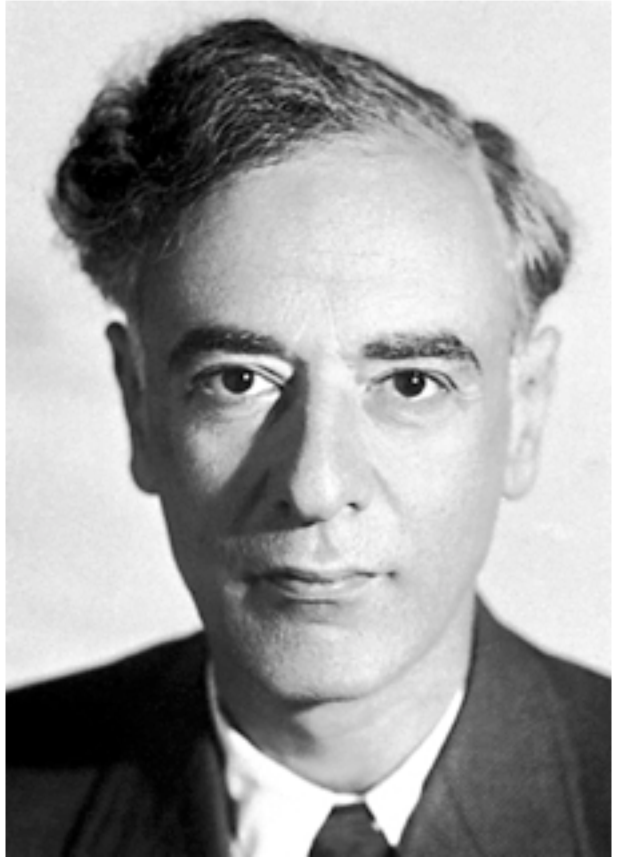
Quantum Field Theory and Condensed Matter Physics: making the vacuum concrete

Fabian Essler (Oxford)

Oxford, June 2013



Lev Landau

“This work contains many things which are new and interesting. Unfortunately, everything that is new is not interesting, and everything which is interesting, is not new.”

The Field
is real!

hbar>0!


The Field
is real!
hbar>0!

Here:

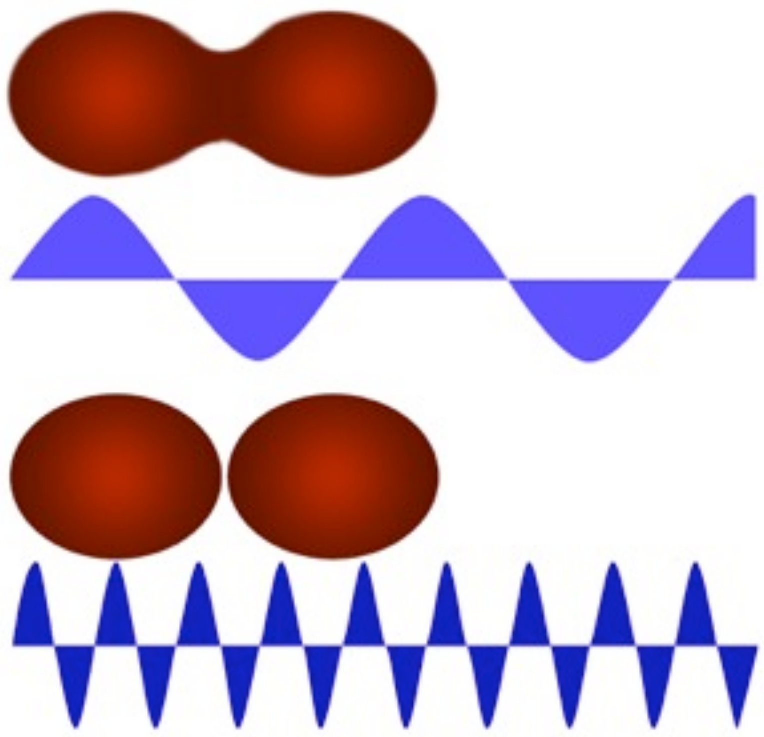
Relativistic QFT as an emergent
(rather than fundamental) phenomenon.

Length/Energy Scales: Organizing Principle of Physics

high energies ⇔ short distances

low energies ⇔ large distances

Think of this in terms of probing a physical system by e.g. light:



to resolve what happens at length scale a, the wavelength λ must be smaller than a.

short distances → small λ → high energies as $E=hc/\lambda$

Length

Classical Mechanics

Quantum Mechanics

Standard Model

ToE?

A Reductionist View of the World

Quantum Field Theory

↓

Quantum Mechanics

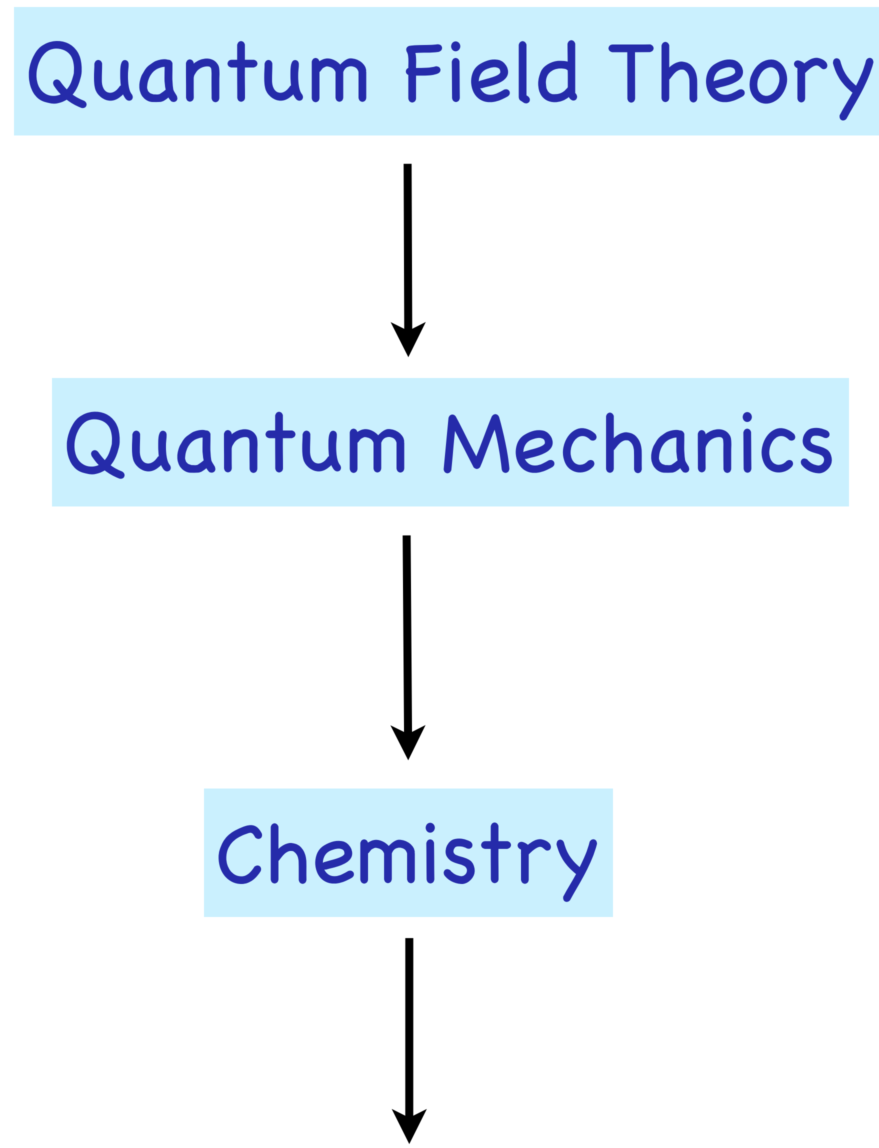
↓

Chemistry

↓

"If we understand QFT/ToE, we understand everything."

A Reductionist View of the World



"If we understand QFT/ToE, we understand everything."

Depends on what "understand" means...

