Gravitational lensing: one of the sharpest tools in an astronomers toolbox

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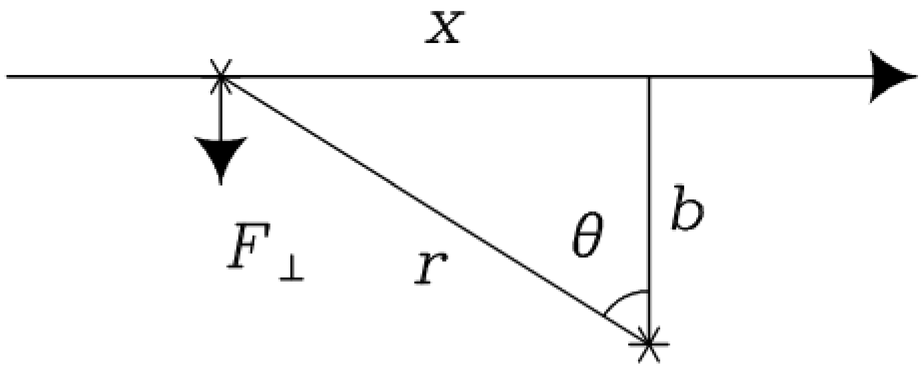
Outline

- Physics of gravitational deflection of light, Newton v. Einstein
  - Weak-field GR – a less geometrical view
  - A non-trivial refractive index for the vacuum
  - A simple relation between deflection and surface density of matter
- Deflection by galaxies and clusters
  - Lensed quasars, weak lensing
- Deflection by stars
  - microlensing
- Eddington 100 years on - Gaia

Light deflection as a consequence of the strong principle of equivalence

- In frame of a freely falling capsule, there's no gravity
- Moving clocks go slow: as v -> c proper time required to whiz past star -> 0
- So photon won't move far during flyby
- So photon trajectory -> trajectory of capsule

Newtonian deflection



$$F_\perp = \frac{GMm}{r^2}\cos\theta = \frac{GMm}{r^2}\frac{b}{r}$$

$$mv_\perp = \int_{-\infty}^{\infty} \mathrm{d}t\, F_\perp = GMmb \int \frac{\mathrm{d}t}{r^3} \simeq GMmb \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{[(vt)^2 + b^2]^{3/2}}$$

$$= \frac{2GMm}{bv} \int_0^\infty \frac{\mathrm{d}\zeta}{(\zeta^2+1)^{3/2}} \qquad (\zeta \equiv vt/b)$$

$$= \frac{2GMm}{bv}$$

$$\alpha = \frac{v_\perp}{v} = \frac{2GM}{bv^2} = \frac{r_s}{b}\frac{c^2}{v^2} \qquad \left(r_s \equiv \frac{2GM}{c^2}\right)$$

For $b = R_\odot = 7 \times 10^5\,\mathrm{km} = 2.3 \times 10^5 r_s$ have $\alpha = 0.875\,\mathrm{arcsec}$

Deflection in GR

- In 1919 Eddington measure *twice* this Newtonian estimate in agreement with Einstein's prediction
- Photons move fast, and gravity like emag is a speed-dependent force

$$m_0 \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} = qF^\mu{}_\nu u^\nu \qquad \text{(emag)}$$
$$m_0 \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} = -m_0 \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda \qquad \text{(gravity)}$$

- You need to use GR when either grav field is strong, or motion is relativistic

Weak-field gravity

- The Einstein field equations are obtained by writing physics in an arbitrary curvilinear coordinate system
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Weak-field gravity

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- So we have complete liberty in choosing our coordinates!
- *Liberty is precious; so precious that it must be rationed* (V.I.Lenin)
- Judicious choice of coordinates lies at the heart of theoretical physics
- *All coordinate systems may be equal, but some systems are more equal than others* (G. Orwell?)

Harmonic coordinates (most =)

- High school mechanics is based on inertial Cartesian coordinates
- In GR these don't exist but harmonic coordinates do:
    - $g^{\mu\nu}\Gamma^{\alpha}_{\mu\nu} = 0$ a 1st order d.e. to be satisfied by g
    - A "gauge condition" like Coulomb gauge div A = 0 in emag
- In harmonic coordinates, metric of a weak field is

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu} \qquad \text{with} \qquad |h| \ll 1$$

Newton's potential

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad h_{00} = -\left(1 + \frac{2\Phi}{c^2}\right) \quad h_{ii} = \left(1 - \frac{2\Phi}{c^2}\right)$$

In harmonic coordinates

Newton's potential

- Metric for a weak gravitational field is

$$h_{00} = -\left(1 + \frac{2\Phi}{c^2}\right) \quad h_{ii} = \left(1 - \frac{2\Phi}{c^2}\right)$$

