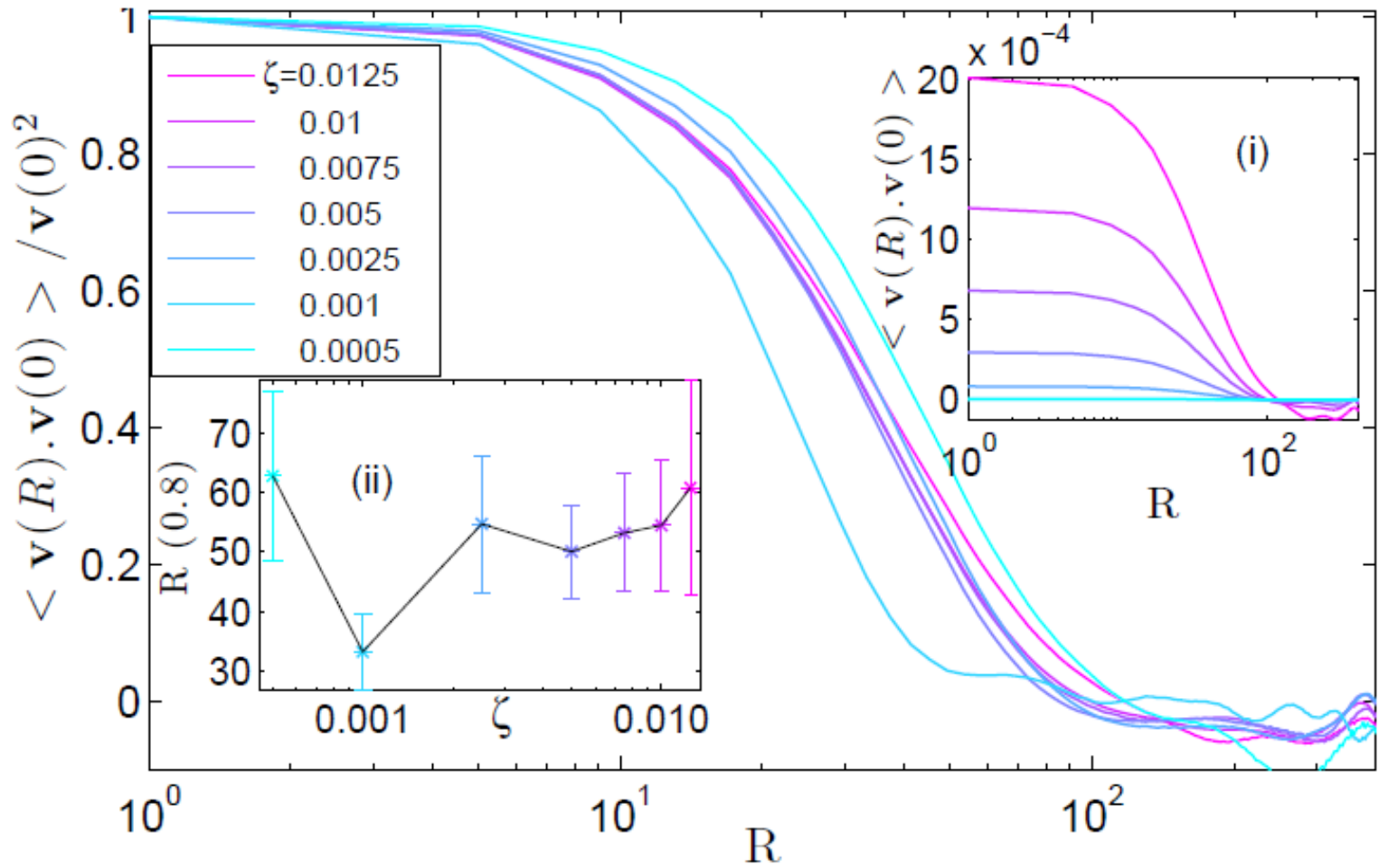
Molecular motors



Velocity increases with activity

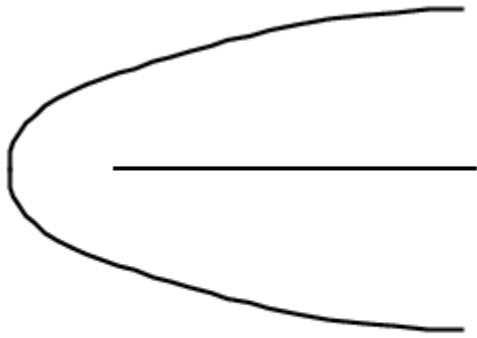
Length scale controlling decay of $\langle vv \rangle$ independent of activity