Following 30 minutes:

Quantum Field Theory

↓

Quantum Mechanics

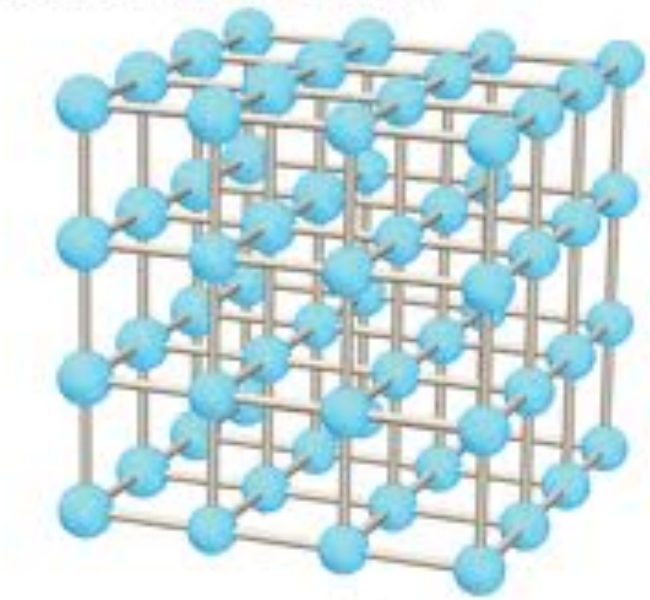
↓

Quantum Field Theory

interesting perspective on QFT (and perhaps the reductionist view)

Example: Lattice Vibrations in Crystals

Crystal: atoms in a periodic array



(b)

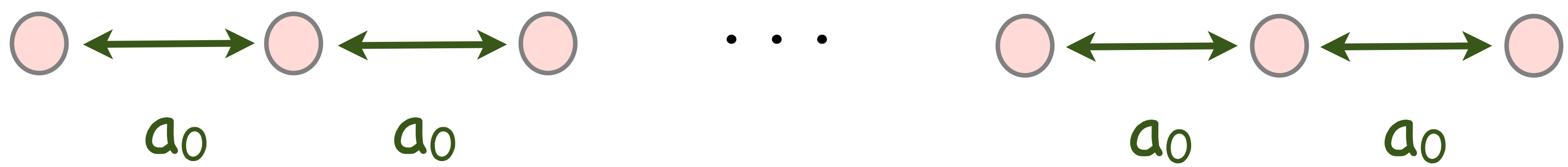
Interaction between the atoms (e.g. Coulomb) $V(\vec{R}_1, \vec{R}_2, \ldots, \vec{R}_N)$

→ causes oscillations of atoms

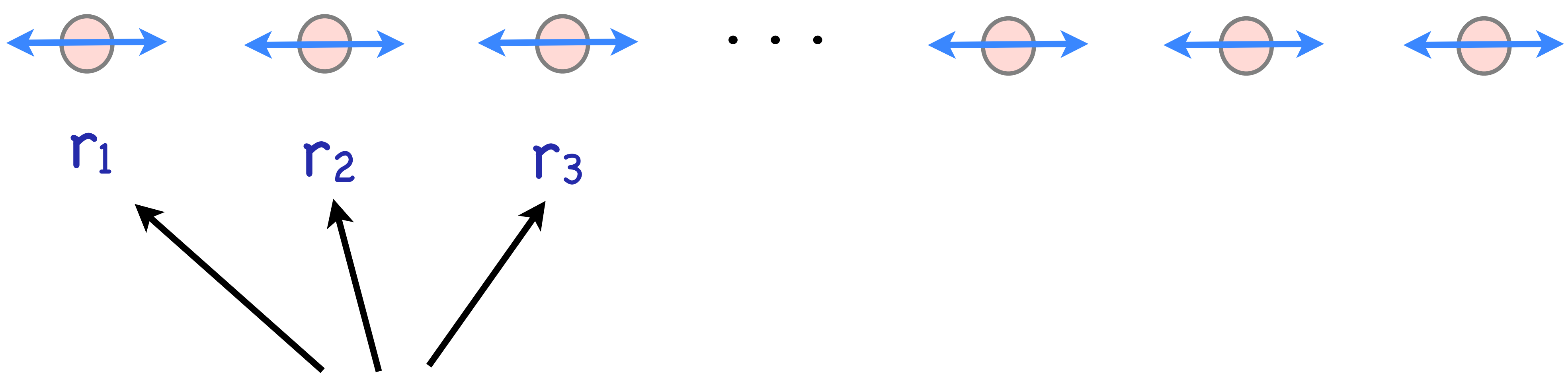
Crystal ↔ mean ("equilibrium") positions $\vec{R}_1^{(0)}, \vec{R}_2^{(0)}, \ldots$

Lattice Vibrations of a Linear Chain

equilibrium positions: $R_j^{(0)} = ja_0$



oscillations: $r_j = R_j - R_j^{(0)}$



deviations from equilibrium positions

- Main interactions between nearest neighbours
- Oscillations around equilibrium positions typically **small:**

$|r_j| \ll a_0$

$$V(R_1, R_2, \ldots, R_N) = V_0 + \frac{\kappa}{2} \sum_{j=1}^{N-1} (r_j - r_{j+1})^2 + \ldots$$

◇ $V_0 = V(R_1^{(0)}, R_2^{(0)}, \ldots, R_N^{(0)})$

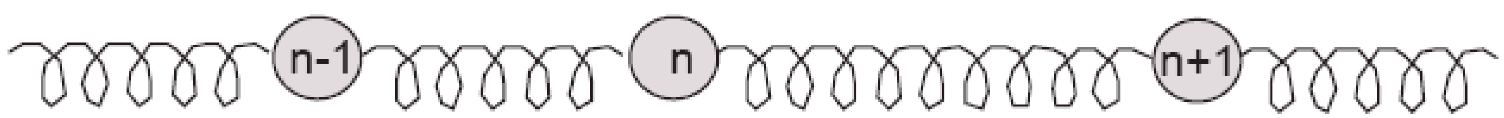
◇ The linear terms add up to zero ↔ equilibrium positions

◇ ...→ small cubic, quartic etc "anharmonic" terms

$$H = \sum_{l=1}^{N} \frac{p_l^2}{2m} + \frac{\kappa}{2} \sum_{l=1}^{N-1} (r_l - r_{l+1})^2 + \dots$$

Kinetic energy   Potential energy

→ N coupled harmonic oscillators!



Quantum mechanically:

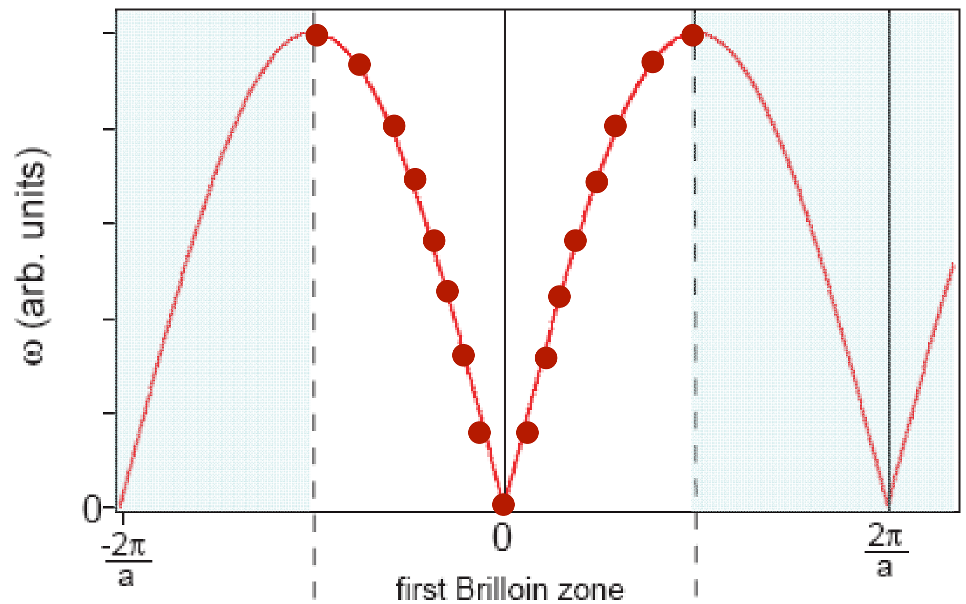
$$p_l = -i\hbar \frac{\partial}{\partial r_l}$$

Solution of the Classical Problem

Newton's equations: $$m\frac{\partial^2 r_l}{\partial t^2} = \kappa(r_{l+1} - r_l) - \kappa(r_l - r_{l-1}) \qquad l = 1, 2, \ldots, N$$

Ansatz: $$r_l(t) = A\cos(kla_0 - \omega t + \delta)$$

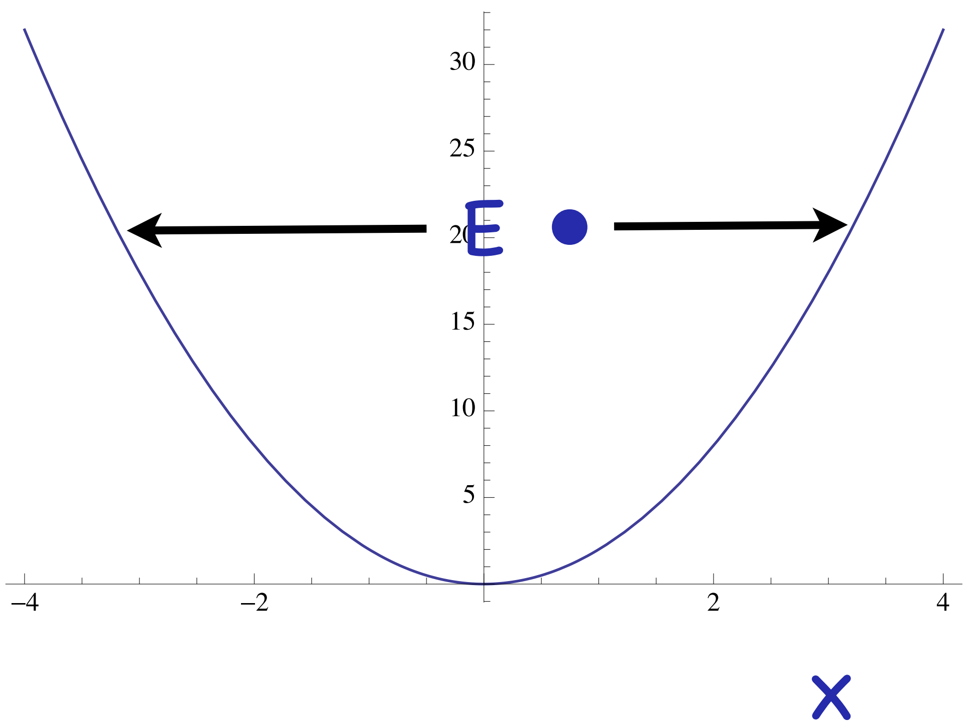
Works if $$\omega(k_l) = \sqrt{\frac{2\kappa}{m}[1 - \cos(k_l a_0)]}$$

