PLASMA: WHAT IT IS, HOW TO MAKE IT AND HOW TO HOLD IT

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Overview

- Why do we need plasmas? For fusion, among other things
- Basic properties of a plasma
- Magnetized plasmas

Burning matter to get energy

- Burning coal = breaking chemical bonds
- Much less energetic than nuclear bonds
  - Chemical bonds due to electromagnetic force
  - Nuclear bonds due to strong force
  - Strong force > electromagnetic force because it overcomes Coulomb repulsion of nucleons
- “Burning” nuclear bonds seems more productive…

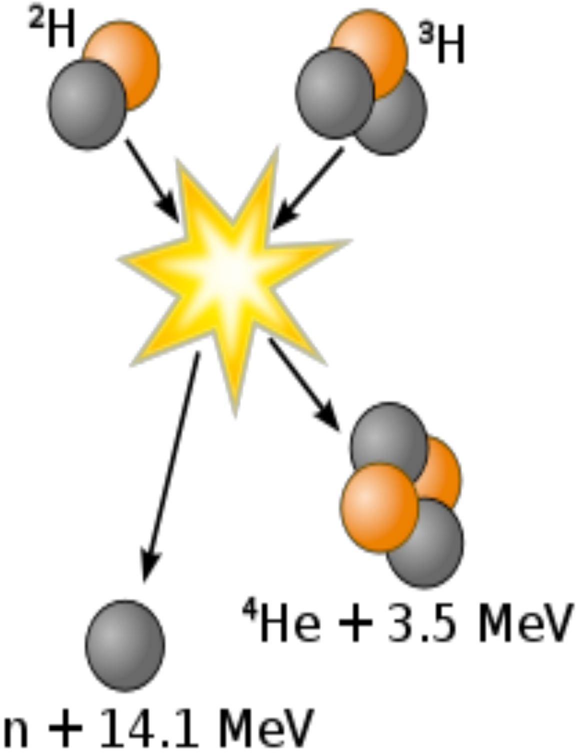
Igniting nuclear fusion reactions

- Initial energy to start reactions
  - Smaller energy barrier for fusion reaction Deuterium-Tritium
  - Sufficient average energy

  = sufficient temperature T

- Use energy from reaction to overcome barrier to produce more reactions
  - Keep energy for sufficiently long time: τ_E [s]
  - Thermal insulation

- Have sufficient number of reactions
  - Fueling $\Rightarrow$ sufficiently dense: n [particles/m^{-3}]



Conditions for fusion

- Temperature: T = 150 millions K
  - At this temperature, matter is ionized = plasma
  - Hottest place in solar system: JET, Oxfordshire
- Density: n = 10^{20} particles/m^3
  - A millionth of the atmospheric density
- Pressure: p = 10 atm
- Energy confinement time: τ_E = 1 – 10 s

- Nuclear fusion is harder than fission, but
  - Fuel is virtually inexhaustible
  - No long-lived radioactive waste

What does plasma mean?

- Langmuir 1929: plasma = Greek for “to mold”

- Misnomer!

  Fusion plasmas at 100 million K do not “mold”

Definition of a plasma

- Ionized gas
- Dominated by long range interactions
- Quasineutral

Ionized gas

- Collisions between energetic electrons and atoms $\Rightarrow$ IONIZATION
- Temperature T > ionization energy ~ eV ~ 10,000 K
  - Sufficient with hot electrons
  - Electrons isolated from ions due to small mass
  - In fusion plasmas T = 100 million K
- How do we ionize? Usually with electric fields

Long range vs. short range

- Long range interactions: plasma currents and charge give magnetic and electric field

- Short range interactions: Coulomb collisions
  - Balance potential energy with kinetic energy

$$\frac{e^2}{4\pi\varepsilon_0 b} \sim T_e \Rightarrow b = \frac{e^2}{4\pi\varepsilon_0 T_e}$$

