Magnets, superfluids and superconductors

Fabian Essler (Oxford)

A Saturday Morning of Theoretical Physics, October 29 2016

Quantum States of Matter

Previous talk: collective behaviour of many particles gives rise to a large variety of states of matter with interesting **emergent properties:** gases, liquids, solids, metals, insulators, magnets, ...

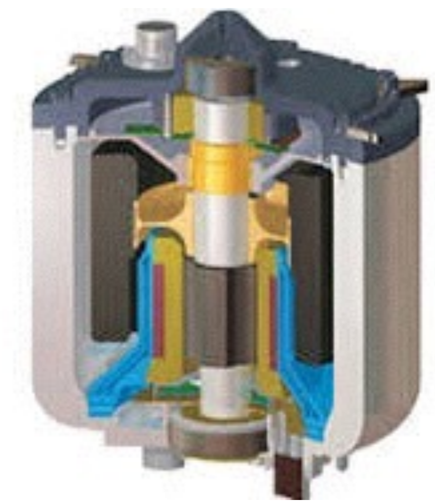
Understanding them in detail is a huge intellectual challenge, **and** is of tremendous practical importance:

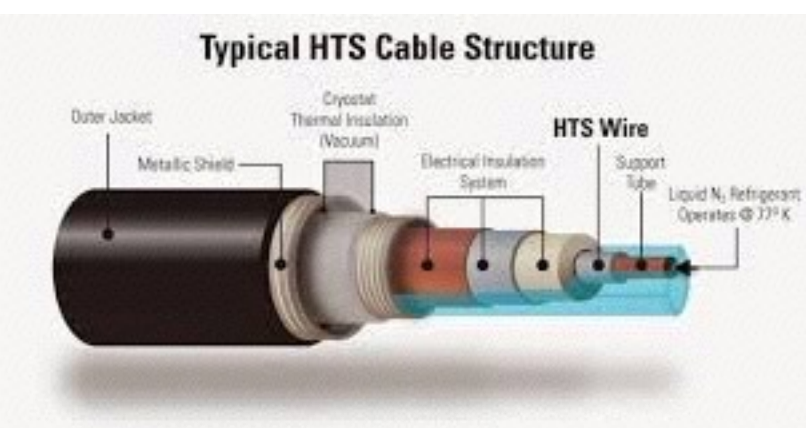
Some applications of superconductors:

Medical

Transport

Storage





Transmission

Power generation

High-energy physics

Quantum theory is the most complete description of nature available to us, and ultimately must explain all of these.

So what's the problem?

$$H\Psi_\alpha(\vec{r}_1,\ldots,\vec{r}_N) = E_\alpha\Psi_\alpha(\vec{r}_1,\ldots,\vec{r}_N)$$

$$H = \sum_{j=1}^{N} -\frac{\hbar^2}{2m_j}\nabla_j^2 + \sum_{k\neq j} V_{\text{int}}(\mathbf{r}_j,\mathbf{r}_k)$$

where $N \sim 10^{23}$!

Even though we basically know V_{int} the problem is **incredibly hard**.

N classical particles: Need to know **6N** numbers $\mathbf{r}_j, \; \dot{\mathbf{r}}_j$

N QM particles: Need to find one particular wave function (e.g. ground state) among all possible wave functions depending on 3N co-ordinates.

Space of wave functions is **gigantic**!

Some simple counting for N spins-1/2:

1 spin: 2 basis states $|\uparrow\rangle \; , |\downarrow\rangle$

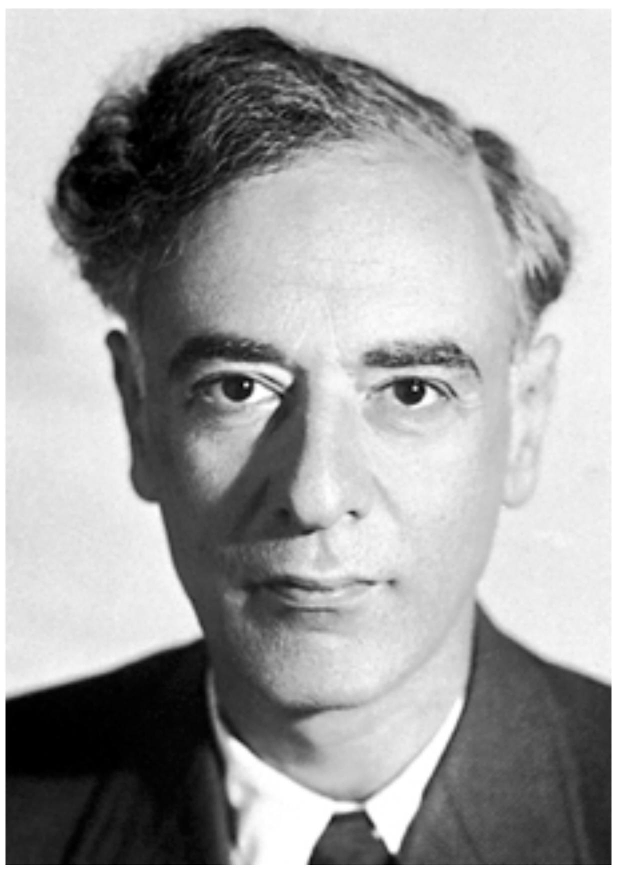
2 spins: 4 basis states $|\uparrow\uparrow\rangle \; , \; |\uparrow\downarrow\rangle \; , |\downarrow\uparrow\rangle \; , |\downarrow\downarrow\rangle$

N spins: 2^N basis states

To find the ground state we must search among vectors with $2^{10^{23}}$ components...

When the going gets tough, change the problem....

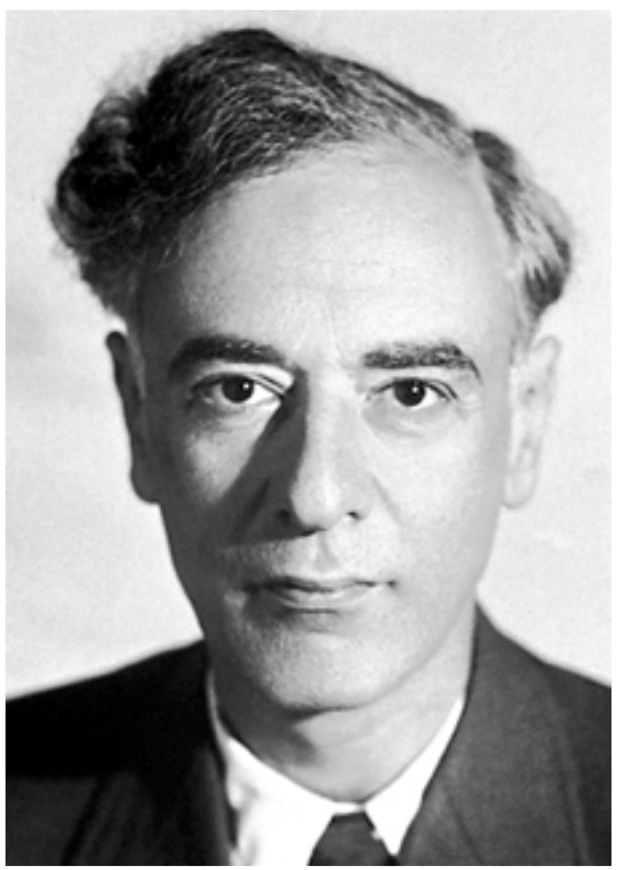
There is way too much information in the wave function -
try instead to understand something about something (rather than nothing about everything).



Я покажу вам путь

When the going gets tough, change the problem….

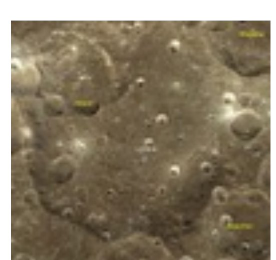
There is way too much information in the wave function -
try instead to understand something about something (rather than nothing about everything).



