Strings and Fields

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Morning of Theoretical Physics

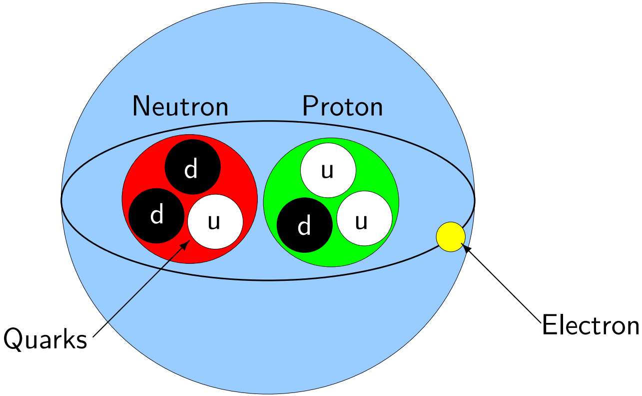
Today

Can we construct a theory that describes all fundamental particles and all their interactions?

- Perhaps the most ambitious (conceptual) enterprise ever attempted!
- We may be like a group of dogs trying to understand the rules of chess...but it doesn't mean we shouldn't try!
- This will lead to a field theoretical 'solution' to the problem of quantum gravity.
- And also to very surprising consequences!

Fundamental particles

Q: What are the fundamental blocks of matter? Look at the atom!



Fundamental particles (fermions)

Fundamental particles (that form tangible matter)

Leptons

e (electron)	ν_e(e-neutrino)
μ (muon)	ν_μ (μ-neutrino)
τ (tau)	ν_τ (τ-neutrino)

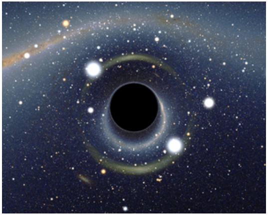
Quarks

u (up)	d (down)
c (charm)	s (strange)
t (top)	b (bottom)

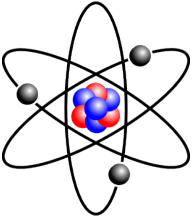
- All these particles form the matter, they are called fermions
- It turns out fundamental forces are also mediated by particles! called bosons

Fundamental forces (bosons)

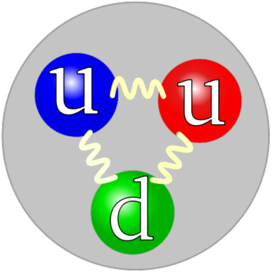
Gravity (graviton)



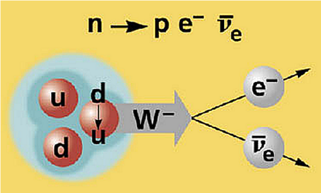
Electromagnetism (photon)



Nuclear Strong (gluons)



Nuclear Weak (W, Z bosons)



Quantum Field Theory

- Gravity is different from the other three, but at sub-atomic level we can forget about it!

$$F_{grav} \sim 10^{-36} F_{weak}$$

- Forgetting about gravity, fundamental particles and their interactions are described by

Quantum Field Theory

A framework that describes how particles interact with each other in a way consistent with special relativity and quantum mechanics.

- Each particle (Fermion or Boson) represented by a field:

$$\text{Photons} \rightarrow F_{\mu\nu}, \quad \text{Leptons} \rightarrow \psi, \cdots$$

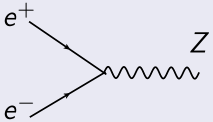
- Their interactions dictated by a Lagrangians for those fields.

Standard Model

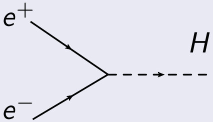
Standard Model of elementary particles

$$\mathcal{L}_{SM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \psi_i y_{ij}\psi_j\phi + h.c. + |D_\mu\phi|^2 - V(\phi)$$

Lagrangian $\rightarrow$ a set of basic rules



$\frac{g}{\cos\theta_W}$



$g\,\frac{m_e}{m_W}$

- Predicts what happens at accelerators with great accuracy!
- Needed an extra boson (the Higgs) which was found!
- The biggest triumph of TP of the last decades!