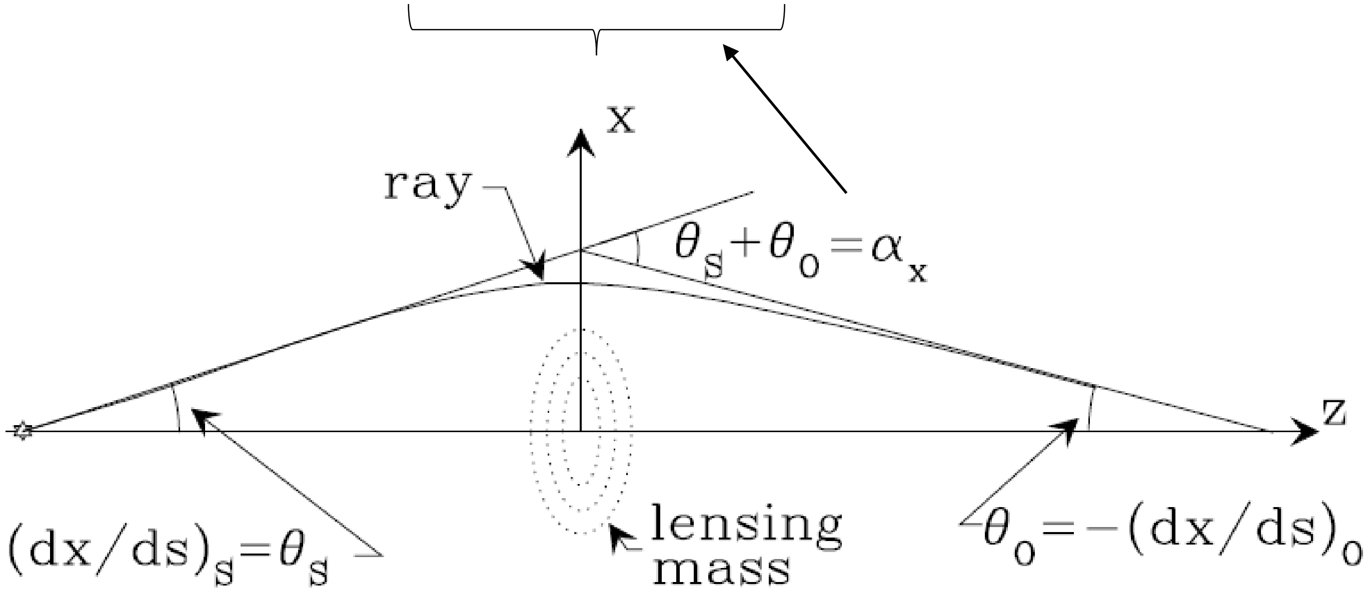
- For photons $ds^2=0$ so "speed" of photon is

$$\frac{\sqrt{\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2}}{\mathrm{d}t} = \sqrt{\frac{1 + 2\Phi/c^2}{1 - 2\Phi/c^2}}c \simeq \frac{c}{1 - 2\Phi/c^2} < c$$

- In glass v=c/n with n>1 refractive index
- So gravity induces a refractive index $1 - 2\Phi/c^2 > 1$
- (Photons don't really slow down but in a potential well there's more space to cover than harmonic coordinates imply)

Fermat’s principle of least time

- The phase of emag field will be stationary, and constructive interference will occur, along paths on which $t = \int \mathrm{d}s\, n/c$ is stationary
- One finds $$\left.\frac{\mathrm{d}x}{\mathrm{d}s}\right|_{\mathrm{obs}} - \left.\frac{\mathrm{d}x}{\mathrm{d}s}\right|_{\mathrm{source}} = \int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}s\, \frac{\partial n}{\partial x}.$$



Bottom line

- Deflection angle is
  - $$\alpha = -\int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}s\, \nabla_\perp n = \frac{2}{c^2}\int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}s\, \nabla_\perp \Phi$$ (a 2-vector)
- Using $\nabla^2\Phi = 4\pi G\rho$ surface density of deflecting matter is

$$\Sigma = \int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}z\, \rho = \frac{1}{4\pi G}\int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}z \left(\nabla_\perp^2 \Phi + \frac{\partial^2 \Phi}{\partial z^2}\right) \simeq \frac{1}{4\pi G}\int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}z\, \nabla_\perp^2 \Phi$$

- Taking divergence of α

$$\nabla_\perp \cdot \alpha = \frac{2}{c^2}\int_{\mathrm{S}}^{\mathrm{O}} \mathrm{d}z\, \nabla_\perp^2 \Phi = \frac{8\pi G}{c^2}\Sigma$$

- Thus: *divergence of α gives surface density of intervening matter on sky*

Stars, galaxies & clusters of galaxies

- Apply $\nabla_\perp \cdot \alpha = \dfrac{8\pi G}{c^2}\Sigma$
- Stars: put $\Sigma = M\delta(\mathbf{r})$, integrate both sides and use 2d divergence theorem