Long range interactions dominate

- Very few particles in sphere of radius b

$$\frac{4\pi}{3} b^3 n_e = \frac{1}{3} \frac{e^6 n_e}{(\varepsilon_0 T_e)^3} << 1$$

- Kinetic energy much larger than Coulomb potential

$$\Gamma = \frac{T_e}{\dfrac{e^2}{4\pi\varepsilon_0 r}} \sim \frac{T_e}{\dfrac{e^2}{4\pi\varepsilon_0 n_e^{-1/3}}} \sim \frac{4\pi\varepsilon_0 T_e}{e^2 n_e^{1/3}} >> 1$$

- Note that $\dfrac{4\pi}{3} b^3 n_e \sim \dfrac{1}{\Gamma^3} << 1$

Continuum approach

- Distribution function

$$f(\mathbf{r},\mathbf{v},t)\,\mathrm{d}^3 v\,\mathrm{d}^3 r =$$ number of particles with velocity $\mathbf{v}$ at $\mathbf{r}$

- Maxwell's equations

$$\nabla\cdot\mathbf{E} = \frac{1}{\varepsilon_0}\sum_s q_s \int f_s\,\mathrm{d}^3 v \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\times\mathbf{B} = \mu_0 \sum_s q_s \int f_s \mathbf{v}\,\mathrm{d}^3 v + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

Particle conservation

- Number of particles must be conserved along trajectories
  - Equation with characteristics = particle trajectories

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_v f_s = \underbrace{\sum_{s'} C_{ss'}[f_s, f_{s'}]}_{\text{Collision operator}}$$

- Weak coupling means only binary collisions matter
- But Coulomb interaction is long range. Do particles infinitely far away matter? No, because…

Quasineutrality

- Natural tendency to neutrality: opposite charges attract
- Not sufficient energy to separate opposite charges

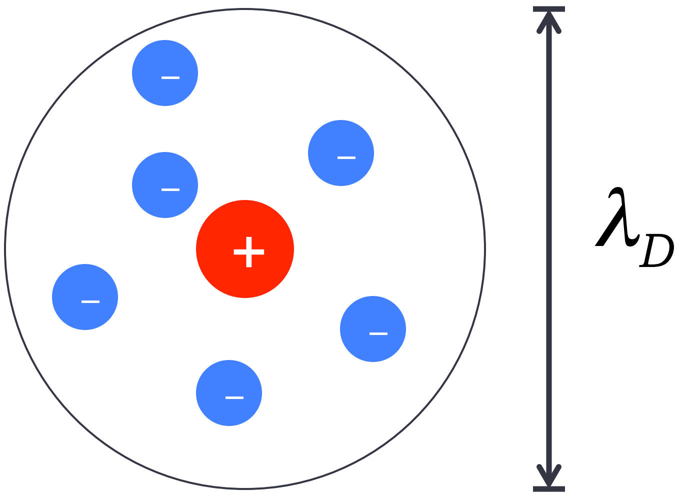
$$L >> \lambda_D$$

$$\omega << \omega_p$$

- Surprisingly, every volume in an ionized gas is almost perfectly neutral!
  - By definition, plasma is quasineutral

Debye length

- Layers of positive and negative charge in uniform plasma



- Sufficient potential difference to attract electrons

$$e\Delta\phi \sim T_e$$

- Poisson's equation

$$-\nabla^2\phi = \frac{e(n_i - n_e)}{\varepsilon_0} \Rightarrow \frac{\Delta\phi}{\lambda_D^2} \sim \frac{en_e}{\varepsilon_0}$$

$$\lambda_D = \sqrt{\frac{\varepsilon_0 T_e}{e^2 n_e}}$$

Validity of quasineutrality

- Plasma size >> $\lambda_D \Rightarrow$ any charge is shielded
- Electrons have to be sufficiently fast to respond

$$\omega << \frac{v_e}{\lambda_D} = \frac{1}{\lambda_D}\sqrt{\frac{2T_e}{m_e}} \sim \omega_p = \sqrt{\frac{e^2 n_e}{\varepsilon_0 m_e}} = \text{plasma frequency}$$