Я покажу вам путь

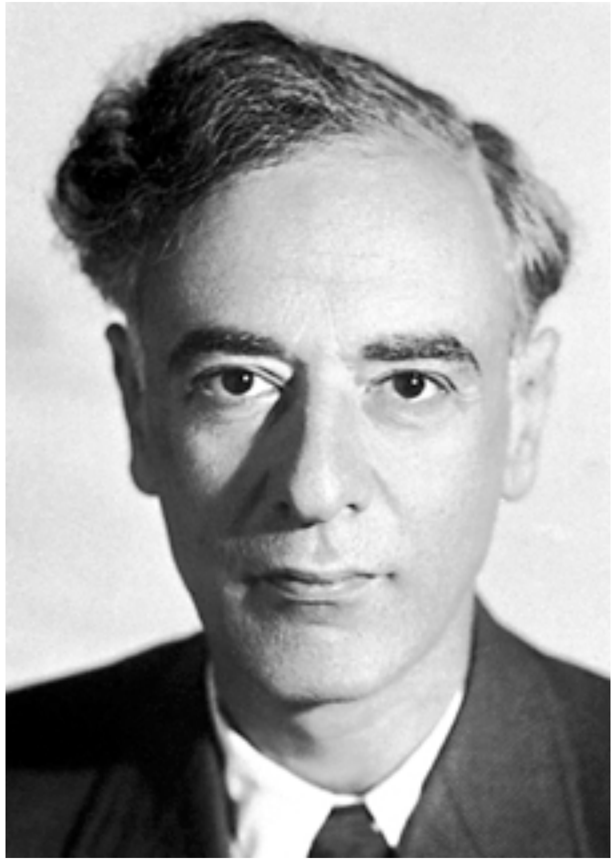
“Landau theory”, “Landau-Ginzburg theory”, “Landau levels”
“Landau-Fermi liquid”, “Landau pole”, “Landau damping”,…

1962



Landau crater

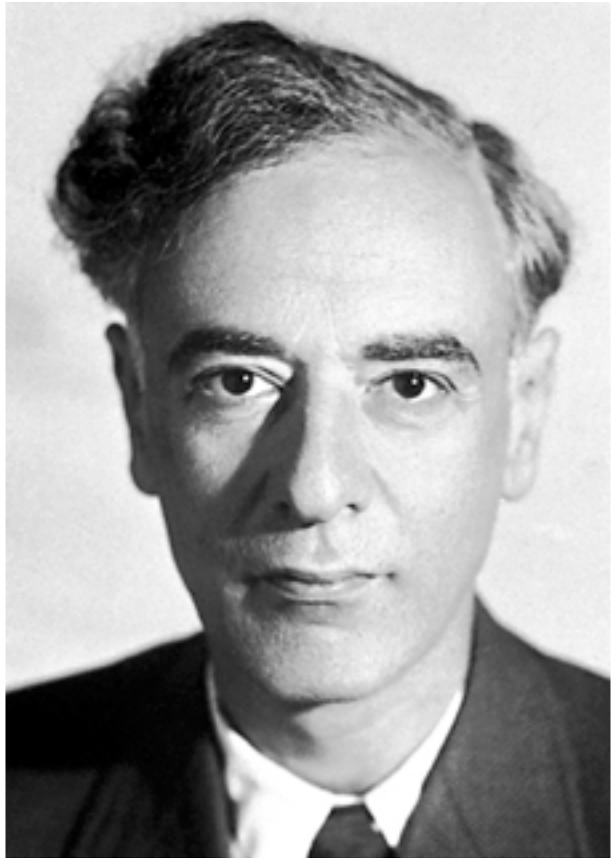
minor planet 2142 Landau



The Landau physicist ranking system

-Log(genius)

0	Newton
0.5	Einstein
1	Bohr, Heisenberg, Schroedinger, Dirac, Bose, Wigner, …
2	Landau (after Nobel prize)
2.5	Landau (before Nobel prize)
4	
5	"Pathologists"