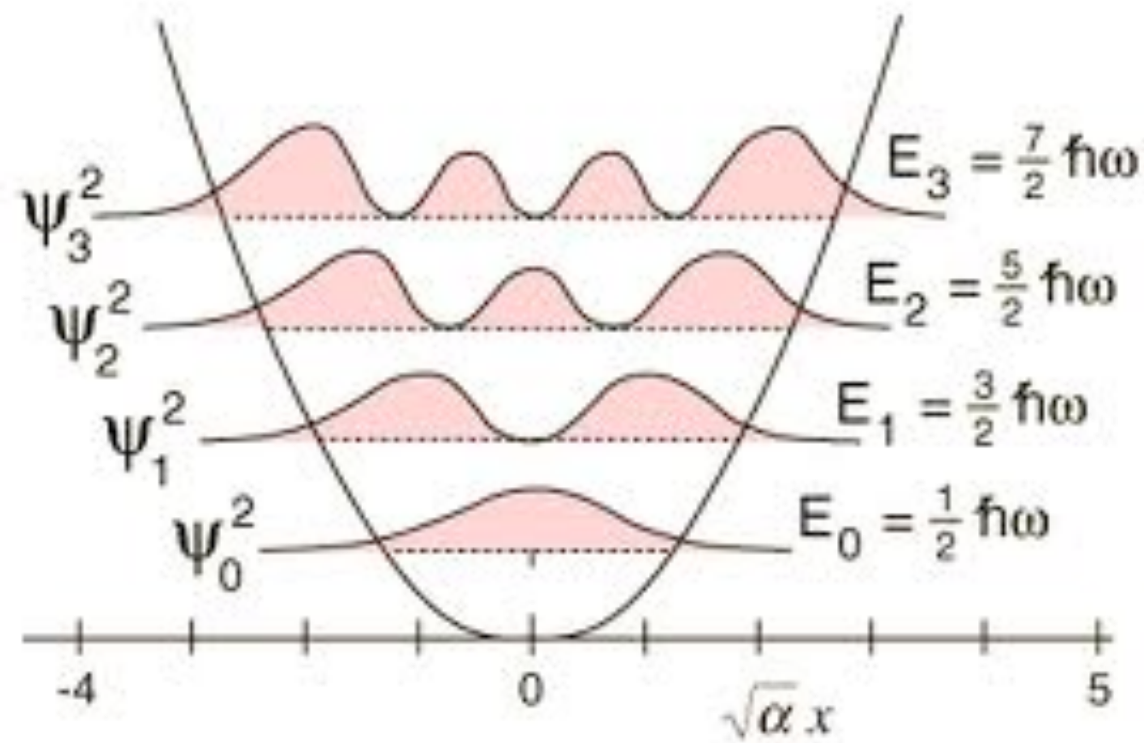


periodicity in k ↔ periodicity of the lattice finite number of atoms → finite number of **normal modes**

Quantum Mechanics

Each ω(k) gives a simple harmonic oscillator
(in an appropriate coordinate) →





quantized energies:

$$E_n(k_l) = \hbar\omega(k_l)\left[n + \frac{1}{2}\right], \quad l = 1, \ldots, N$$

Ground State ("Vacuum"):

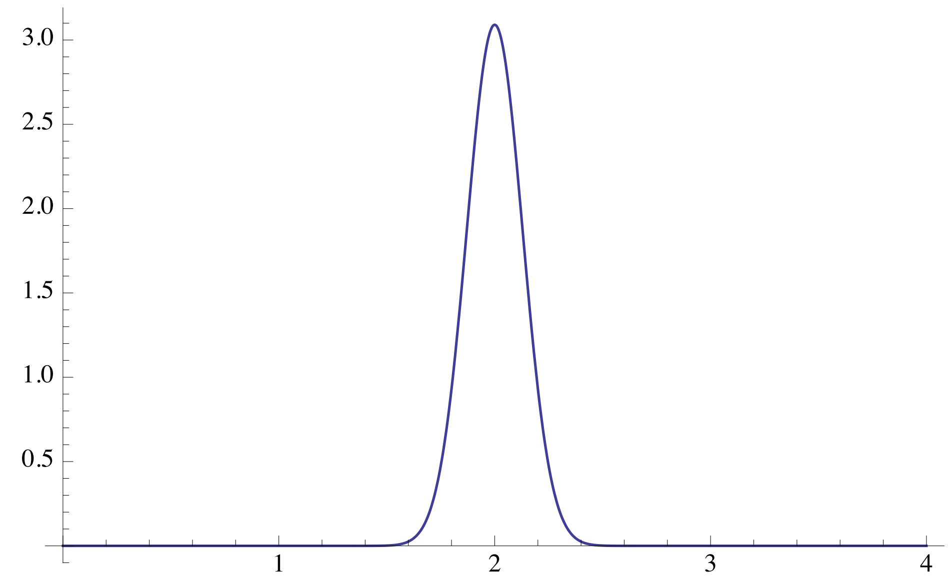
zero-point energy:

$$E_{\mathrm{GS}} = \sum_{l=1}^{N} \frac{\hbar\omega(k_l)}{2}$$

wave function:

$$\Psi(r_1, \ldots, r_N) \propto \exp\left(-\frac{1}{2} \sum_{j,k} r_j M_{jk} r_k \right)$$

Probability distr. of 2^{nd} atom



Excited states:

Single particles ("Phonons")

$$E = E_{\mathrm{GS}} + \hbar\omega(k_l)$$

wave function:

$$\Psi(r_1, \ldots, r_N) \propto \left[\sum_{n=1}^{N} \cos(k_l n) r_n\right] \Psi_{\mathrm{GS}}$$

Two phonons:

$$E = E_{\mathrm{GS}} + \hbar\omega(k_l) + \hbar\omega(k_n)$$

Quantum Field Theory

Recall that

$$H = \sum_{l=1}^{N} \frac{p_l^2}{2m} + \frac{\kappa}{2} \sum_{l=1}^{N-1} (r_l - r_{l+1})^2 + \dots$$

$$p_l = -i\hbar \frac{\partial}{\partial r_l}$$

Quantum Field Theory

Define $$\Phi(la_0) = \frac{1}{\sqrt{a_0}} r_l$$

Hamiltonian becomes $$H = a_0 \sum_{l=1}^{N} \frac{m}{2} \left[\frac{\partial}{\partial t} \Phi(la_0) \right]^2 + \frac{\kappa}{2} \left[\Phi([l+1]a_0) - \Phi(la_0) \right]^2$$

Now consider the limit $$N \to \infty \,, \quad a_0 \to 0 \,, \quad \kappa \to \infty$$

$$L = N a_0 \text{ and } \bar{\kappa} = \kappa a_0^2 \text{ fixed}$$

↑ volume

↑ vibrations remain ≪a_0

$$l a_0 \longrightarrow x$$

$$\Phi(l a_0) \longrightarrow \Phi(x)$$

$$a_0 \sum_l \longrightarrow \int_0^L dx$$

$$H \rightarrow \int_0^L dx \left[\frac{m}{2} \left(\frac{\partial \Phi}{\partial t} \right)^2 + \frac{\bar{\kappa}}{2} \left(\frac{\partial \Phi}{\partial x} \right)^2 \right]$$