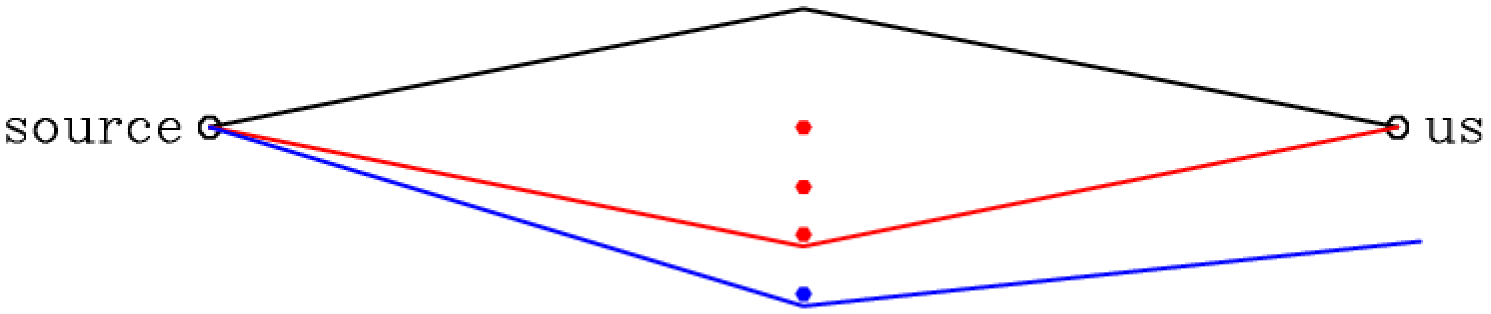
$$\int \mathrm{d}^2\mathbf{r}\,\nabla\cdot\alpha = 2\pi r\alpha(r) = \frac{8\pi G}{c^2}M \quad\to\quad \alpha(r) = \frac{4GM}{rc^2} = 2\frac{r_s}{r}$$

- Galaxies etc: $v_c = \mathrm{const}$, $\rho = v_c^2/(4\pi G r^2)$, $\Sigma(r) = v_c^2/(4Gr)$, so

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}(r\alpha_r) = \frac{2\pi v_c^2}{c^2 r} \quad\Rightarrow\quad \alpha = 2\pi\frac{v_c^2}{c^2} \quad \text{a constant}$$

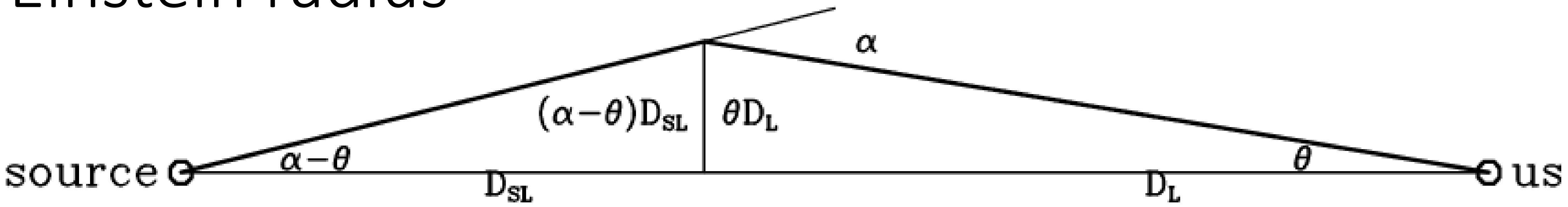
Lensing by galaxies (~isothermal spheres)

- For singular isothermal sphere, only one image until lens close to line of sight:



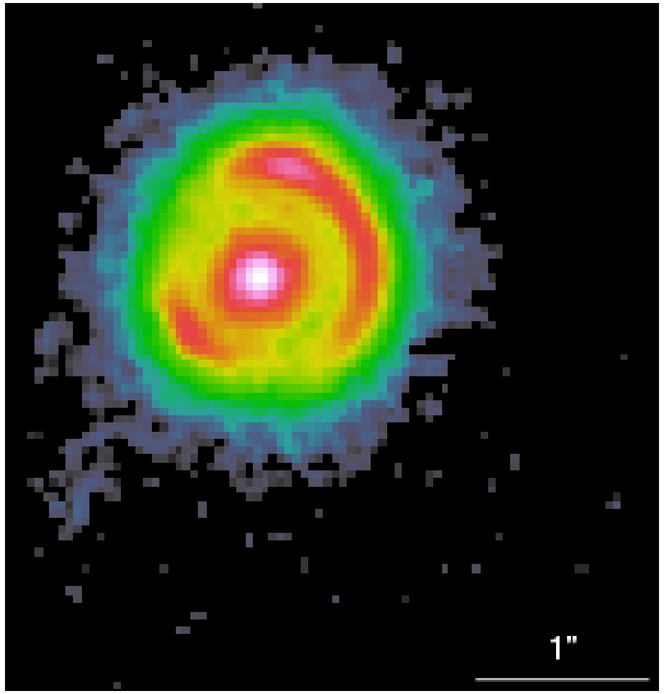
- Thereafter 2 images separated by constant angle but lower image increases in brightness as lens approaches line of sight
- When lens exactly in front of source, image becomes "Einstein ring"