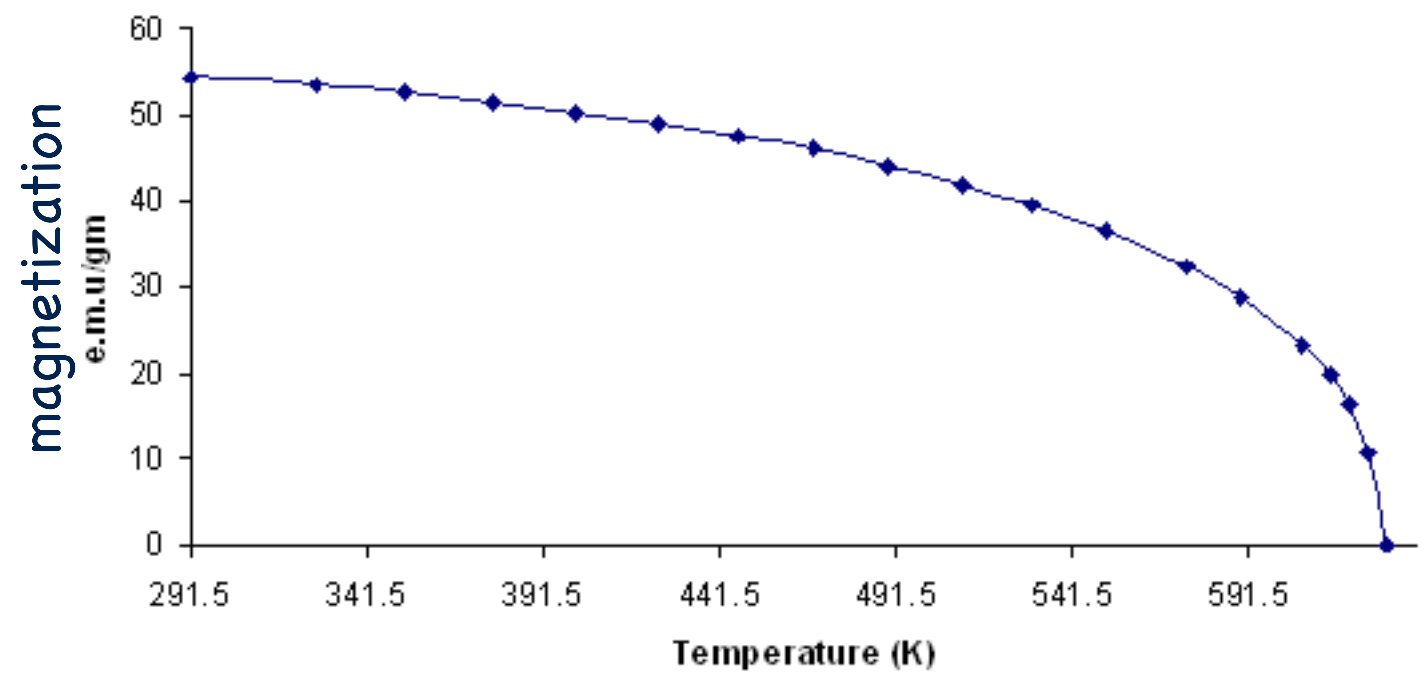
The Landau physicist ranking system

-Log(genius)

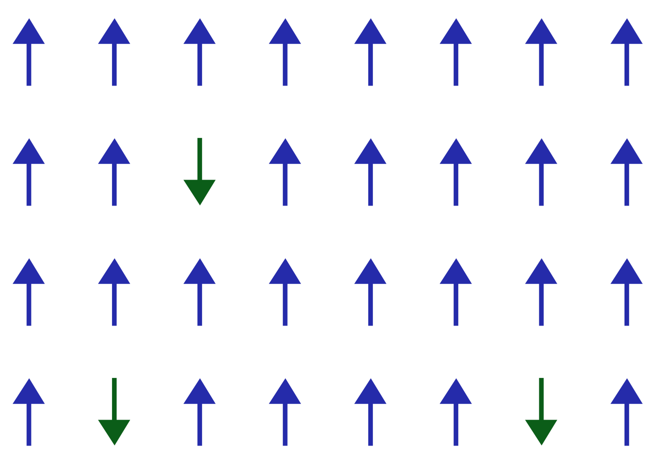
0	Newton
0.5	Einstein
1	Bohr, Heisenberg, Schroedinger, Dirac, Bose, Wigner, …
2	Landau (after Nobel prize)
2.5	Landau (before Nobel prize)
4	
5	“Pathologists”
…	
∞	current political class