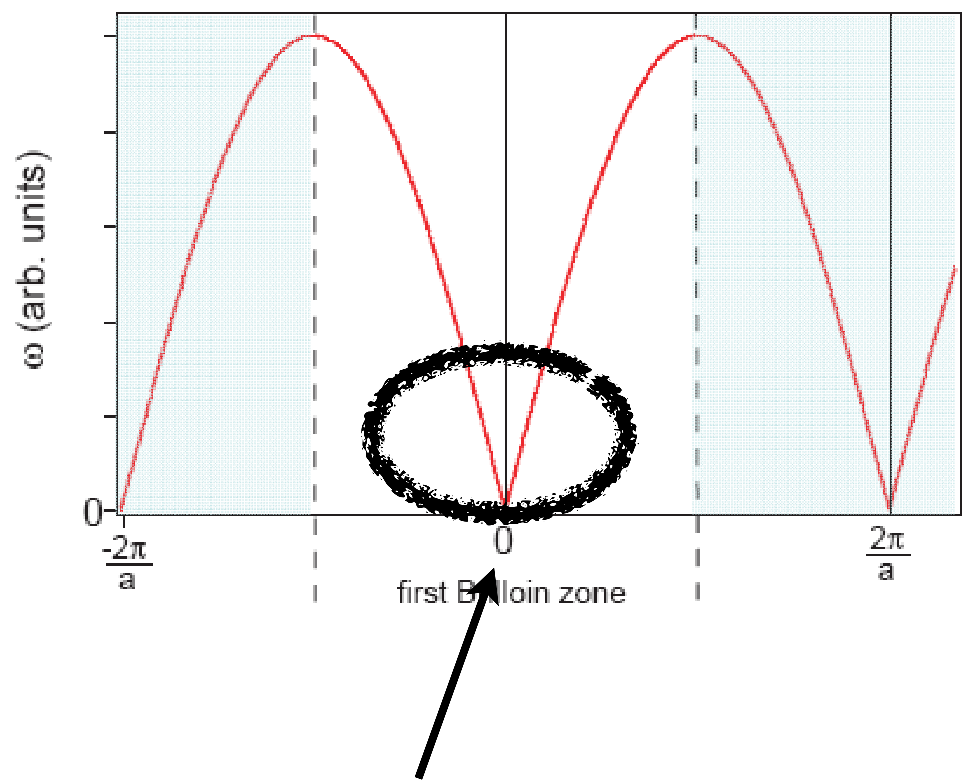
corresponding Lagrangian density

$$\mathcal{L}(t,x) = \left[\frac{m}{2} \left(\frac{\partial \Phi(t,x)}{\partial t} \right)^2 - \frac{\bar{\kappa}}{2} \left(\frac{\partial \Phi(t,x)}{\partial x} \right)^2 \right]$$

Massless (relativistic) Scalar Field

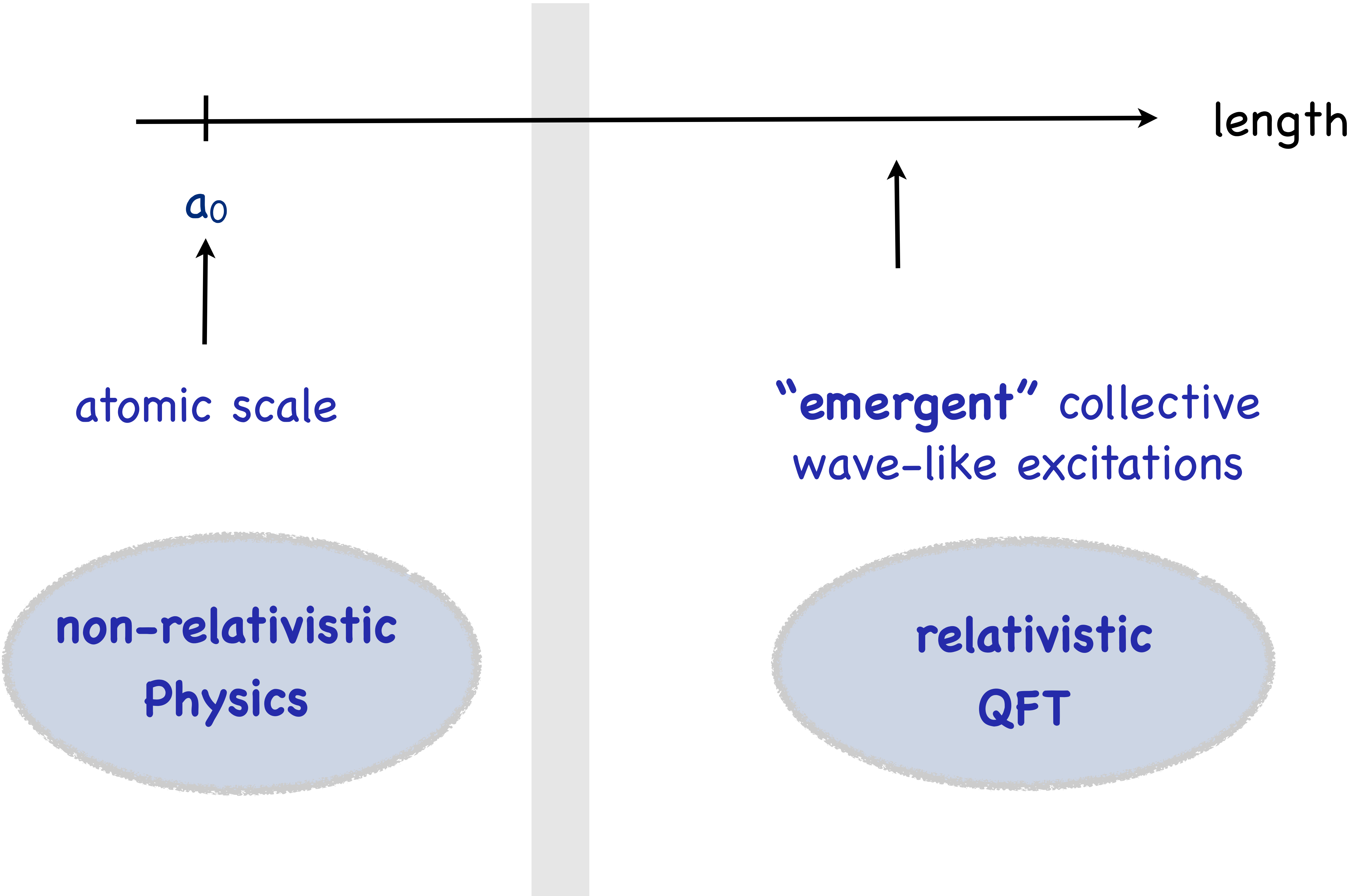
Which part of the physics does **QFT** describe?

- **Large distances** compared to "lattice spacing" a_0
- Small frequencies $\omega(k)$, i.e. **low energies**