$\langle vv \rangle$: simulations

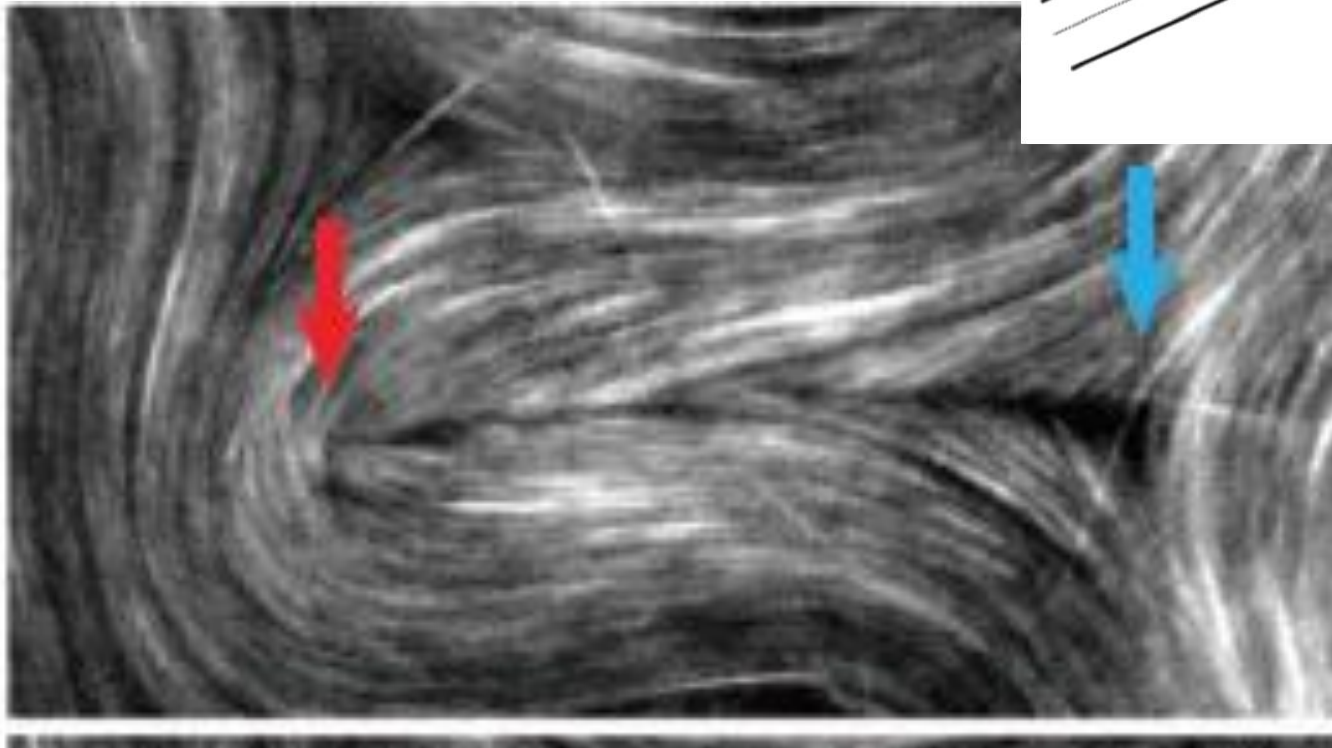
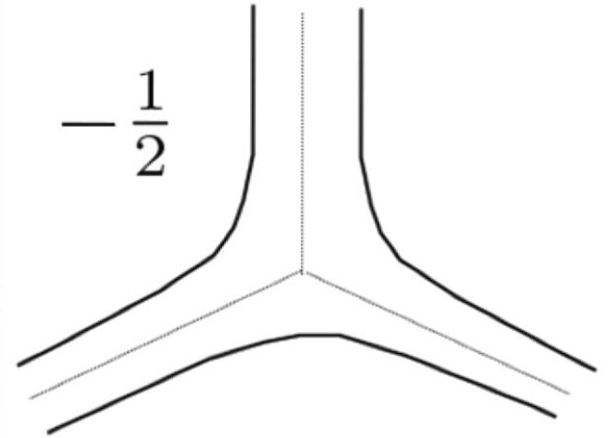