We know that matter can be in different **phases:**

Magnetization of Ni:

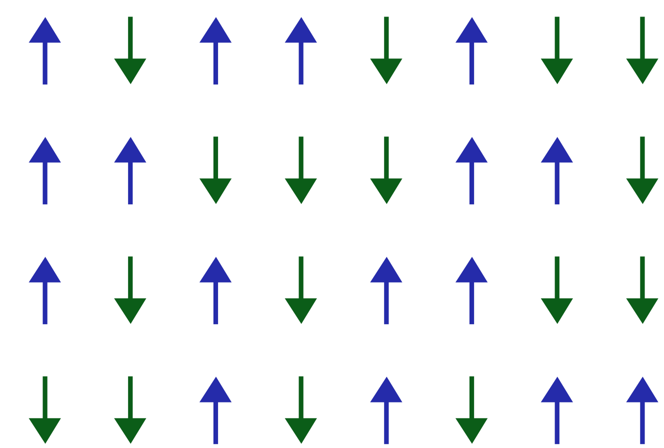


$T<T_c$



net magnetization

$T>T_c$



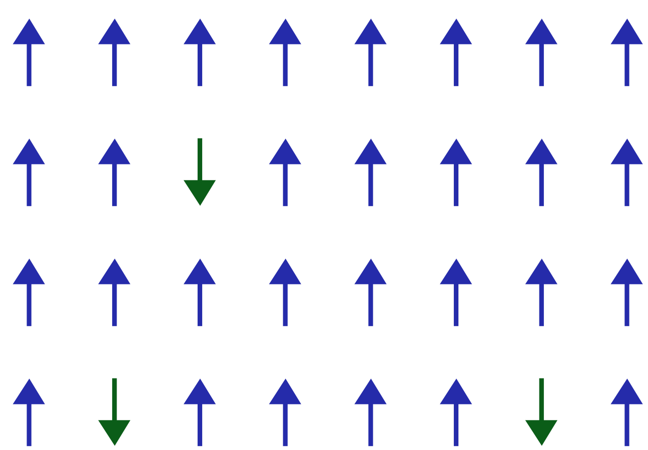
no net magnetization

The transition is related to a **symmetry**

No symmetry under "spin reversal"

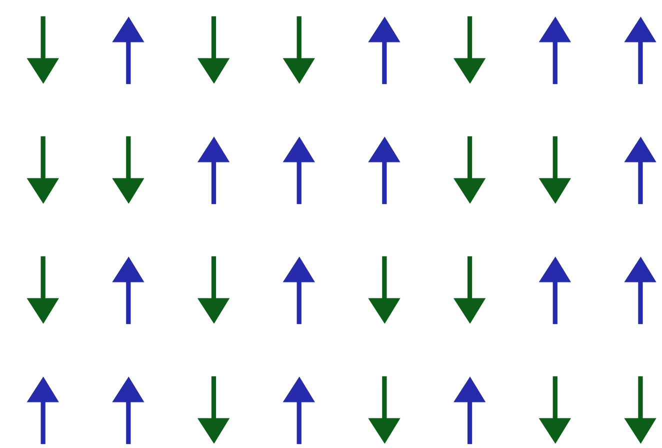
Symmetry under "spin reversal"

$T<T_c$



net magnetization

$T>T_c$



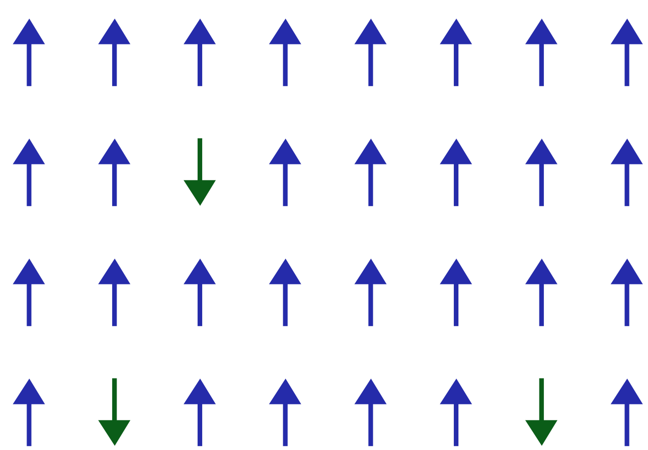
no net magnetization

The transition is related to a **symmetry**

No symmetry under "spin reversal"

