Matter Emerges From the Vacuum

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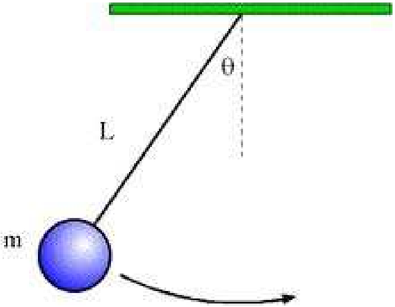
22nd June 2013

Oxford Physics

Structure of Talk

- From Classical Fields to Quantum Fields
- The Vacuum
- The Excitations
- Interactions

The Simple Pendulum



"The career of a young theoretical physicist consists of treating the harmonic oscillator in ever-increasing levels of abstraction."

Sidney Coleman

The Harmonic Oscillator

The simple harmonic oscillator is the most important system in physics.

The kinetic energy is $\frac{1}{2}m\dot{x}^2$ and the potential energy $\frac{1}{2}m\omega^2 x^2$.

The classical equations of motion are

$$m\ddot{x} = -m\omega^2 x$$

with an oscillating solution $x = A\cos(\omega t + \phi)$

The harmonic oscillator Lagrangian and Hamiltonian are

$$\begin{aligned} L &= \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2, \\ \mathcal{H} &= \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2. \end{aligned}$$

Classical Fields

Classical fields have values at all points in space and time.

Examples are

- The electromagnetic vector potential $A_\mu(\mathbf{x}, t)$.
- The gravitational potential field $\Phi(\mathbf{x}, t)$.

The classical vacuum solution is simple:

$$A_\mu(\mathbf{x}, t) = 0,$$

Classical fields without sources satisfy the wave equation:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) (\text{FIELD}) = 0.$$

Classical Fields and Wave Equation

Equation of motion is

$$\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \phi}{\partial z^2} = 0$$

This equation is solved by the mode ($\mathbf{k}$ wavenumber/momentum)

$$\phi(\mathbf{x}, t) = \phi_k(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \qquad \mathbf{k} = (k_1, k_2, k_3)$$

The equation for this mode reduces to

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi_k(t) + |\mathbf{k}|^2 \phi_k(t) = 0.$$

This is the equation of motion for a harmonic oscillator with frequency $\omega(k) = c|k|$.

Classical Fields and Wave Equation

We can decompose a classical field as

$$\phi(x,t) = \int d^3\mathbf{k}\, \phi_{\mathbf{k}}(t) e^{-i\mathbf{k}\cdot\mathbf{x}}.$$

The dynamics of $\phi(x,t)$ is encoded by the dynamics of all $\phi_{\mathbf{k}}(t)$.

Each individual $\phi_{\mathbf{k}}(t)$ satisfies the harmonic oscillator equation

$$\ddot{\phi}_{\mathbf{k}}(t) = -c^2 |\mathbf{k}|^2 \phi_{\mathbf{k}}(t)$$

with frequency $\omega(k) = c|k|$.

The dynamics of a continuous classical field is the dynamics of an infinite number of classical harmonic oscillators.

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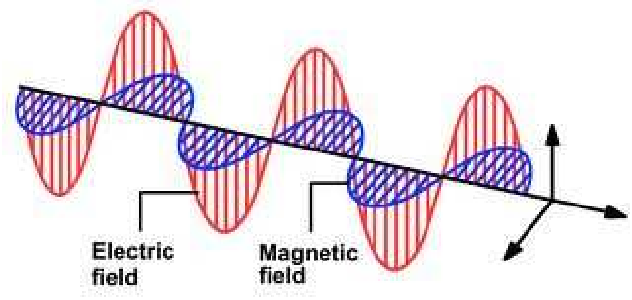
A propagating mode of the electromgnetic field, with oscillating **E** and **B** fields.