Topological defects in active systems

$\frac{1}{2}$

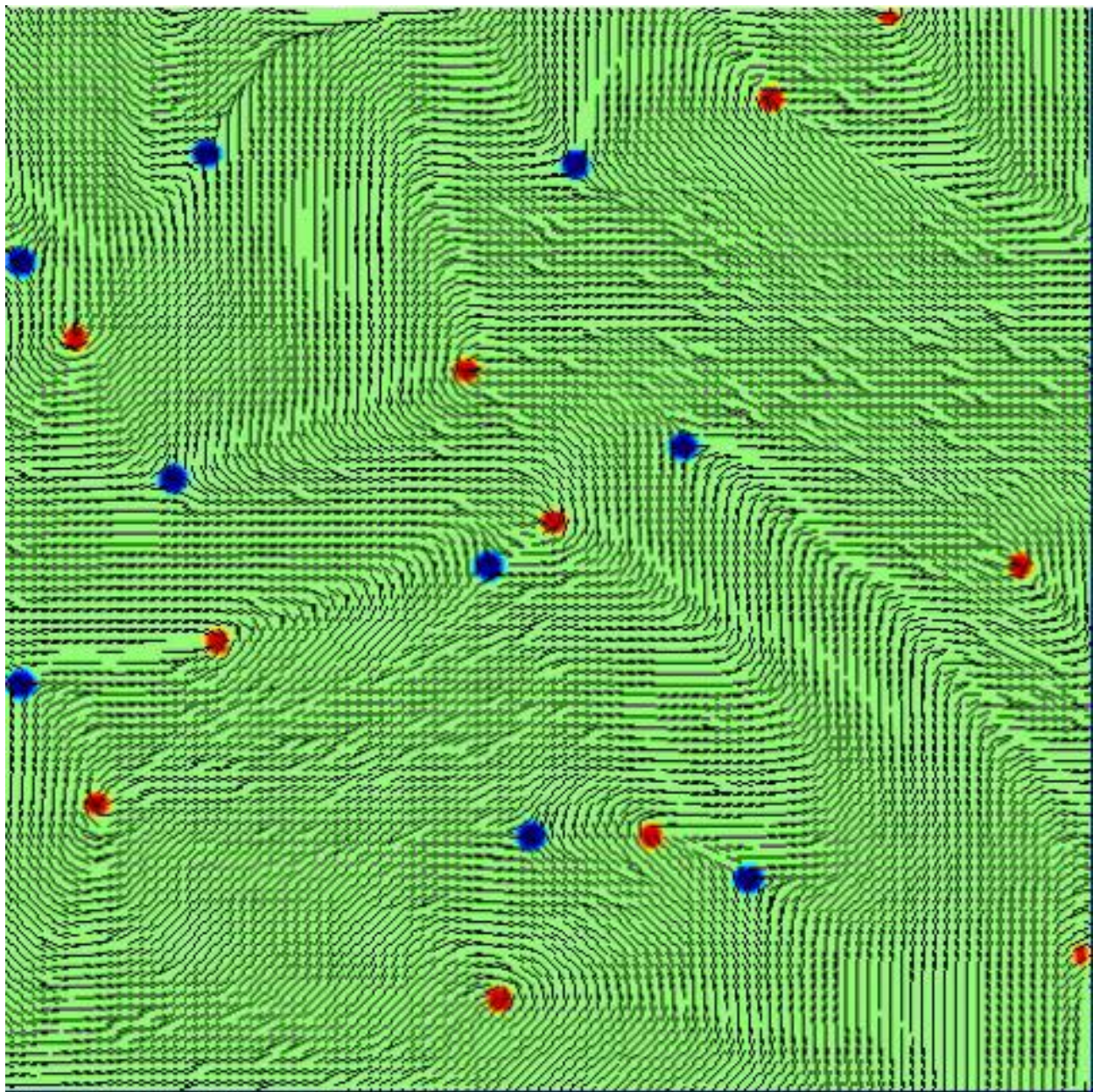


$-\frac{1}{2}$



microtubule bundles driven by
molecular motors

L. Giomi, M.J. Bowick, Ma Xu, M.C. Marchetti, PRL 110, 228101
Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012



Defect dynamics: steady state

$$\alpha \frac{\zeta}{K} = \beta \frac{\sigma \zeta l Q n^2}{\mu}$$