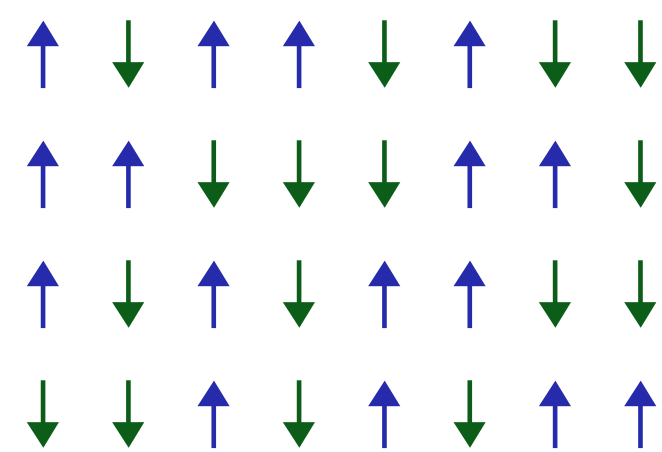
Symmetry under "spin reversal"

$T<T_c$



net magnetization

$T>T_c$



no net magnetization

Observation: magnetization = "order parameter"

= physical quantity that distinguishes between phases

Landau's idea:

write down a theory for the order parameter **in the vicinity of the phase transition** (⇒m small!)

Landau L.D., Zh. Eksp. Teor. Fiz. 7, 19 (1937)

$$F = \text{const} - hm + \alpha_2 m^2 + \alpha_3 m^3 + \alpha_4 m^4 + \dots$$

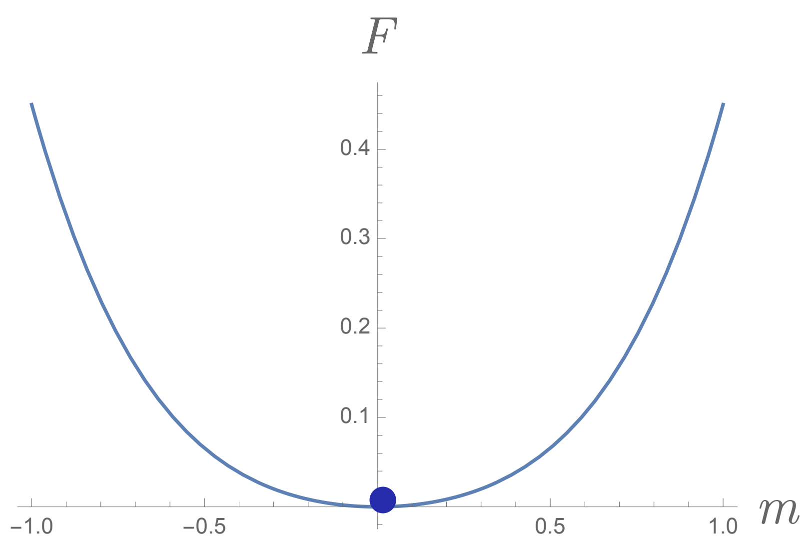
↑ magnetic field

Symmetry: For h=0 we must have $\alpha_3=0$.