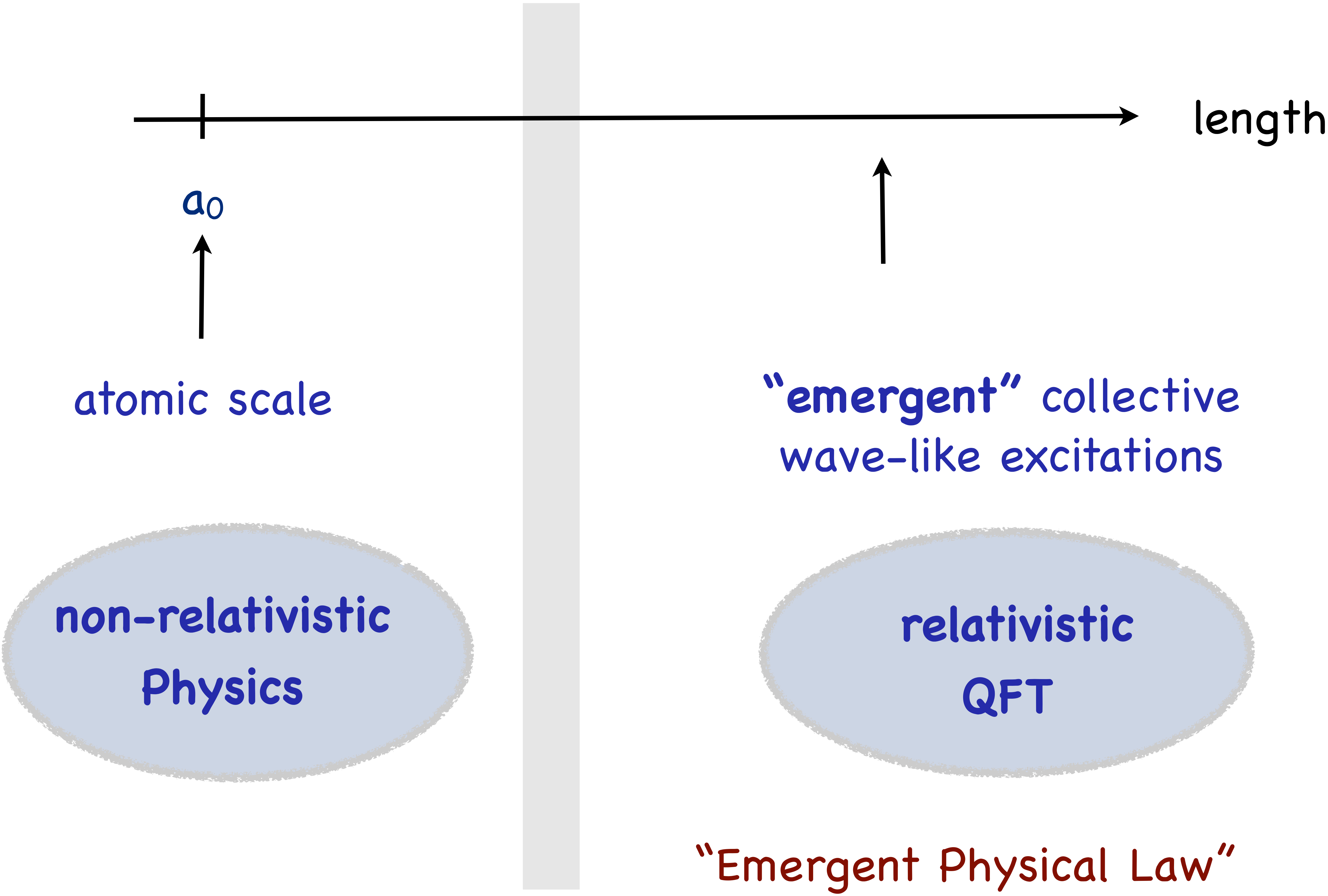


i.e. normal modes in **this** region

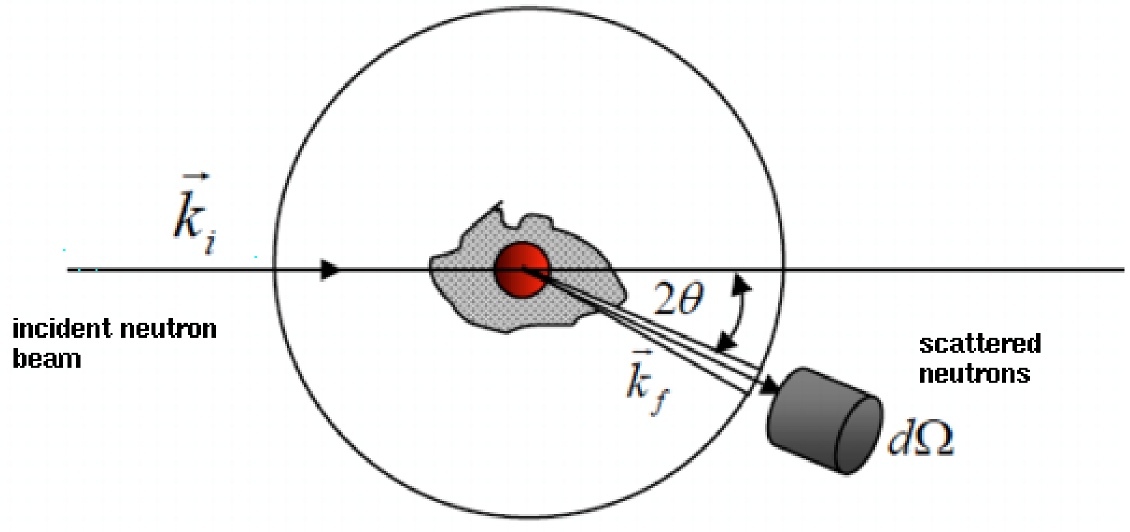
Which part of the physics does **QFT** describe?



Which part of the physics does **QFT** describe?

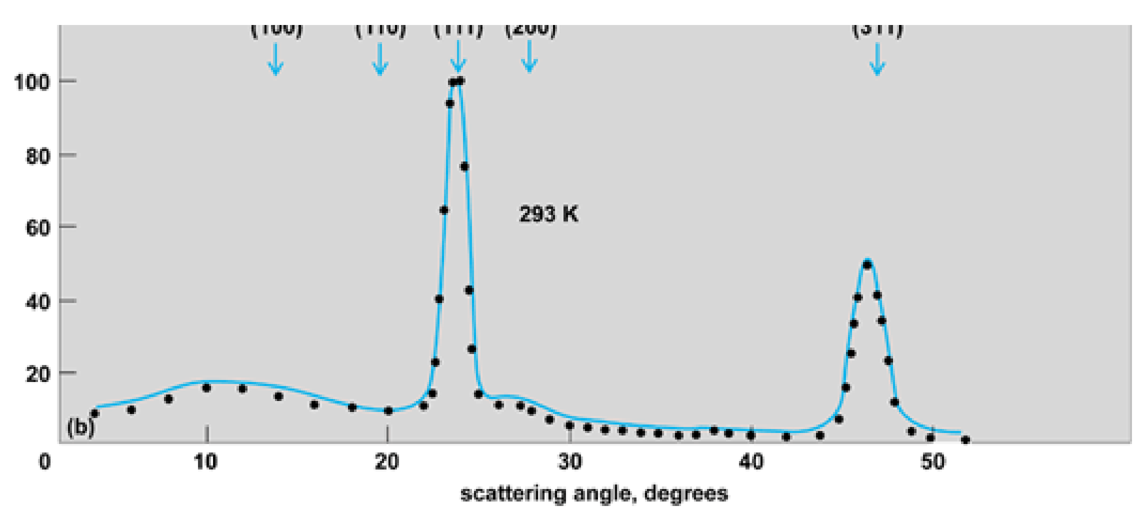


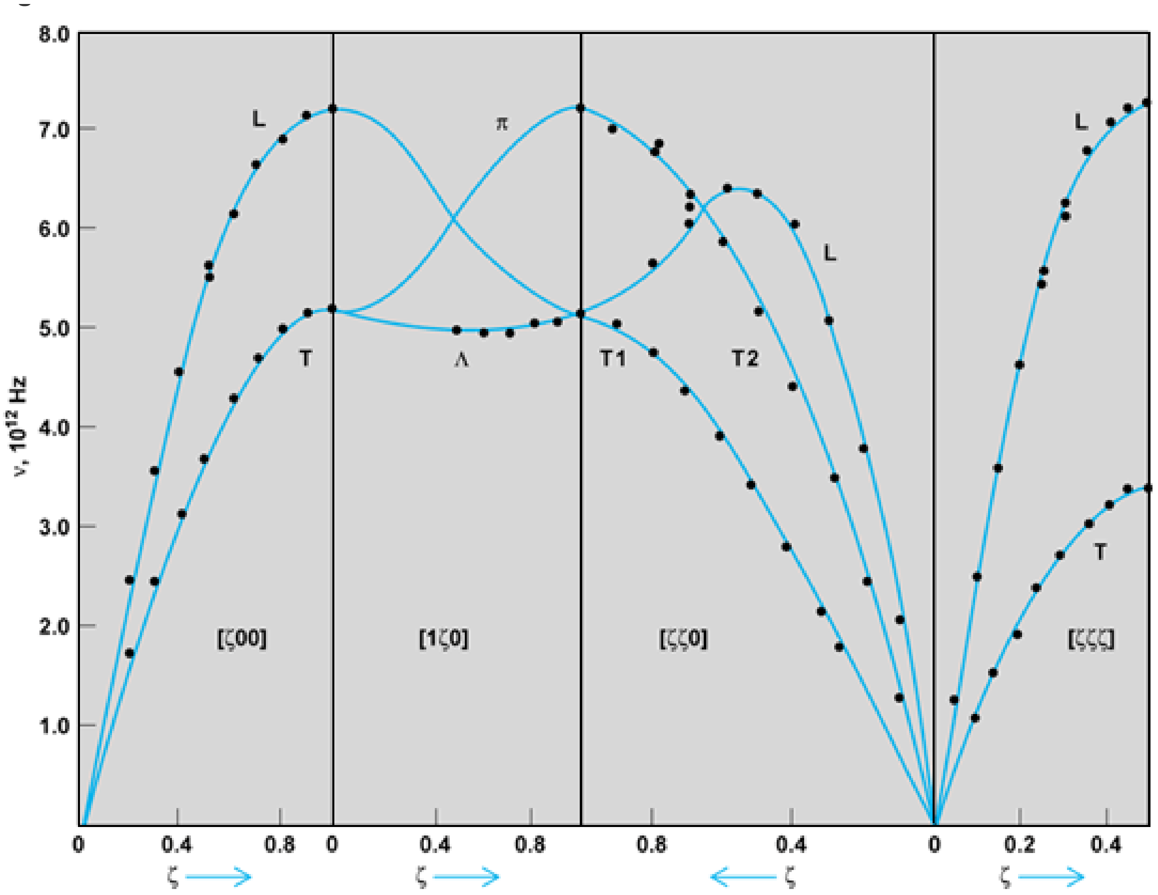
Measuring the collective modes: inelastic neutron scattering



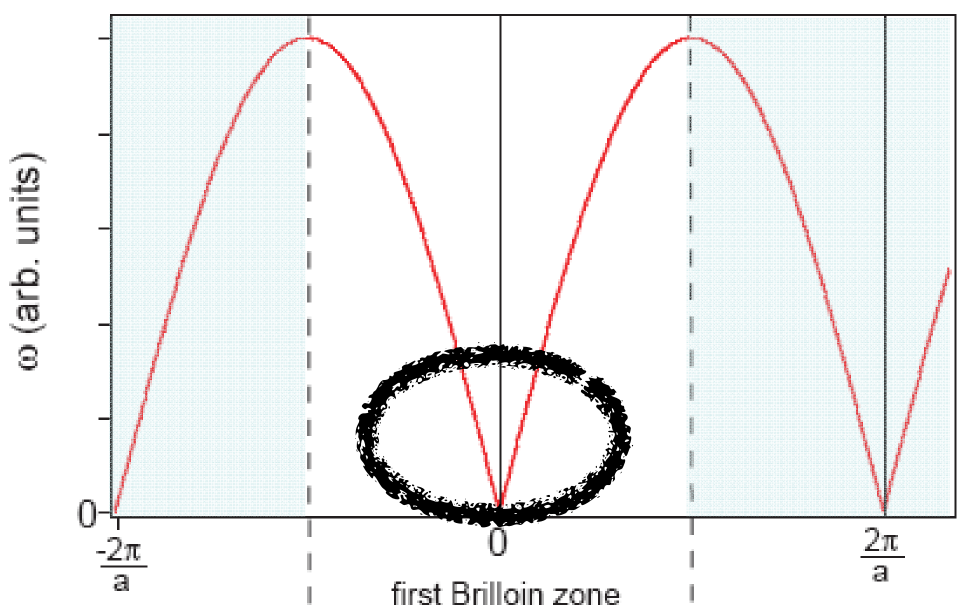
neutron gives energy ω and momentum k to the crystal