The Classical Harmonic Oscillator

The harmonic oscillator Lagrangian and Hamiltonian are

$$\begin{aligned} L &= \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2, \\ \mathcal{H} &= \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2. \end{aligned}$$

The classical equations of motion are

$$m\ddot{x} = -m\omega^2 x$$

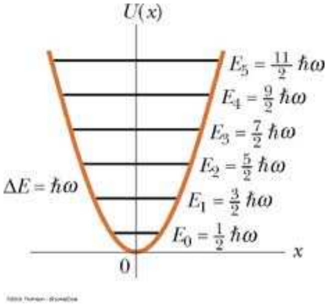
with an oscillating solution

$$x = A\cos\left(\omega t + \phi\right)$$

The Quantum Harmonic Oscillator

The world is quantum, and we should instead work with a quantum harmonic oscillator.

In quantum mechanics there are *discrete* energy levels.



The Quantum Harmonic Oscillator

The spectrum of the quantum harmonic oscillator is

$$\begin{aligned} E_0 &= \frac{1}{2}\hbar\omega \\ E_1 &= \left(1 + \frac{1}{2}\right)\hbar\omega \\ E_2 &= \left(2 + \frac{1}{2}\right)\hbar\omega \\ E_n &= \left(n + \frac{1}{2}\right)\hbar\omega. \end{aligned}$$

Key points:

- The 'vacuum' ground state has a zero-point energy $\frac{1}{2}\hbar\omega$.
- There are excited states: the nth excited state has energy $E_n = (n + \frac{1}{2})\hbar\omega$.

Lots of Quantum Harmonic Oscillators

The Lagrangian of lots of quantum harmonic oscillators is

$$L = \sum_{i=1}^{N} \left(\frac{1}{2} m_i \dot{x}_i^2 - \frac{1}{2} m_i \omega_i^2 x_i^2 \right).$$

The energy spectrum for lots of quantum harmonic oscillator is

$$E = \sum_{i=1}^{N} \left(n_i + \frac{1}{2} \right) \hbar \omega_i.$$

This is the sum of the energies of each individual quantum harmonic oscillator.

There is a ground state energy from the sum of the ground state energy of each individual harmonic oscillator.

Lots of Quantum Harmonic Oscillators

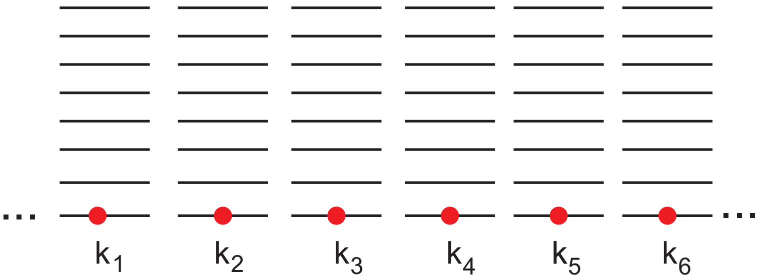
A continuous classical field decomposes into an infinite number of classical harmonic oscillators, each associated to a wavenumber $k = (k_1, k_2, k_3)$ and a frequency $\omega(k) = c|k|$.

A continuous quantum field decomposes into an infinite number of quantum harmonic oscillators, each associated to a wavenumber $k = (k_1, k_2, k_3)$ and a frequency $\omega(k) = c|k|$.

The possible states of a continuous quantum field are the possible states of an infinite number of quantum harmonic oscillators.

These are the ground state and many excited states.

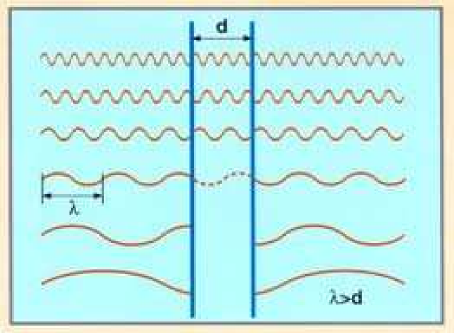
The Vacuum



The quantum field theoretic vacuum - no particles, all harmonic oscillators in their ground state.