- Need sufficient electrons in Debye sphere

$$\frac{4\pi}{3}\lambda_D^3 n_e = \frac{4\pi}{3}\frac{\left(\varepsilon_0 T_e\right)^{3/2}}{e^3 n_e^{1/2}} = \frac{1}{3\sqrt{4\pi}}\Gamma^{3/2} >> 1$$

  - Weak coupling = sufficient particles for shielding

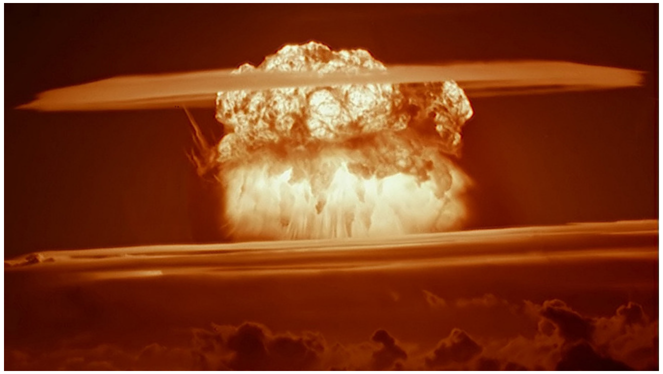
Coulomb collisions

- Coulomb interaction is long range, but it does not reach beyond λ_D!

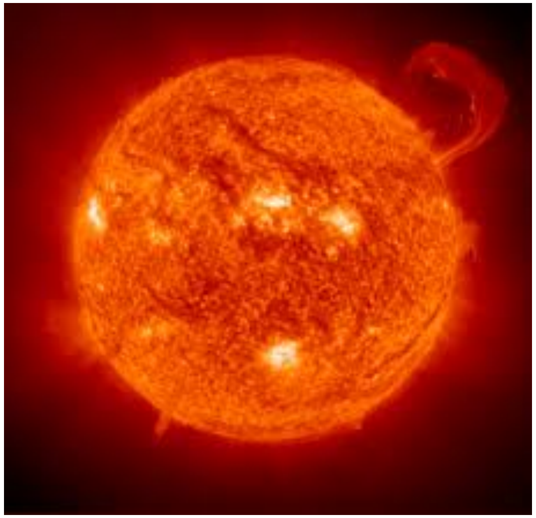
- Equation with binary collisions valid

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s} \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_v f_s = \sum_{s'} C_{ss'} \left[f_s, f_{s'} \right]$$

Confining very hot plasmas



- Do not confine: just get fusion faster than plasma life time
  - H bombs
  - Inertial confinement fusion
- Within a physical container
  - Lots of plasma energy lost to the wall
  - Very low temperatures (at most 100,000 K)



- With gravity
  - Only star-size entities can achieve fusion
- Magnetic confinement

Magnetized plasmas

- For strong magnetic field plasmas show a particular behavior

- Quasineutrality helps magnetization: makes $\mathbf{E}$ small
  - In principle,