Thermodynamic stability: must have $\alpha_4>0$.

$$F = \alpha_2 m^2 + \alpha_4 m^4$$

$\alpha_2 > 0$

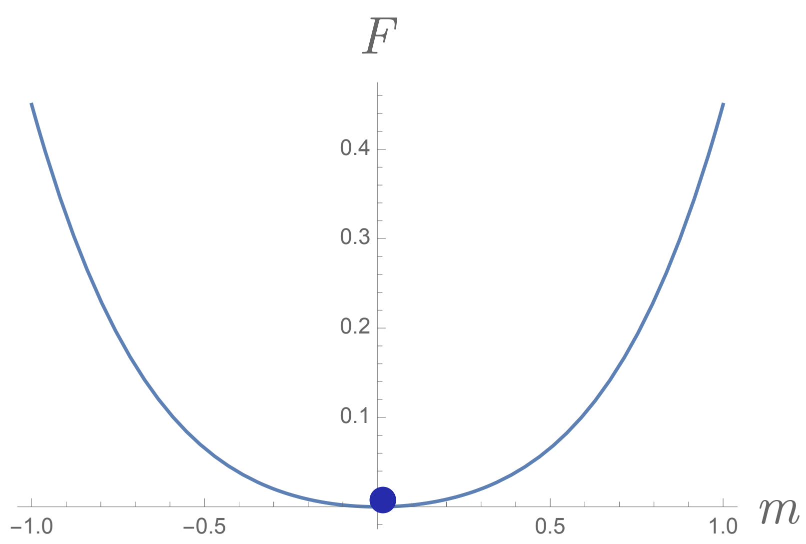


Minimum at 0:
no magnetic order

high temp. $T>T_c$

$$F = \alpha_2 m^2 + \alpha_4 m^4$$

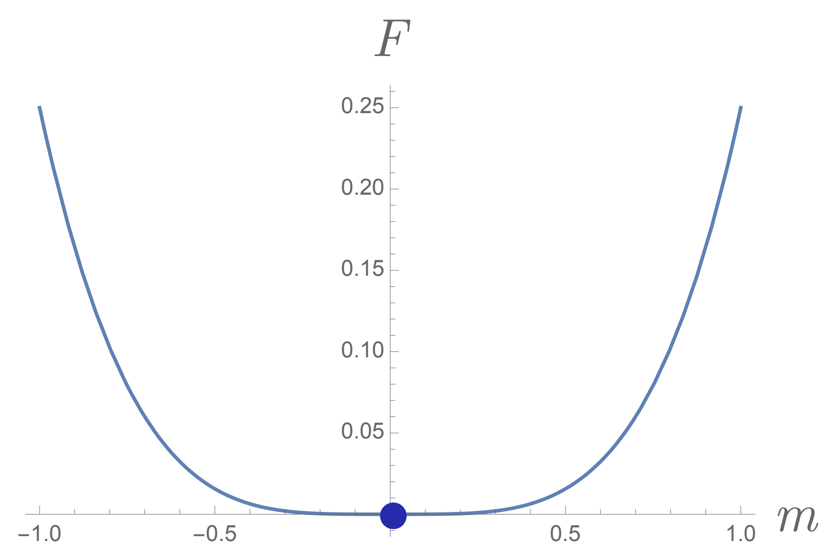
$\alpha_2 > 0$



Minimum at 0:
no magnetic order

high temp. $T>T_c$

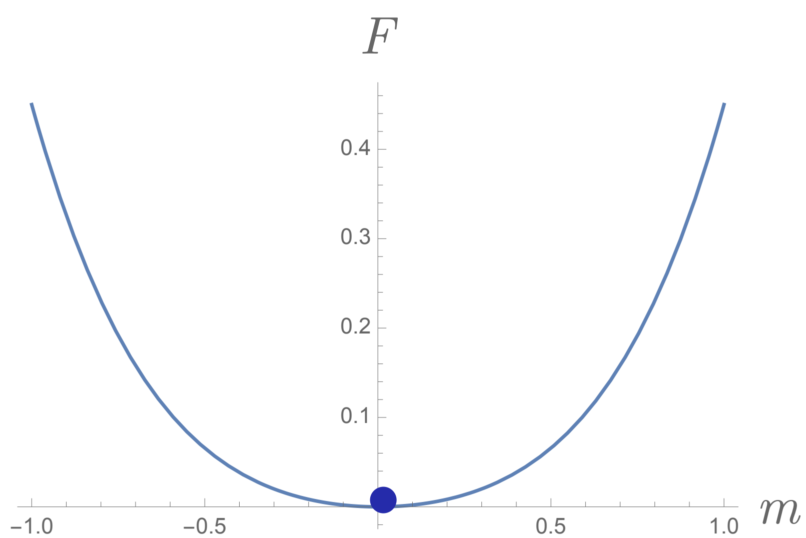
$\alpha_2 = 0$



$T=T_c$

$$F = \alpha_2 m^2 + \alpha_4 m^4$$

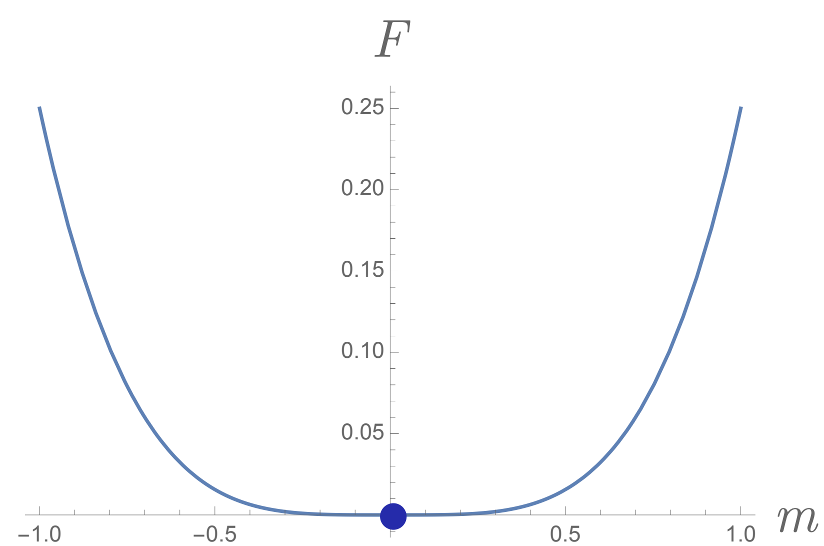
$\alpha_2 > 0$



Minimum at 0: no magnetic order