Einstein radius



$$(\alpha - \theta) D_{\rm SL} = \theta D_{\rm L}$$

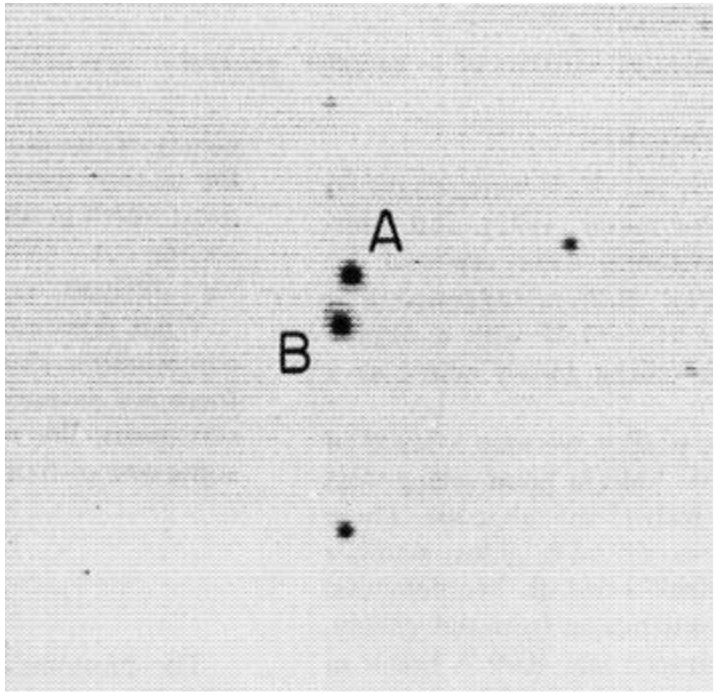
$$\Rightarrow \quad \theta = \frac{D_{\rm SL}}{D_{\rm S}} \alpha = \frac{D_{\rm SL}}{D_{\rm S}} 2\pi \frac{v_c^2}{c^2} = \frac{2 D_{\rm SL}}{D_{\rm S}} \left(\frac{v_c}{1000\,{\rm km\,s^{-1}}} \right)^2 \times 7.2\,{\rm arcsec}$$



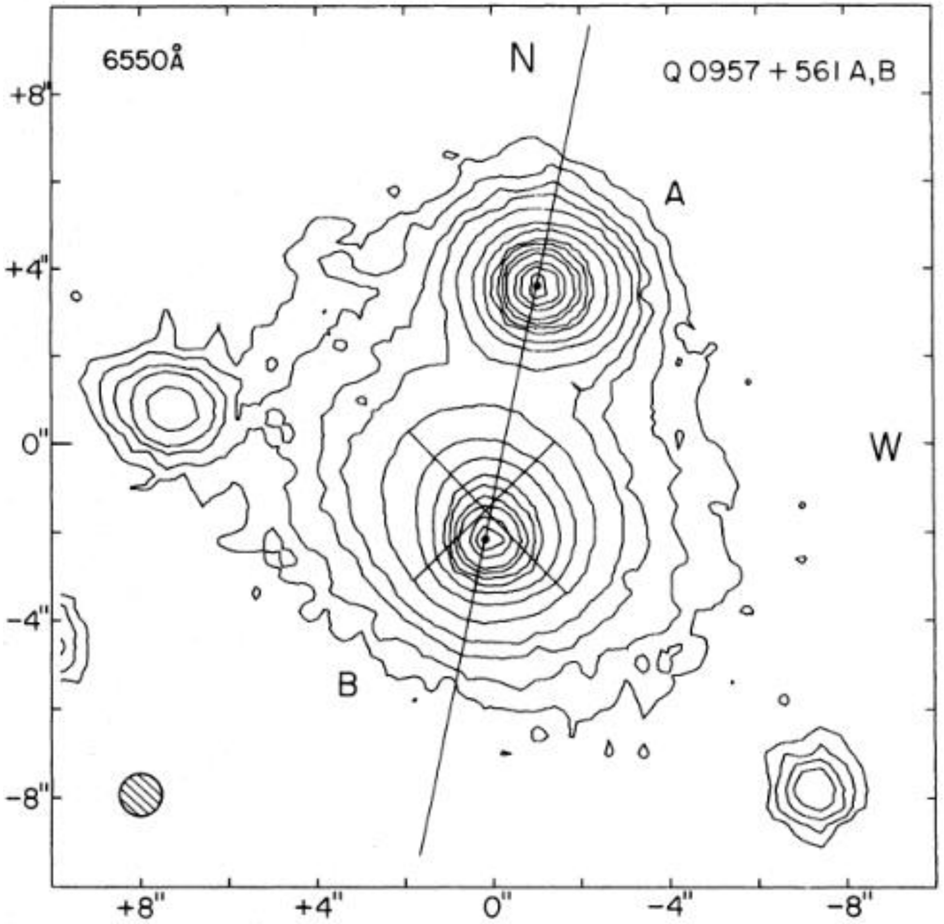
B0631+519

First lensed object: Q0957+561

- 1979 Walsh, Carswell & Weymann (Manchester/Cambridge/AZ)
- Quasars 5.7” apart, both at z=1.405
- Lensed by cluster at z=0.39

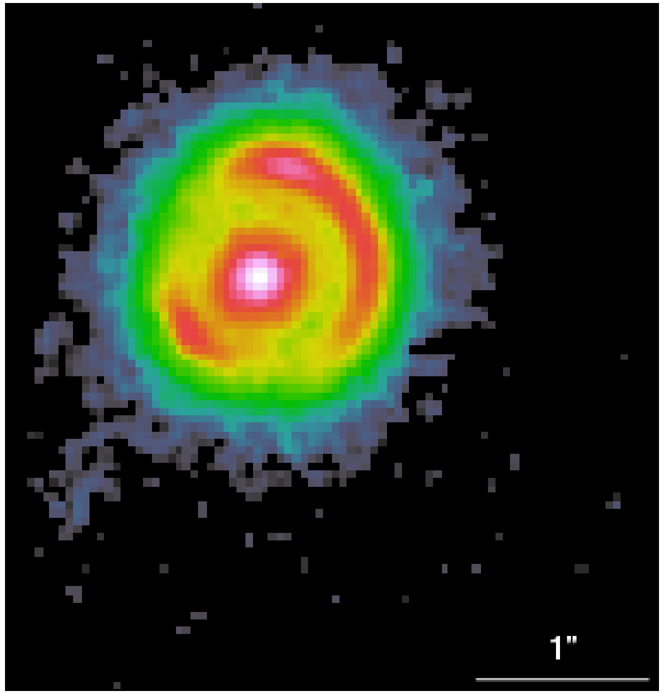


Young et al 1980



Following up Walsh et al (1979)