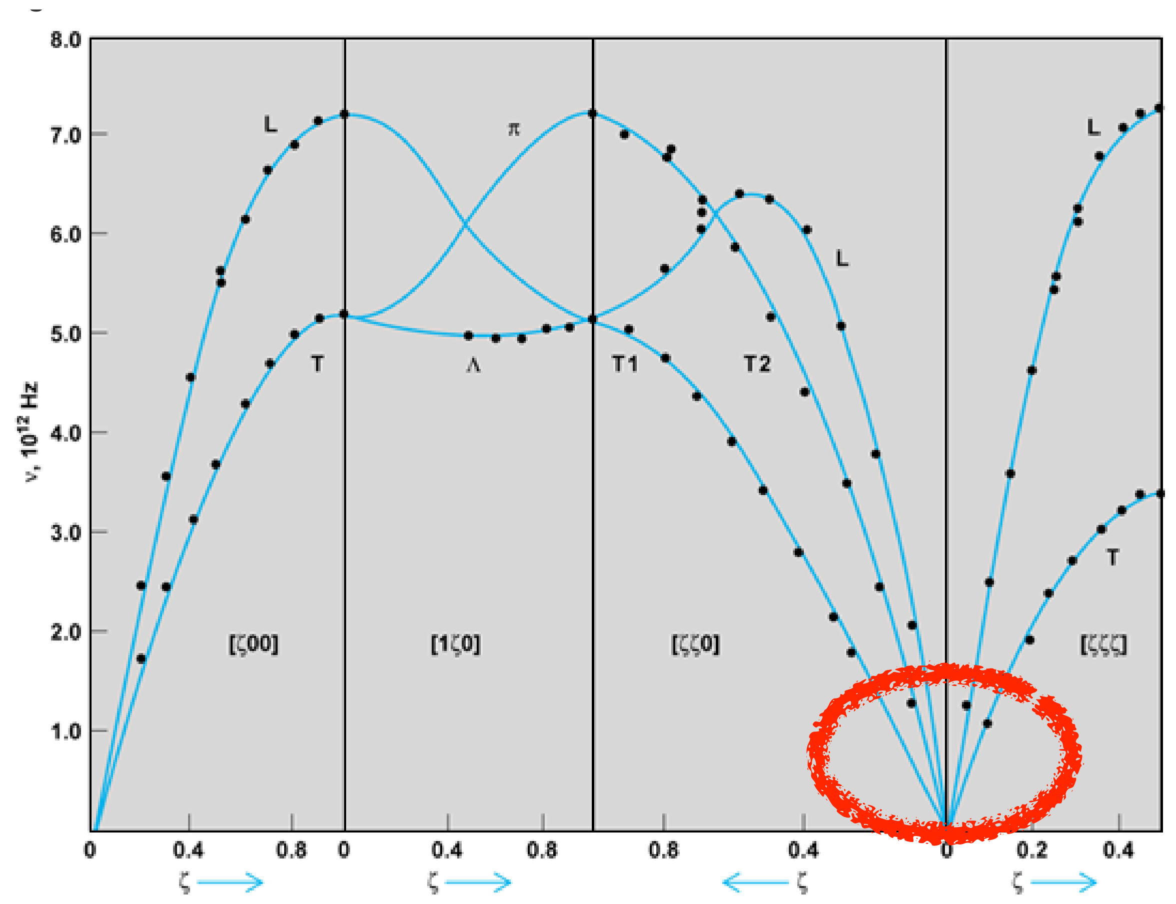
→ excites a **single** normal mode if ω and k "match"





Phonon spectrum of Cu


ω (arb. units)
0
-2π/a
0
2π/a
first Brillouin zone


8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
ν, 10^12 Hz
L
T
π
Λ
T1
T2
L
L
T
[ζ00]
[1ζ0]
[ζζ0]
[ζζζ]
0
0.4
0.8
0
0.4
0.8
0.8
0.4
0
0.2
0.4
ζ
ζ
ζ
ζ

Main Points so far

- Quantum mechanics of atoms in solids can give rise to **collective excitations**, which are described by a QFT
- The lowest energy state ("vacuum") has a non-trivial wave function.

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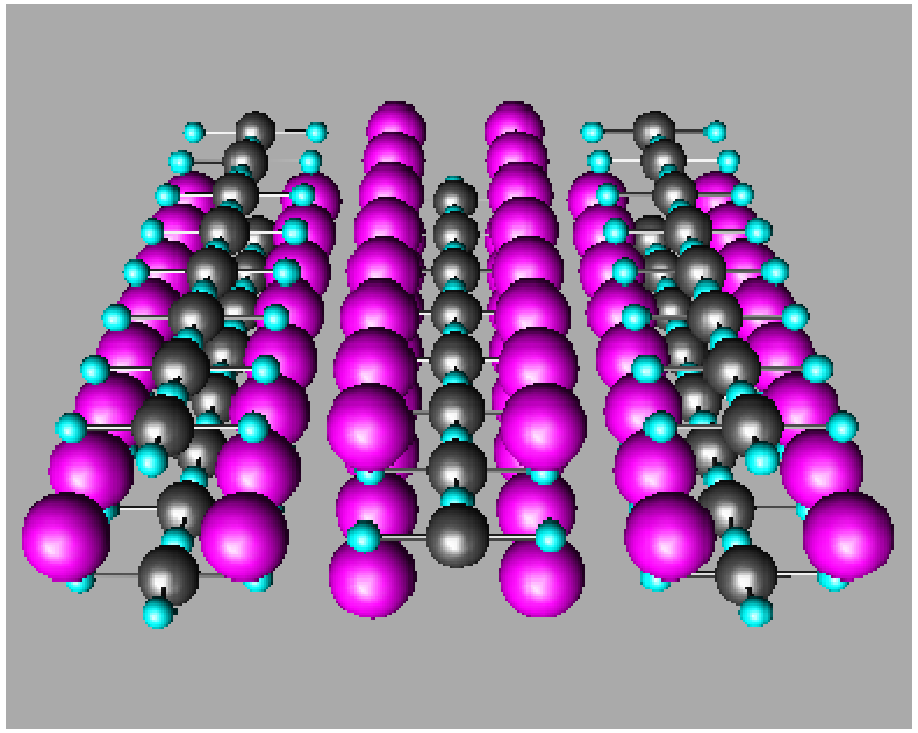
- Quantum mechanics of atoms in solids can give rise to **collective excitations**, which are described by a QFT
- The lowest energy state ("vacuum") has a non-trivial wave function.

Much more exotic physics and QFTs can arise in this way!

"Splitting the Electron"

Consider electronic degrees of freedom in "quasi-1D crystals"

Sr_2CuO_3

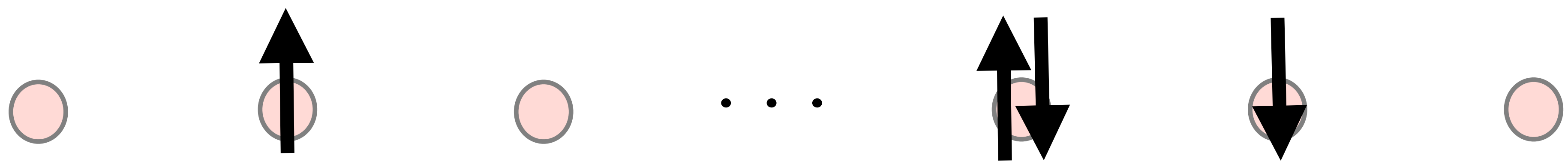


low energy electronic physics due to outer electrons on Cu atoms (black)

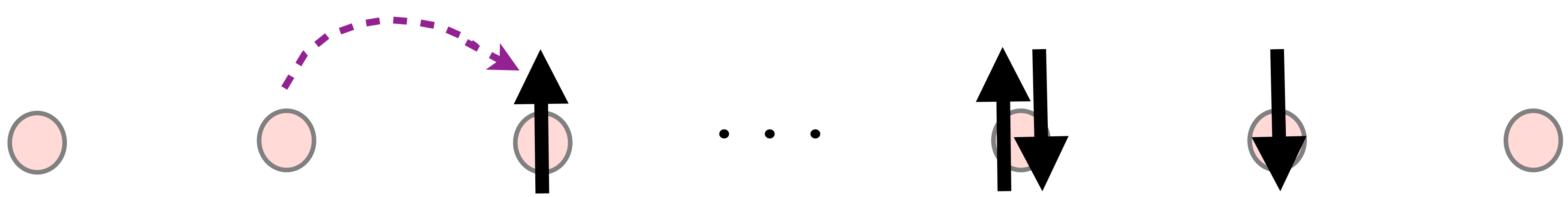
anisotropy → e^- move essentially only along 1D chains