l : independent of activity

Active nematics

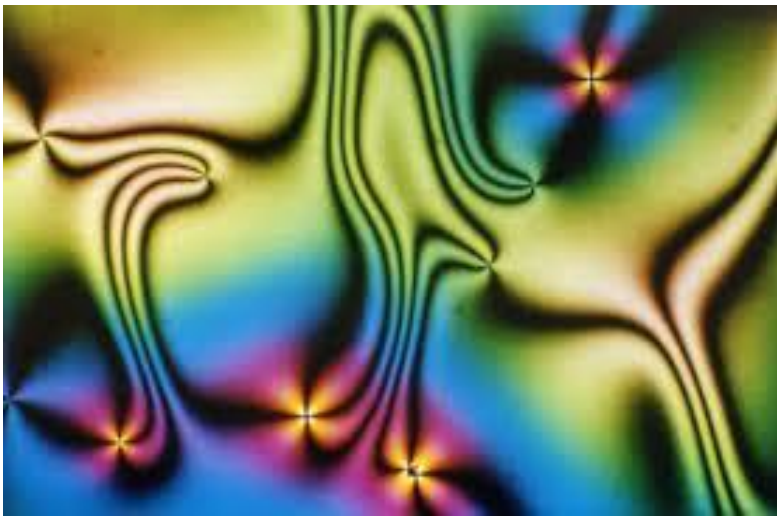
Experimental and simulation evidence that defects control active turbulence

Predicting the rate of defect formation?

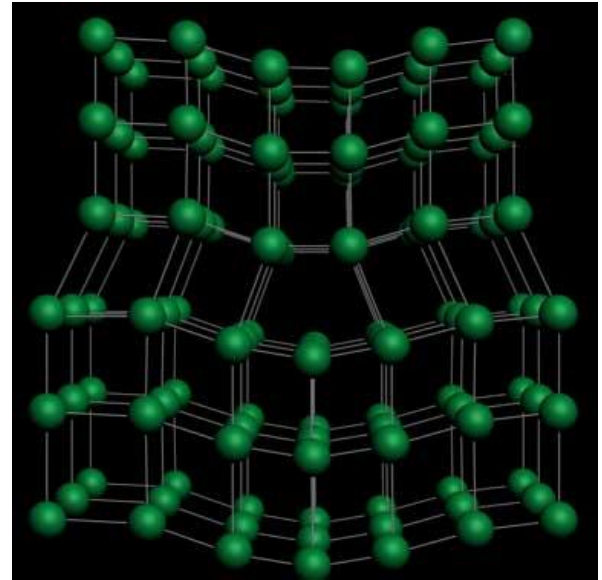
Predicting the cross section for annihilation?

Varying concentration?

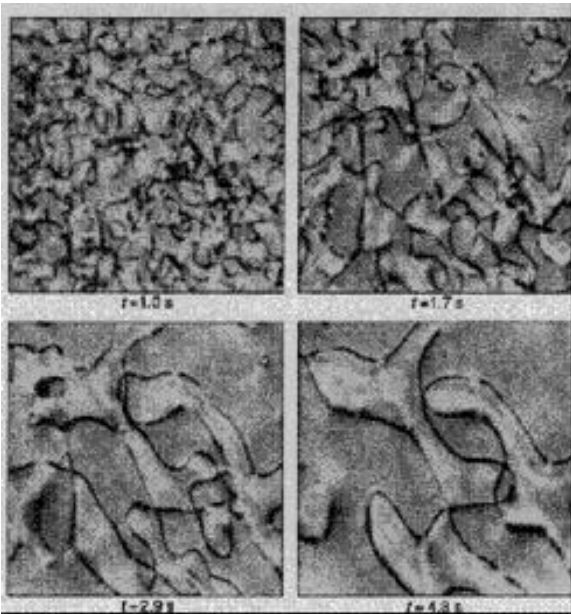
How generic is this picture?