Standard Model problems

The Standard model is incredibly accurate but...

It cannot exist by itself!

- The particles of the standard model do couple to gravity.
- At some point gravity will become important and the SM will break down!

$$M_{pl} = \sqrt{\frac{\hbar c}{G_N}} \sim 1.22 \times 10^{19} \, GeV$$

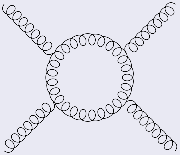
- This corresponds to very small distances! ($\sim 1.61 \times 10^{-33}$ cm)

Quantum gravity problems

- Gravity at large scales is well described by Einstein's theory of gravity (General relativity). But this fails at very small scales.
- If we try to construct a quantum field theory for gravitons we run into infinities.

Gravitons are point-like and can get too close to each other!

Scattering of two gravitons

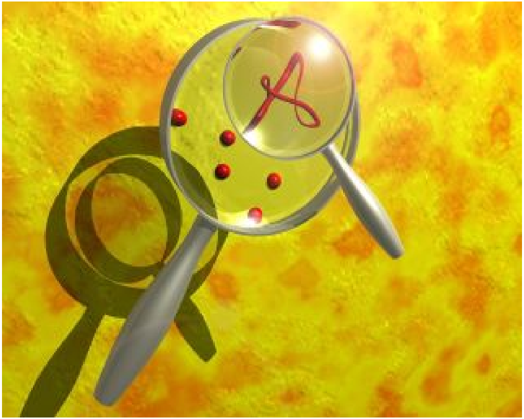


$$\sim \quad \int_0^\infty d^4p \frac{p^6}{(p^2)^4} \rightarrow \text{divergent!}$$

Note: we have infinities in the SM too...but those we can cure. The infinities in the case of gravity are much worse!

String Theory

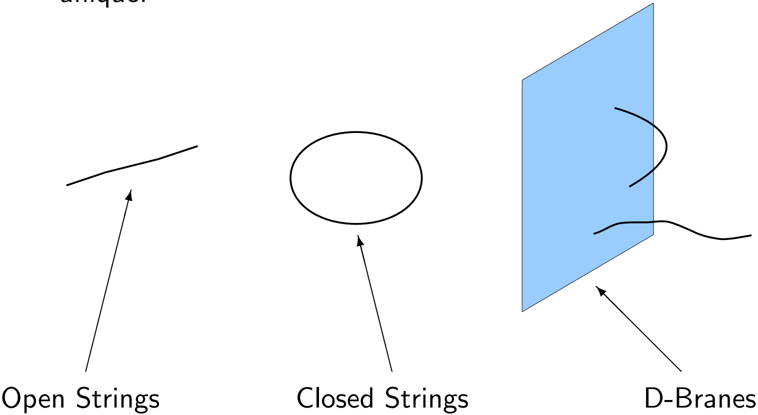
- What is the "form" of the elementary particles?
- Propose that they are little (really tiny!) strings!



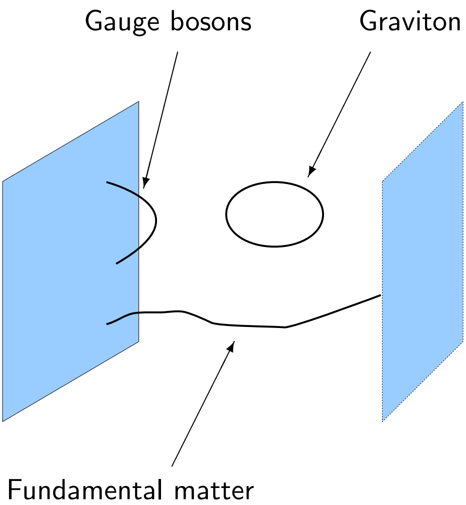
String Theory

Let's sit down and construct a relativistic theory of strings (as opposed to point particles) consistent with quantum mechanics...