$$\frac{|\mathbf{v}\times\mathbf{B}|}{|\mathbf{E}|} \sim \frac{v}{c} << 1$$

- Need to satisfy

$$L >> \rho$$

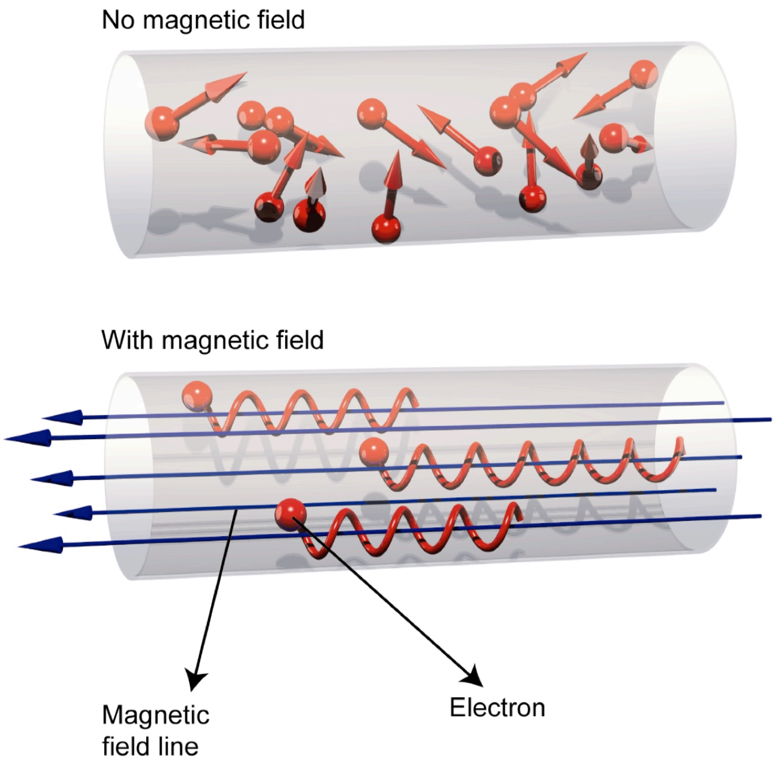
$$\omega << \Omega$$

Gyromotion

- Gyromotion or Larmor motion ($\partial \mathbf{B}/\partial t = 0 = \nabla \mathbf{B}$)
- Free motion parallel to $\mathbf{B}$
- Perpendicular force balance

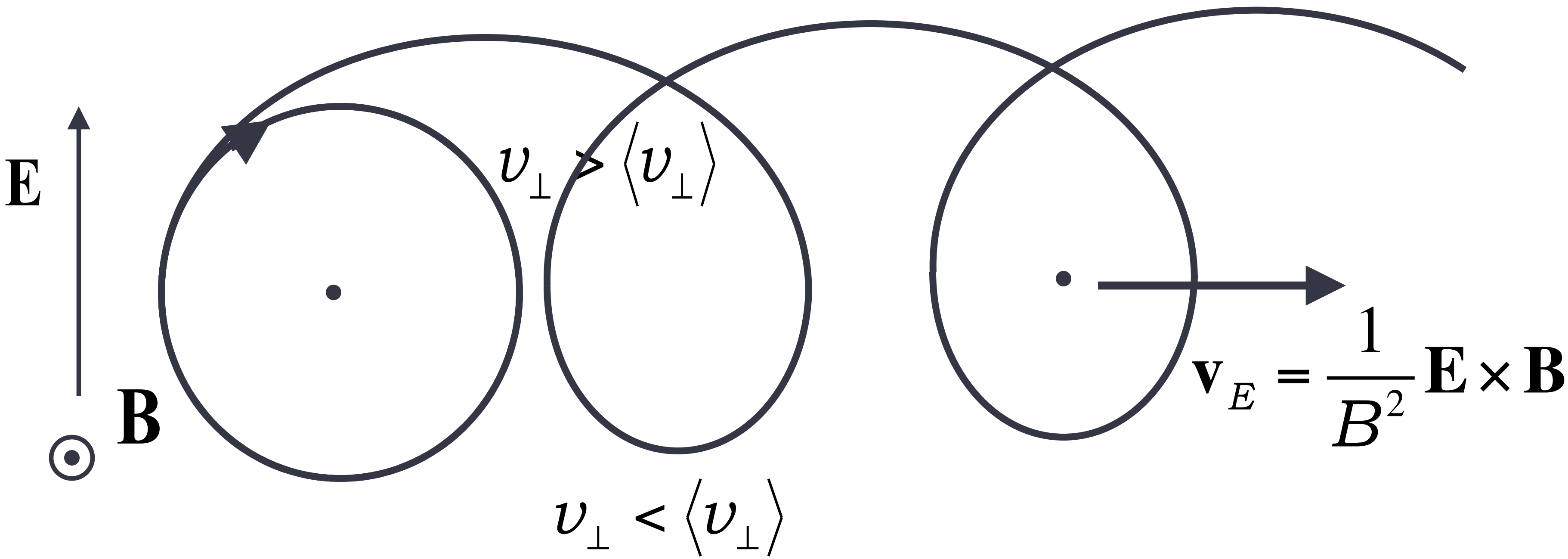
$$\frac{mv_\perp^2}{\rho} = ev_\perp B$$