liquid crystals



crystal dislocations



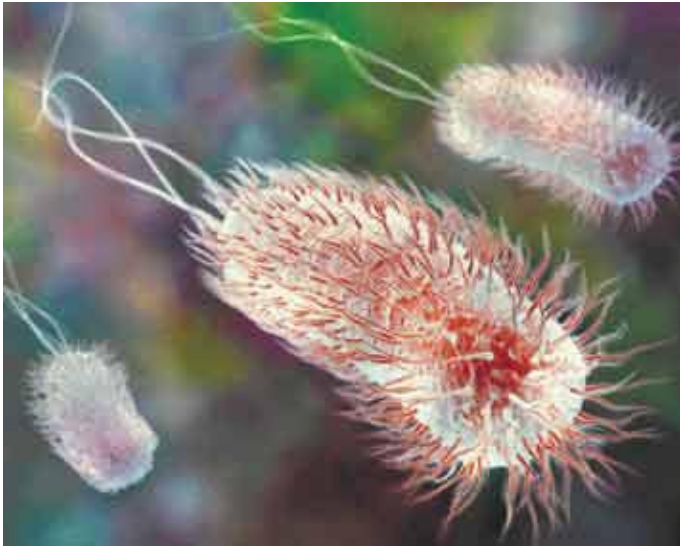
cosmic strings in the early universe

magnetic monopoles in spin ice

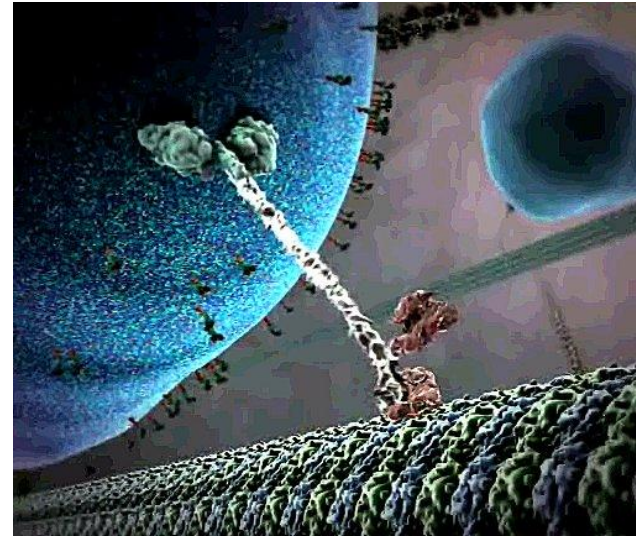
topological insulators

quantum vortex in a superfluid

Active matter

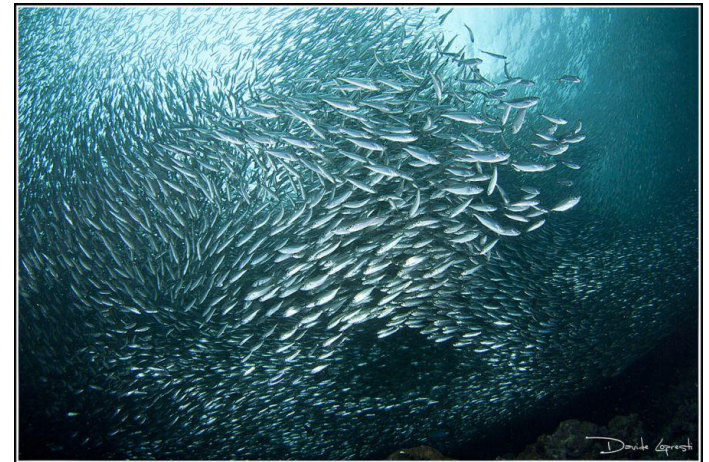


bacteria, self-propelled colloids



molecular motors, cells

active systems operate out of
thermodynamic equilibrium



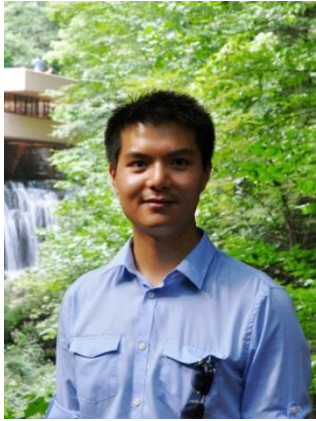
birds, fish



Dmitri (Mitya) Pushkin
University of Oxford



Sumesh Thampi
University of Oxford



Henry Shum
University of Pittsburgh



Jorn Dunkel
MIT



Ramin Golestanian
University of Oxford

Pushkin, Shum, Yeomans, J. Fluid Mechanics **726** (2013) 5
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