- Actually you don't have much choice... the theory is *pretty* unique!



String theory

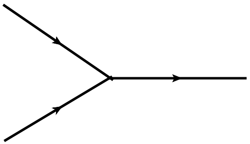


- All particles and forces under a unified framework!

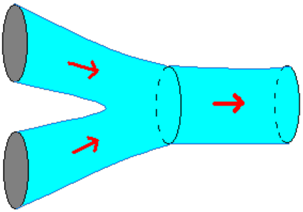
- We don't have infinities anymore!

Point particles

Strings



VS



"Features" of String Theory

- Predicts Super-Symmetry: a remarkable symmetry between fermions and bosons.
- It predicts the number of dimensions. (no other physical theory does!)
- Is extremely hard to make computations with!
- (Probably) Has the richest physical/mathematical structure among any theory invented by humankind.

Lets have a glimpse at the last three!

Dimensionality of space-time

- Mass of the graviton in ST? A bunch of 'harmonic oscilators' very much as in QM!

Quantum mechanics → Mass of the graviton/string

$$M^2 = (D-2)\sum_{n=1}(-1)^{n+1}n - 2$$

- How we regularise the divergent sum?

$$1 - 2 + 3 - 4 + \cdots$$

- Introduce a regulator ϵ and then take it to zero!

$$\sum_{n=1}(-1)^{n+1}n = \lim_{\epsilon\to 0}\sum_{n=1}(-1)^{n+1}ne^{-\epsilon n} = \lim_{\epsilon\to 0}\frac{e^{\epsilon}}{(e^{\epsilon}+1)^2} = \frac{1}{4}$$

Dimensionality of space-time

- Plugging $1 - 2 + 3 - 4 + \cdots = \frac{1}{4}$ into the mass formula we get

$$M^2 = \frac{D-2}{4} - 2$$

- But general relativity implies the graviton must be massless:

$$M^2 = \frac{D-2}{4} - 2 = 0 \to D = 10$$

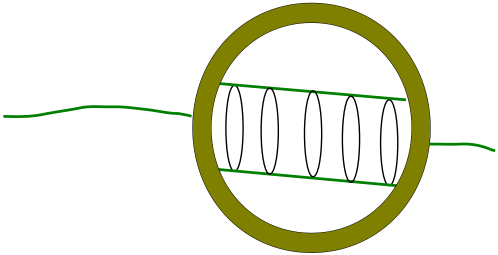
The theory is consistent with QM and GR only in **ten dimensions**!!

We expected $3 + 1 = 4$, length, width, and height; plus time.

Extra dimensions

Q: If string theory predicts 10 dimensions, why do we see only 4?

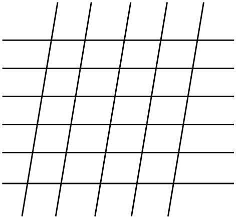
A: Not all of them have to be large!



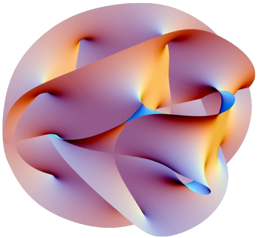
$$\text{2d Hosepipe} = \underset{\text{Large dimension}}{\mathbb{R}} \times \underset{\text{Compact dimension}}{S^1}$$

String compactification

- String Theory on 10D space-times of the form $M^{3,1} \times M_6$



$\times$



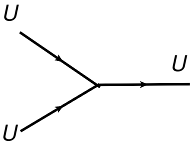
$M^{3,1}$: (3+1)d flat space-time
(large dimensions)

M_6: 6d Compact manifold
(compact dimensions)

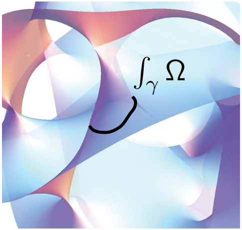
4d Physics vs 6d Geometry

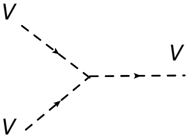
- There are bazillion ways to curl up six dimensions! and the predictions of ST in four-dimensions depends on that.