- Gyroradius: $\rho = \dfrac{mv_\perp}{eB} = \dfrac{v_\perp}{\Omega}$
- Gyrofrequency: $\Omega = \dfrac{eB}{m}$



E×B drift

- Gyromotion + perpendicular electric field?
- Perpendicular drift



Gyrokinetic motion

- Two-scale expansion [Catto, Plasma Phys. 1978]
  - Separate fast and slow times $t_g = \Omega t$, $t_t = \omega t \sim \rho_* t_g$ to calculate particle position $\mathbf{r}(t_g, t_t)$
  - Simplified for approximately circular gyromotion

$$\mathbf{r}(t_g, t_t) \cong \mathbf{R}(t_t) + \rho(t_t)\left(\sin t_g \hat{\mathbf{e}}_1 + \cos t_g \hat{\mathbf{e}}_2\right) + \ldots$$

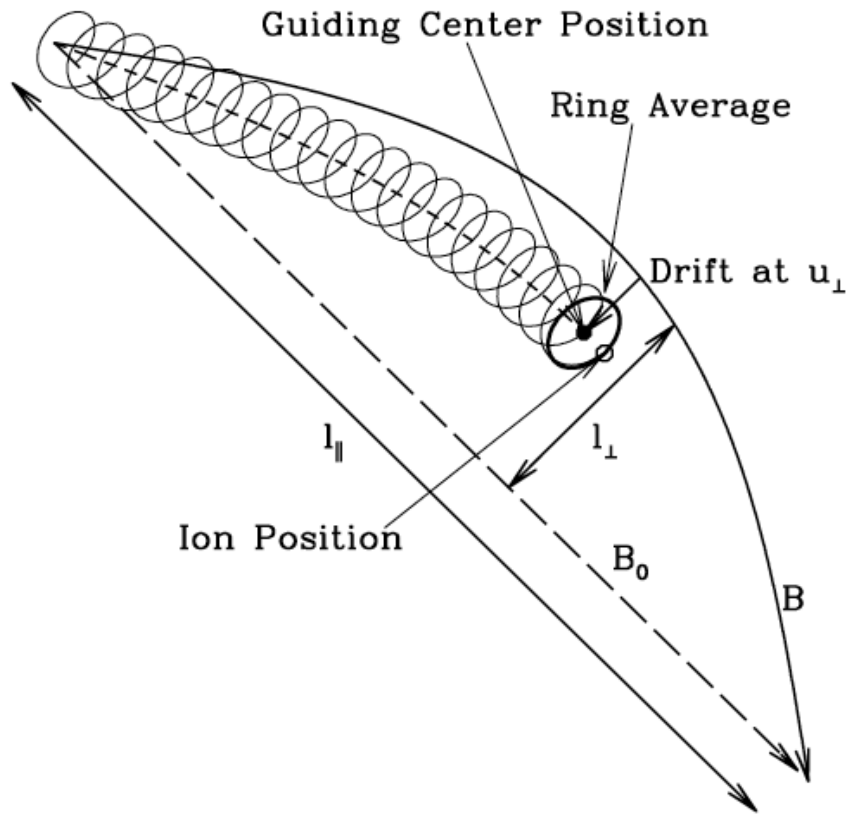
- Gyrokinetic equations of motion
  - To lowest order perpendicular average motion $= 0$

$$\frac{d\mathbf{R}}{dt} = \underbrace{v_\parallel \hat{\mathbf{b}}}_{O(v_t)} + \underbrace{\mathbf{v}_d}_{O(\rho_* v_t)} + \underbrace{\text{other drifts}}_{O(\rho_*^2 v_t) + \ldots}; \quad \frac{d\rho}{dt} = \ldots; \quad \frac{dv_\parallel}{dt} = \ldots$$