The Ground State: The Casimir Effect

Is the zero point energy physical?



Zero point energy for two metal plates comes from normal modes between these plates.

The number of normal modes - and the zero point energy - reduces as the plate separation d reduces.

The Ground State: The Casimir Effect

The attractive Casimir force between metal plates of area A separated by d is

$$F = \frac{\pi^2}{240} \frac{\hbar c}{d^4} A$$

This force has been measured in agreement with the theoretical results.

Forces come from a system adjusting to a state of lower energy.

The Casimir force comes from the lowering of zero point energy as the plates approach.

The Ground State: The Cosmological Constant

Each harmonic oscillator has a ground state energy of $\frac{1}{2}\hbar\omega$ (can be negative for fermions) and formally there are an infinite number of harmonic oscillators.

'Infinite' will become 'finite' due to an underlying granularity, but quantum field theory still gives an enormous number of harmonic oscillators.

The vacuum has been weighed to have a 'zero point energy' of $\sim 10^{-11}\,\mathrm{J\,m^{-3}}$.

The naive quantum field theory estimate is *at least* $\sim 10^{49}\,\mathrm{J\,m^{-3}}$!

This mismatch is the biggest open problem in theoretical physics and may be telling us something very deep about gravity and quantum mechanics.

Excited States and Particles

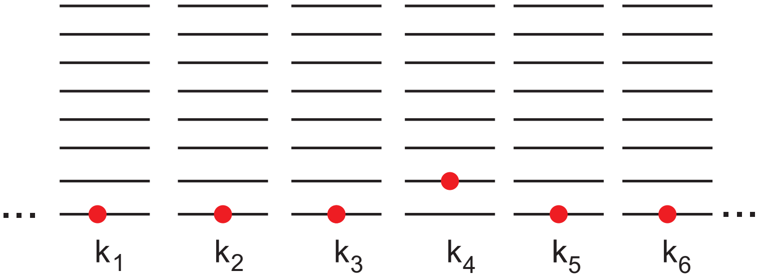
A field has an infinite number of harmonic oscillators each associated to a momentum $\mathbf{k}$.

The quantum vacuum has all harmonic oscillators in the ground state and no particles.

Each excitation of a harmonic oscillator with momentum $\mathbf{k}$ corresponds to the existence of a particle with momentum $\mathbf{k}$.

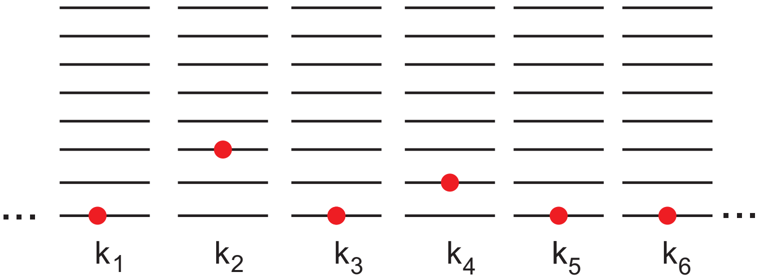
The spectrum of the 'fermionic harmonic oscillator' is different and only has a single excitation. This reflects the fact that two fermions cannot occupy the same quantum state.

Excited States and Particles



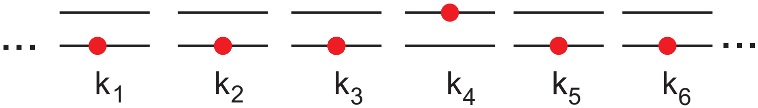
A quantum mechanical state describing one particle with momentum k_4.

Excited States and Particles



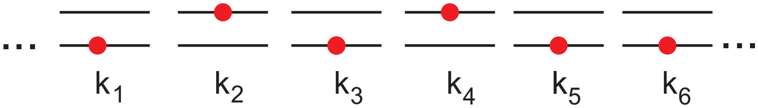
A quantum mechanical state describing one particle with momentum k_4 and two particles with momentum k_2.