- Systematic searches for lensed AGN have found a few dozen
- VLA + HST most powerful combination
- See CASTLe site www.cfa.Harvard.edu/castles
- >2 images because galaxies & clusters have ellipsoidal mass distributions



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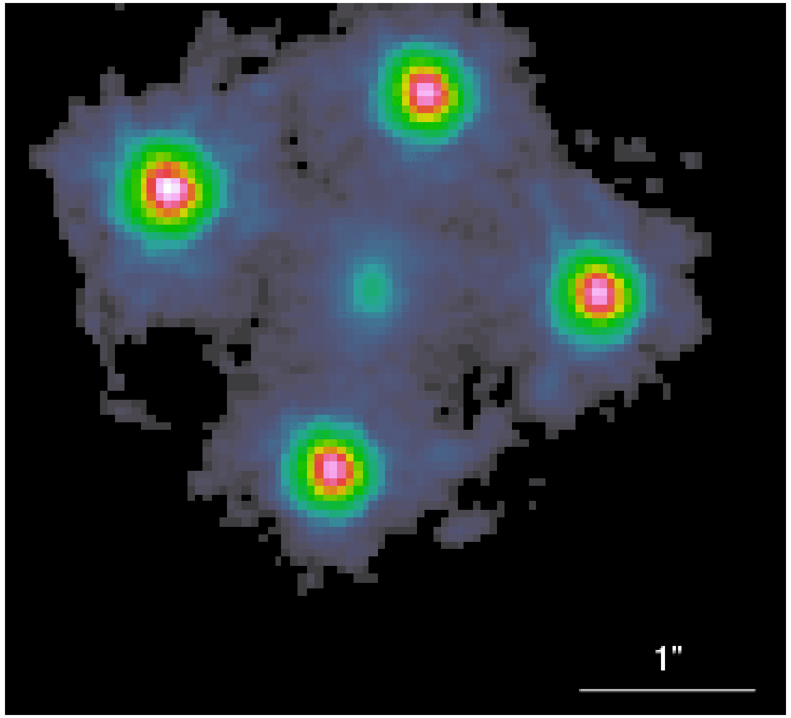

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Weak lensing

- Outside Einstein radius there's only 1 image
- But it's stretched perpendicular to the local gravitational field
- This example is extreme
- By measuring small distortions of millions of galaxy images, we hope to learn how clustering has proceeded
- A key probe of "dark energy"

Cluster A2218 imaged by HST

Deflection by stars

- Since α = 2r_s/r a point-mass deflector will always produce 2 images – light can pass either side of the deflector
- Angles from observer-deflector line are solutions of a quadratic

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta + 4\theta_{\rm E}}\right) \qquad \text{where} \qquad \theta_{\rm E} \equiv \sqrt{\frac{2D_{\rm SL}r_s}{D_{\rm S}D_{\rm L}}}$$

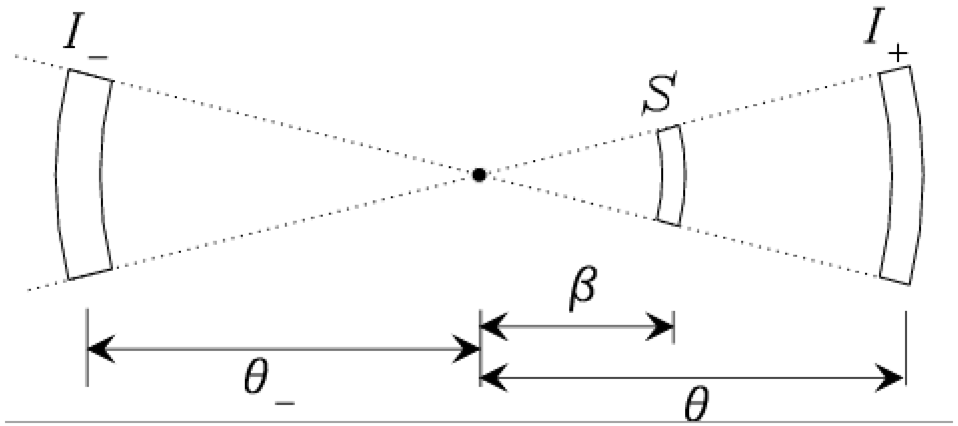
and β is the angle between source & deflector

- Unless β <~ θ_E, one image v faint

$$\theta_{\rm E} \simeq \sqrt{r_s/D_{\rm L}} \simeq \sqrt{3\,{\rm km}/10\,{\rm kpc}} \sim 0.7\,{\rm mas}$$

- α~1 mas, too small for even HST to resolve images
- But towards Galactic centre ~1 star in a million is being lensed