Basic model:

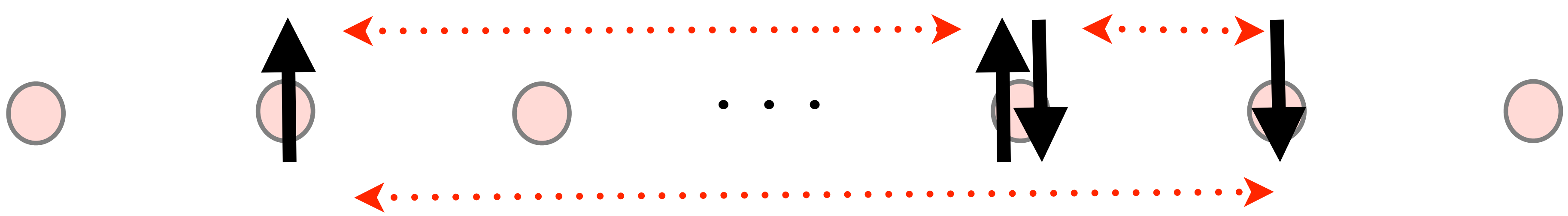
Lattice: on each site either 0, 1 or 2 electrons (spin!)



Electrons can hop to neighbouring sites



Electrons repel through Coulomb interaction



Field Theory Limit: description in terms of  

$$\Psi_1(t,x) = \begin{pmatrix} R_\uparrow(t,x) \\ L_\uparrow(t,x) \end{pmatrix}, \qquad \Psi_2(t,x) = \begin{pmatrix} R^\dagger_\downarrow(t,x) \\ L^\dagger_\downarrow(t,x) \end{pmatrix}$$

fermionic quantum fields

2 species of 2-dimensional Dirac spinors

$$\mathcal{L}(t,x) = \sum_{a=1}^{2} \bar{\Psi}_a(t,x) \left[i\gamma^0 \frac{\partial}{\partial t} - i\gamma^1 \frac{\partial}{\partial x} \right] \Psi_a(t,x) + g \sum_{\alpha=1}^{3} J_1^\alpha(t,x) J_1^\alpha(t,x) - J_0^\alpha(t,x) J_0^\alpha(t,x)$$

$$\bar{\Psi}(t,x) = \Psi^\dagger(t,x)\gamma^0$$

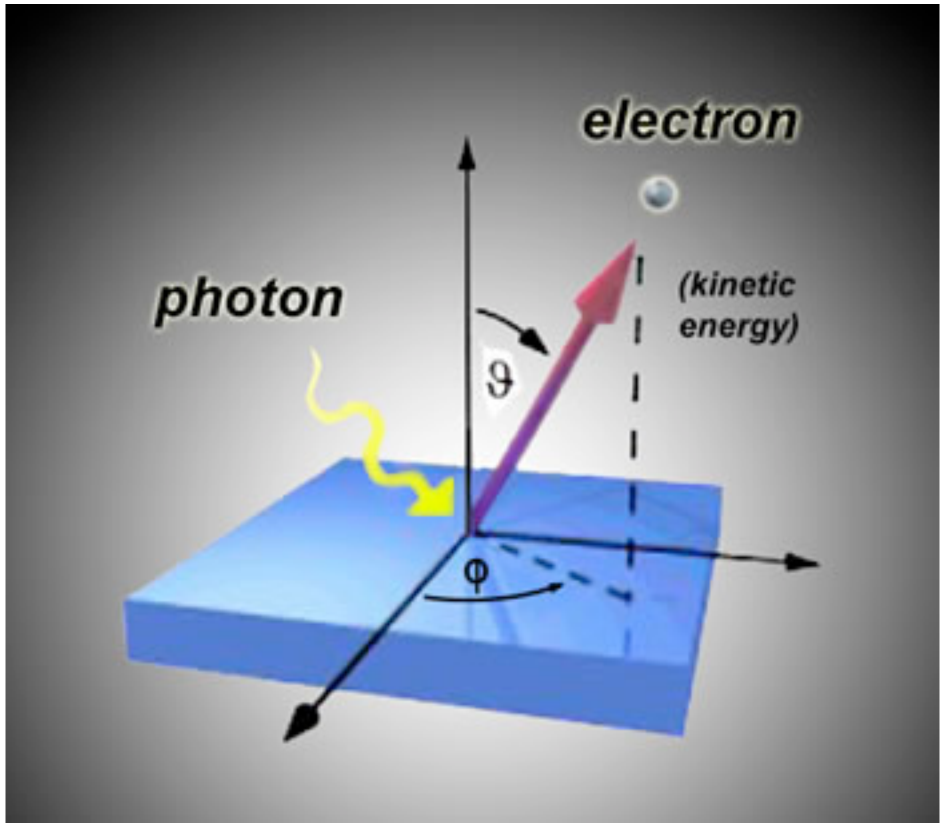
$$J_\mu^\alpha = \frac{1}{2} \bar{\Psi}_a \gamma_\mu \sigma_{ab}^\alpha \Psi_b \,, \qquad \mu = 0,1$$

$$\gamma^0 = -\gamma_0 = i\sigma^y = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\gamma^1 = \gamma_1 = \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

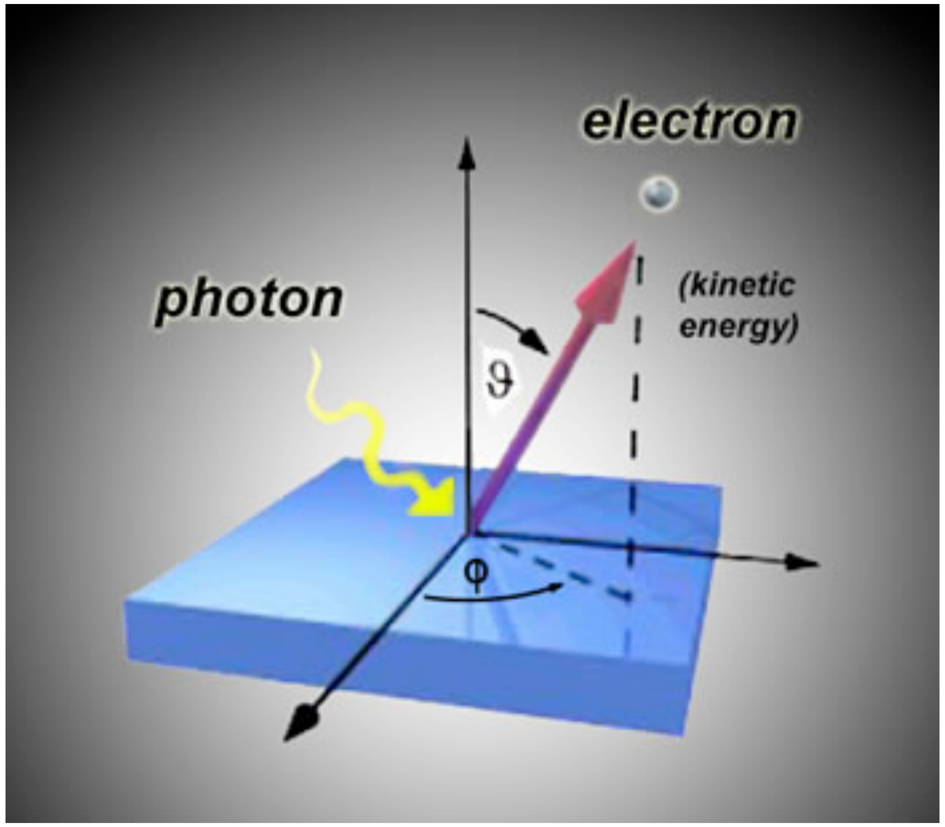
"SU(2) Thirring Model"

Measure excitations by scattering photons
(**A**ngle **R**esolved **P**hoto **E**mission **S**pectroscopy)

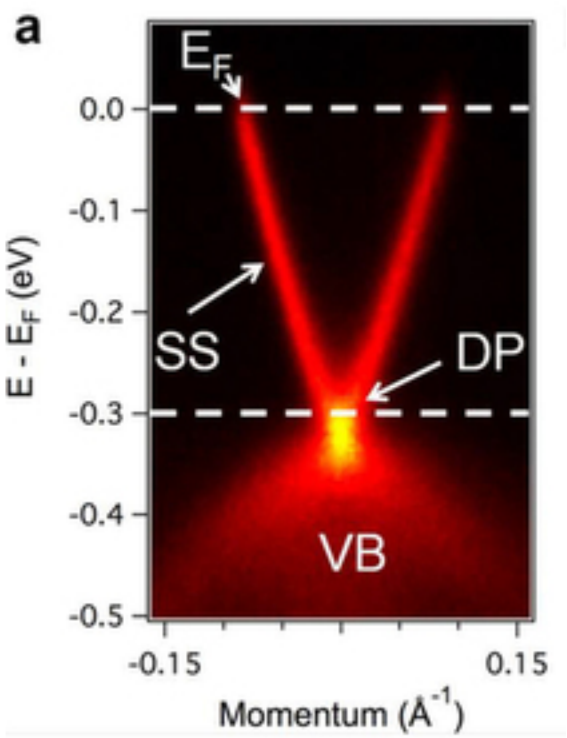


Count emitted electrons with given energy and momentum (↔angles)