Excited States and Particles



The quantum mechanical state describing one fermionic particle with momentum k_4.

Excited States and Particles

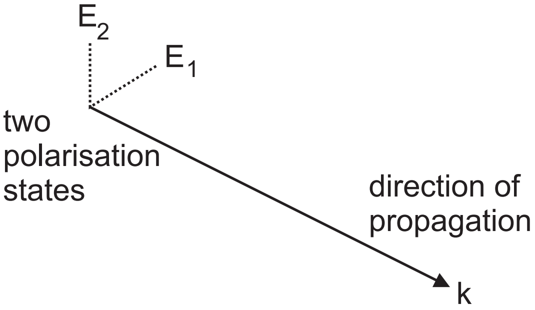


The quantum mechanical state describing one fermionic particle with momentum k_4 and one fermionic particle with momentum k_2.

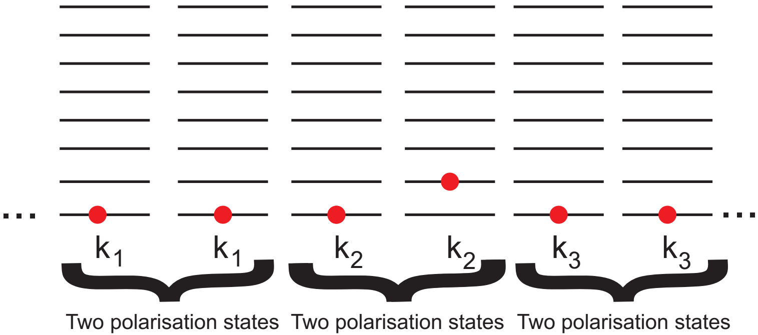
What is a photon?

The electromagnetic field is described by the vector potential $A_\mu(\mathbf{x}, t)$.

As a quantum field, this has two harmonic oscillators associated to each momentum value **k** - one for each polarisation.



What is a photon?



A single photon of given polarisation and momentum **k** is the quantum state of the A_μ field with a single excitation for that momentum and that polarisation.

The Standard Model

The Standard Model is a particular choice of quantum field theory. It contains many different types of fields. Different types of particle corresponds to excitations of different types of field.

- The photon is an excitation of the electromagnetic field.
- The Higgs boson is an excitation of the Higgs field.
- The electron is an excitation of the electron field.
- The gluon is an excitation of the gluon field.

What about interactions?

Interactions

Non-interacting particles correspond to states of simple harmonic oscillators.
Real harmonic oscillators are not simple:

$$\begin{aligned} \ddot{x} &= -\omega^2 \sin x \\ &= -\omega^2 \left(x - \frac{x^3}{6} + \dots \right) \\ &\simeq -\omega^2 x \qquad \text{for } x \text{ small} \end{aligned}$$

$$L = \frac{1}{2} m \dot{x}^2 - m\omega^2 (1 - \cos x) = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2 + m\omega^2 \frac{x^4}{24} + \dots$$

Higher-order non-quadratic interaction terms.

Interactions

The full Lagrangian for quantum fields also has anharmonic terms.

$$\mathcal{L} = \underbrace{\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2}_{\text{SHO}} - \underbrace{\frac{\lambda}{24} \phi^4}_{\text{Anharmonic term}}$$

These anharmonic terms cause interactions between the differrent harmonic oscillators and therefore between different particles.

The dynamics of a free quantum field theory is the dynamics of an infinite number of decoupled quantum harmonic oscillators.

The dynamics of the Standard Model is the dynamics of an infinite number of coupled harmonic oscillators.

Summary

1. The dynamics of a field decomposes into the dynamics of an infinite number of harmonic oscillators.

2. A quantum field consists of an infinite number of harmonic oscillators, each labelled by wavenumber/momentum (k_1, k_2, k_3).

3. Particles are nothing more than the excited states of these harmonic oscillators.

4. Interactions arise from anharmonic terms.