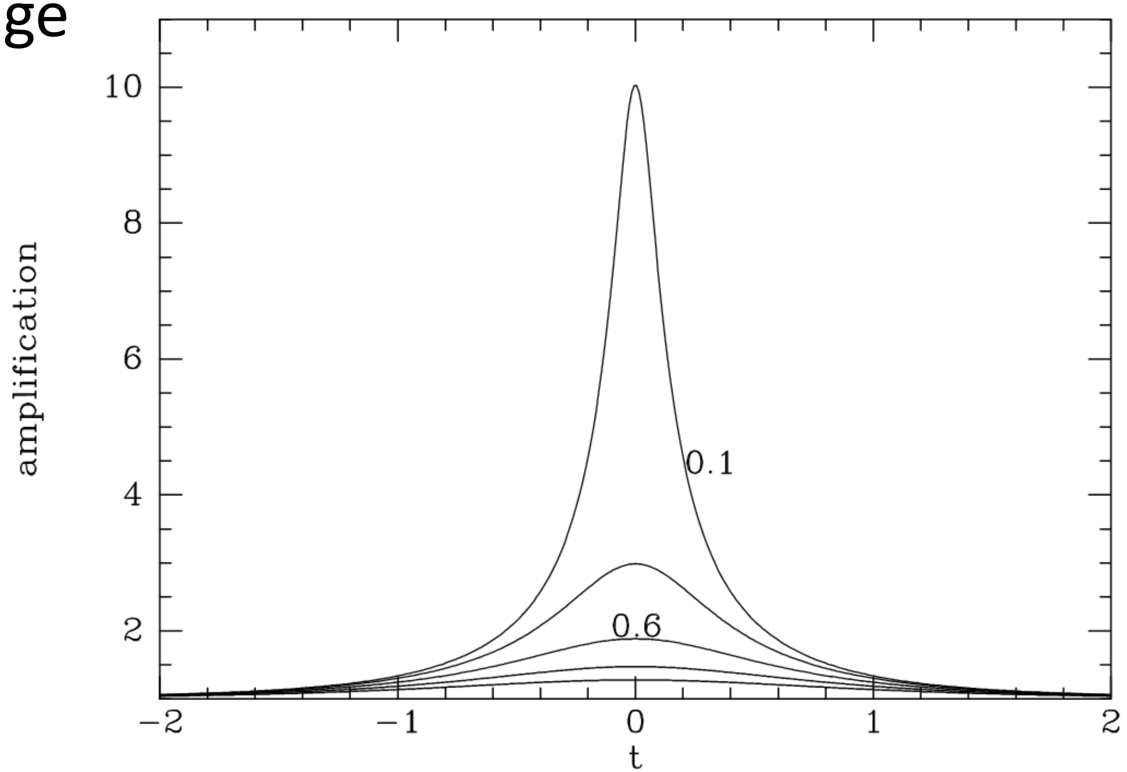
Brightness of images



- Surface brightness of objects unchanged by lensing
- So brightness proportional to area of image

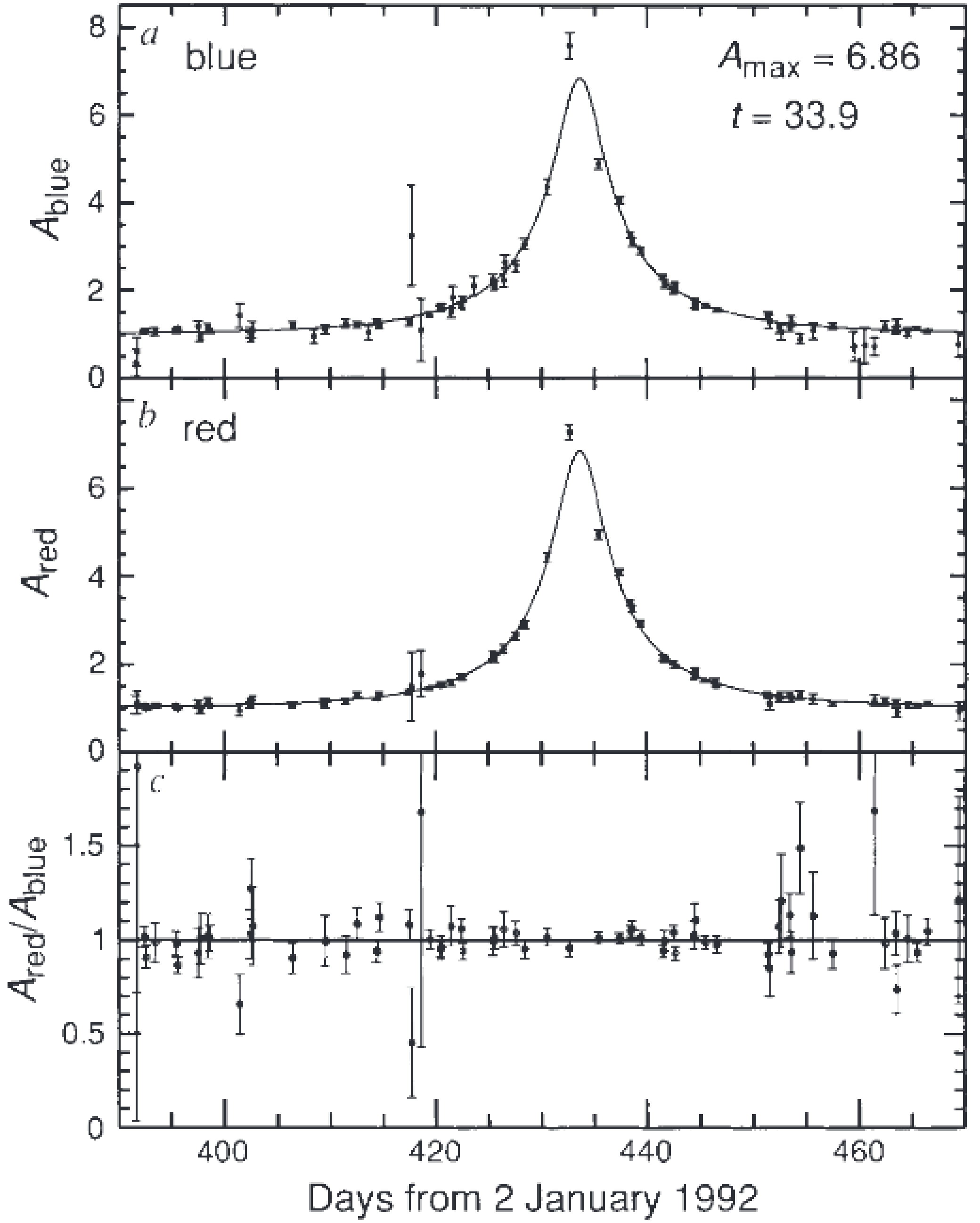
$$\frac{A_\pm}{A_S} = \frac{\theta_\pm \, \mathrm{d}\theta_\pm}{\beta \, \mathrm{d}\beta}$$
$$= \frac{\theta_\pm}{2\beta}\left(1 \pm \frac{\beta}{\sqrt{\beta^2 + 4\theta_{\mathrm{E}}^2}}\right)$$