Measure excitations by scattering photons
(**A**ngle **R**esolved **P**hoto **E**mission **S**pectroscopy)



If the collective excitations are electrons, we expect to see something like this

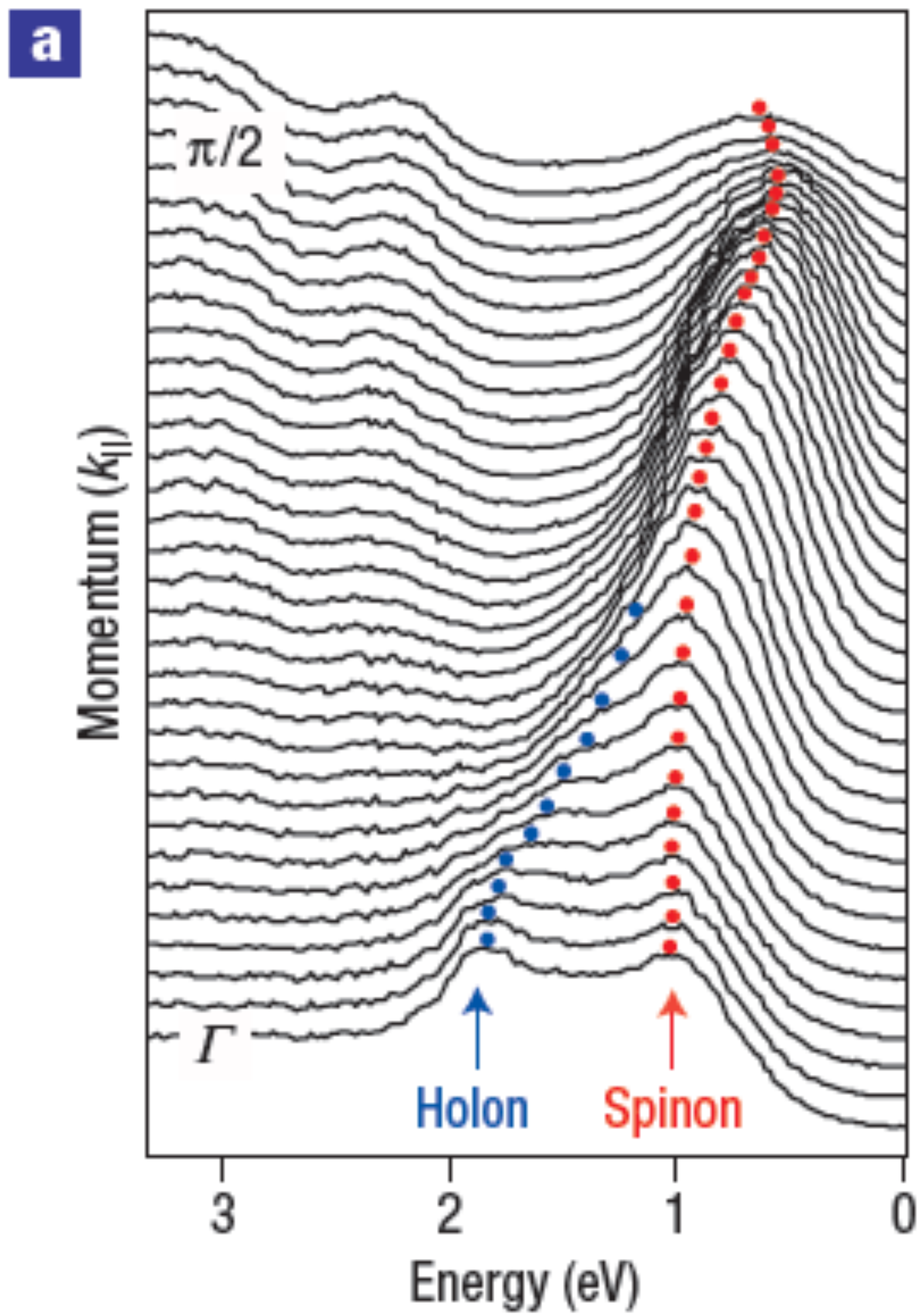


Spectral Function : ARPES on $SrCuO_2$

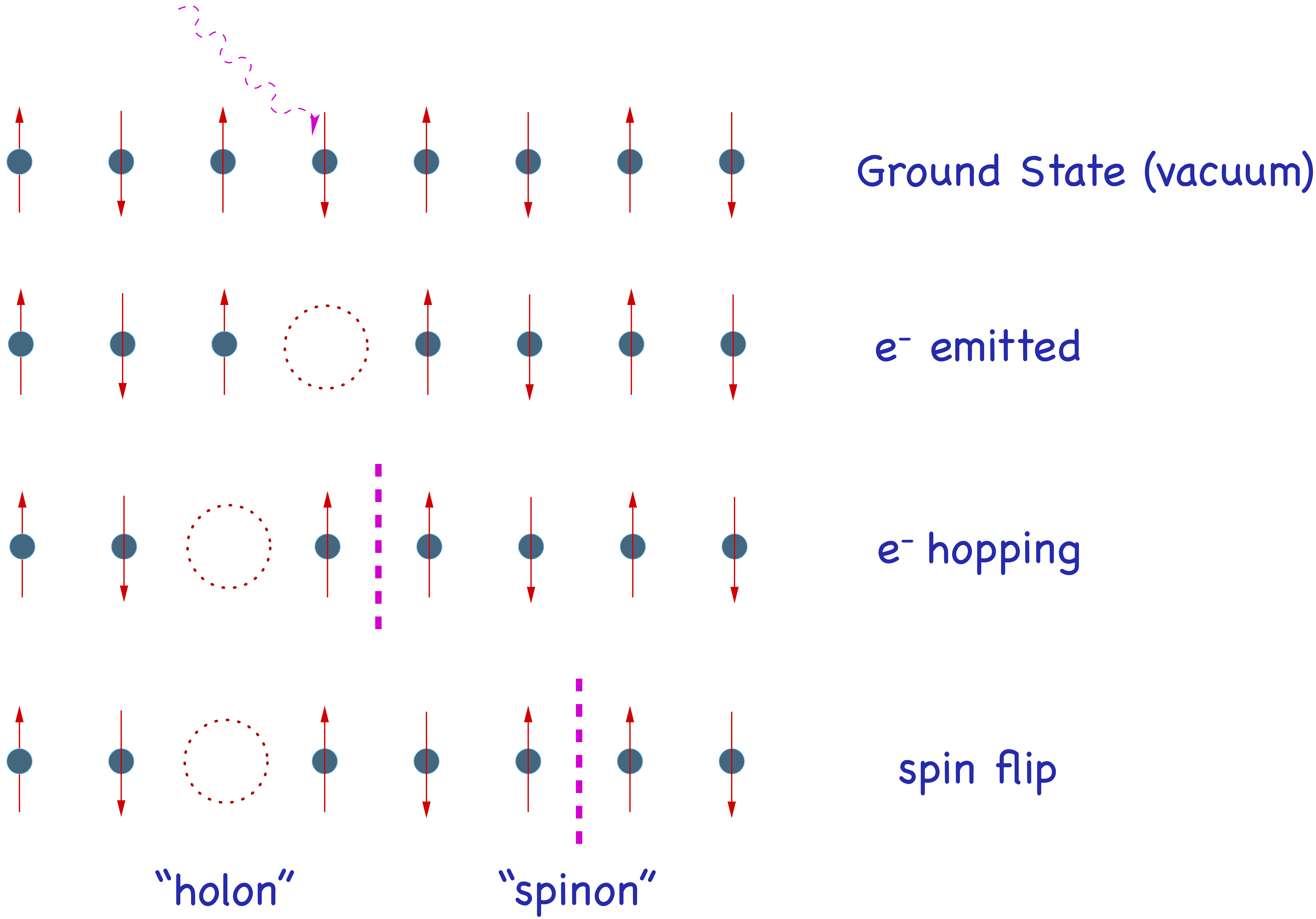
(Kim et al '06)

Instead, for quasi-1D systems one observes

The electron has fallen apart! ("spin-charge separation")



How to understand this? In Sr_2CuO_3 we have 1 electron per site



The origin for this "splitting of the electron" is the highly non-trivial nature of the ground state (vacuum)!!

Summary

- Quantum Field Theories often describe **collective** properties of solids at large distances/low energies.
- Lorentz covariance can be an **emergent** feature in this regime.
- Low-energy physics can be very exotic because the "vacuum" is highly non-trivial.
- In the Cond. Mat. context it is possible to vary the spatial dimensionality D=1,2,3 (anisotropy!).