4d Particles and their interactions $\leftrightarrow$ shape of M_6−Manifold



$\leftrightarrow$





$\leftrightarrow$

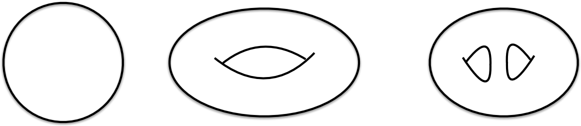
Number of straight lines you can fit on M_6

4d Physics vs Geometry

Q: Is there a way of wrapping the six extra dimensions, such that in 4d the theory looks like the standard model?

A: Super hard to get models that in 4d look exactly like the Standard model (but too easy to get similar models!)

- 2d compact manifolds are a piece of cake:



- 6d Manifolds are extremely rich and hard to study...the necessary mathematics hasn't been developed yet!

Still, we have learnt a lot from this perspective!

Physics in 4d $\leftrightarrow$ Geometry in 6d

Predictability in String Theory

- The theory is 'almost' unique, except for a discrete choice

Type I | Type IIA | Type IIB | $SO(32)$ | $E_8 \times E_8$

- Believed to be different corners of a single eleven-dimensional theory! called M-theory.

However

- Many classical solutions with spaces of the form $M^{3,1} \times M_6$.
- The metric is one of the fields of string theory, but we have others and we have an abundance of vacua.

Q: How many? A rough estimate gives... 10^{500}!

- We need to understand whether there are further constraints!

String theory, achievements and problems

Achievements

- Unifies all known fundamental particles and forces under one framework.
- Cures the divergence problems of gravity, providing a quantum theory of gravity.
- Other achievements (not explained)
  - Explanation of black hole entropy.
  - Precise implementation of the holographic principle.
  - Dualities that led to new results in gauge theories, CFT's, etc

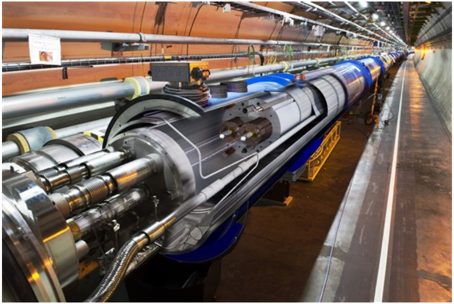
Problems

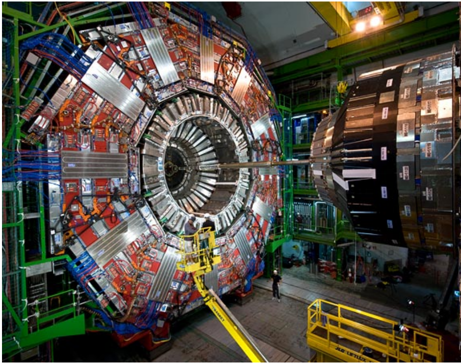
- Supersymmetry has not been yet observed in nature.
- Extra dimensions.
- Abundance of vacua.
- The mathematics we need, has not been developed yet!

Is string theory really true?

Testing string theory

- The LHC, the biggest machine ever built by humankind, might provide some evidence for the ideas that feature in String Theory!





String Theory

The maths are incredibly hard, and this enterprise may take hundreds of years, but if we succeed we would have found the

Final Theory

Unifies all known fundamental particles and forces under one framework, and all laws of Nature must follow from it!!

- A new conception of space and time.
- We could understand the quantum mechanics of black holes, the origin of the universe, etc.
- We could finally answer the oldest and deepest scientific questions.
- Applications that we can't even imagine!