Gyrokinetic equations

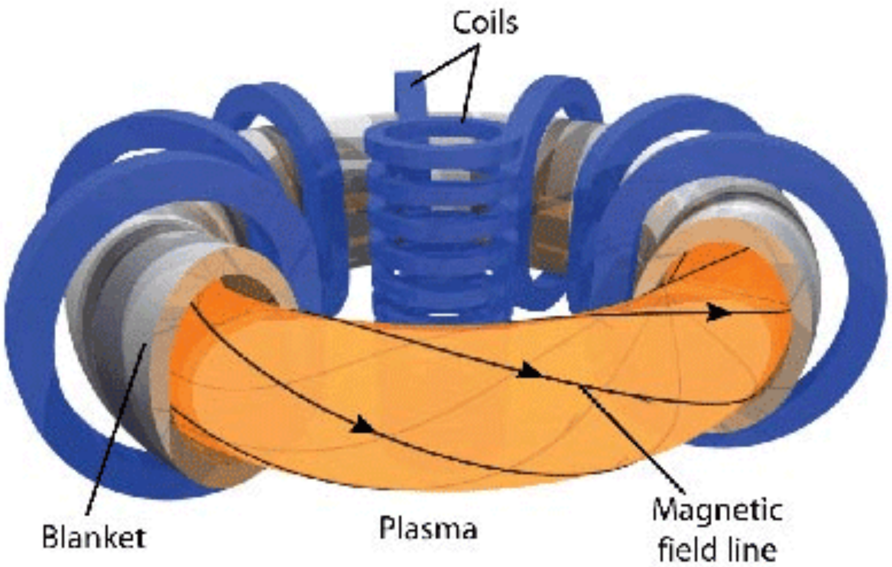
- Moving rings of charge

$$\frac{\partial f_s}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \nabla_{\mathbf{R}} f_s + \frac{d\rho}{dt}\frac{\partial f_s}{\partial \rho} + \frac{dv_{\parallel}}{dt}\frac{\partial f_s}{\partial v_{\parallel}} = \left\langle \sum_{s'} C_{ss'}\left[f_s, f_{s'}\right] \right\rangle$$

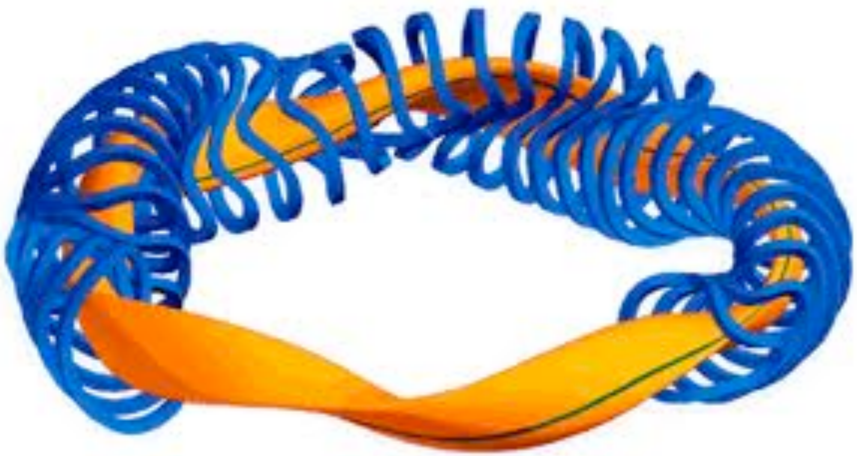


Tokamaks and stellarators

- Need to account for drifts in design



Tokamak



Stellarator