- Net magnification $a=(A_+ + |A_-|)/A_S$ can be large



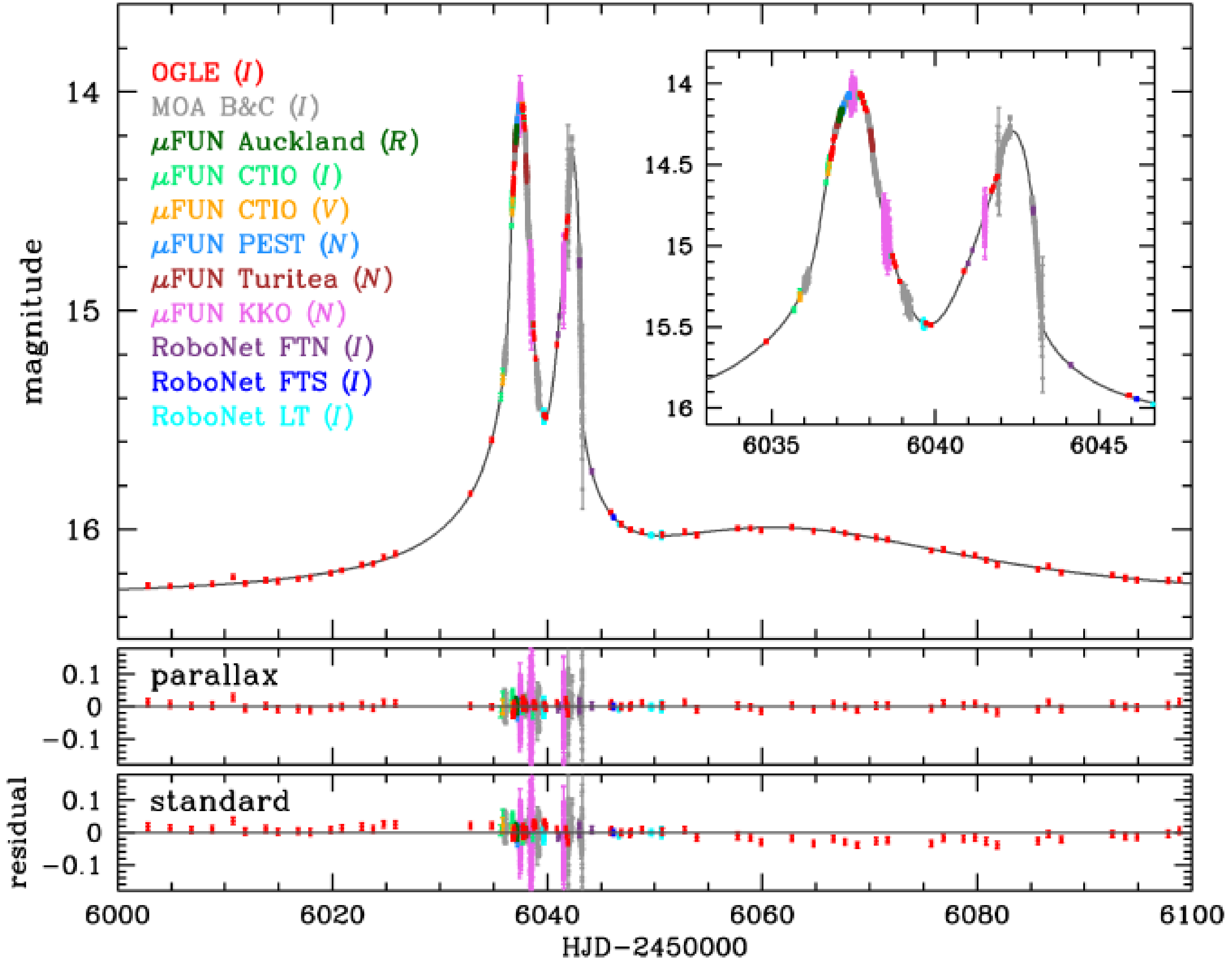
Microlensing

- By monitoring brightnesses of ~100 million stars find hundreds of events p.a.
- First detection 1993
- Duration of event betrays typical mass of lensing star because proper motions of stars known
- Reveals: typical deflector mass is ~0.8Msun and most mass inside ~3 kpc has to be in stars (not DM or interstellar gas)



Planets

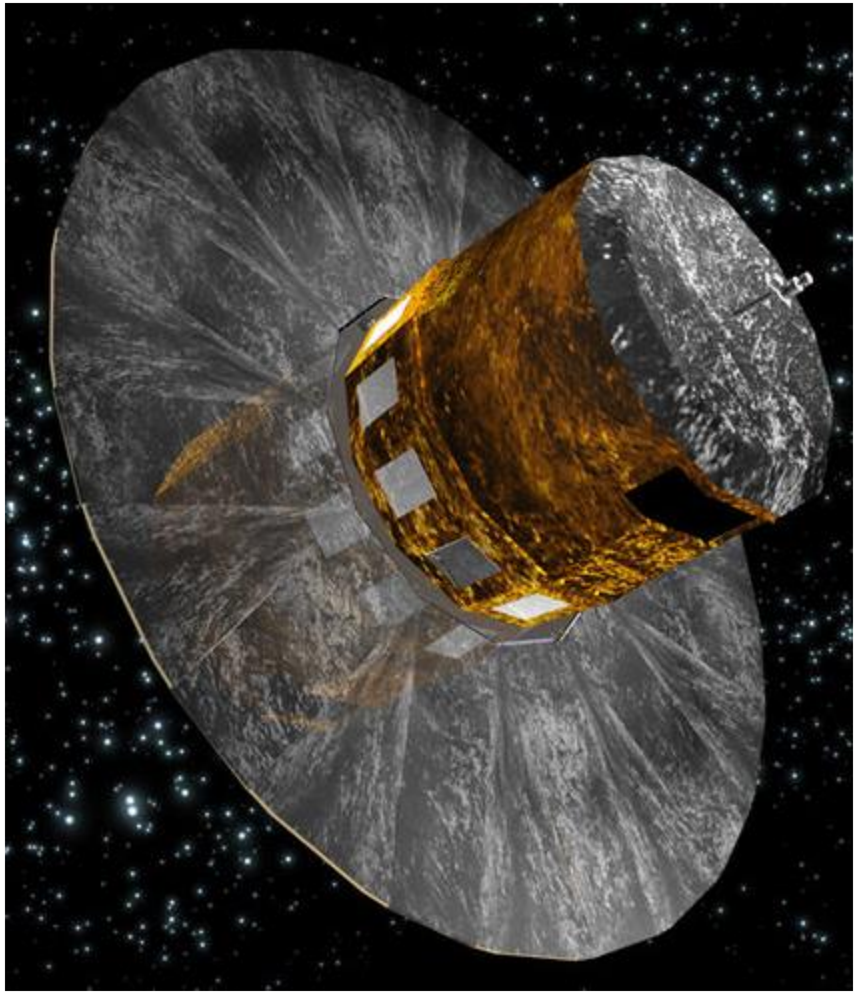
- In high-amplification events, planets may show up
  - e.g. 5Mearth at 2.6AU around star near GC (Beaulieu+ 2006)
  - 0.22Msun + 1.9MJ planet at 0.87AU (Han+ 2013)



Han+ 2013

Eddington 100 years on

- Gaia currently tracking motions of a billion stars from a station 2 Mkm from Earth
  - For millions of stars precision better than 20 μas
  - 45 deg from Sun, deflection is 10 mas
- Consequently
  - Need to model grav fields of planets & major asteroids
  - Obtain precision test of Einstein



Conclusions

- Deflection of light by gravity provides a powerful probe of gravitational fields
- The divergence of deflection ~ surface density of mass on sky
- If a scatterer is within one Einstein radius of a source on the sky, it produces multiple images
- Strongly lensed quasars often have 4 images
- Weaker gravitational field stretch images perpendicular to the field; measuring this effect in millions of galaxies should map dark energy over time
- In microlensing the images are unresolvably close, but the lensing is detectable through overall brightening
- Several extrasolar planets have been detected through their contributions to microlensing
- Deflection by the Sun & planets a major effect for Gaia all over the sky – analysis of this effect will strongly test GR

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  - Obtain precision test of Einstein
    - Taylor expand in Φ/c2: $g_{00} = -(1 + 2\Phi/c^2 + 2\beta(\Phi/c^2)^2)$ $g_{ii} = 1 - 2\gamma\Phi/c^2$
    - Is $\beta = 0$ and $\gamma = 1$?