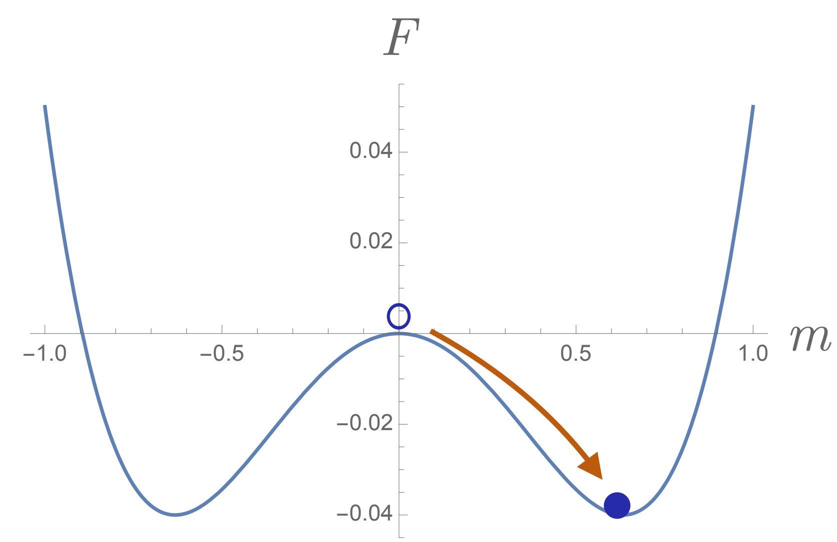
high temp. $T>T_c$

$\alpha_2 = 0$



$T=T_c$

$\alpha_2 < 0$



Minimum at $\pm m_0$: magnetic order

low temp. $T<T_c$

$$\alpha_2(T) = A\frac{T-T_c}{T_c} + \ldots$$

Spontaneous Symmetry Breaking

For $T<T_c$ the order parameter "**spontaneously**" picks one of the two possible values $\pm m_0$. **This breaks the $m \to -m$ symmetry of the free energy!** (same principle as the Higgs mechanism)

In practice tiny imperfections (e.g. stray fields, boundary conditions) can select one of the two minima.

Captures many states of matter:

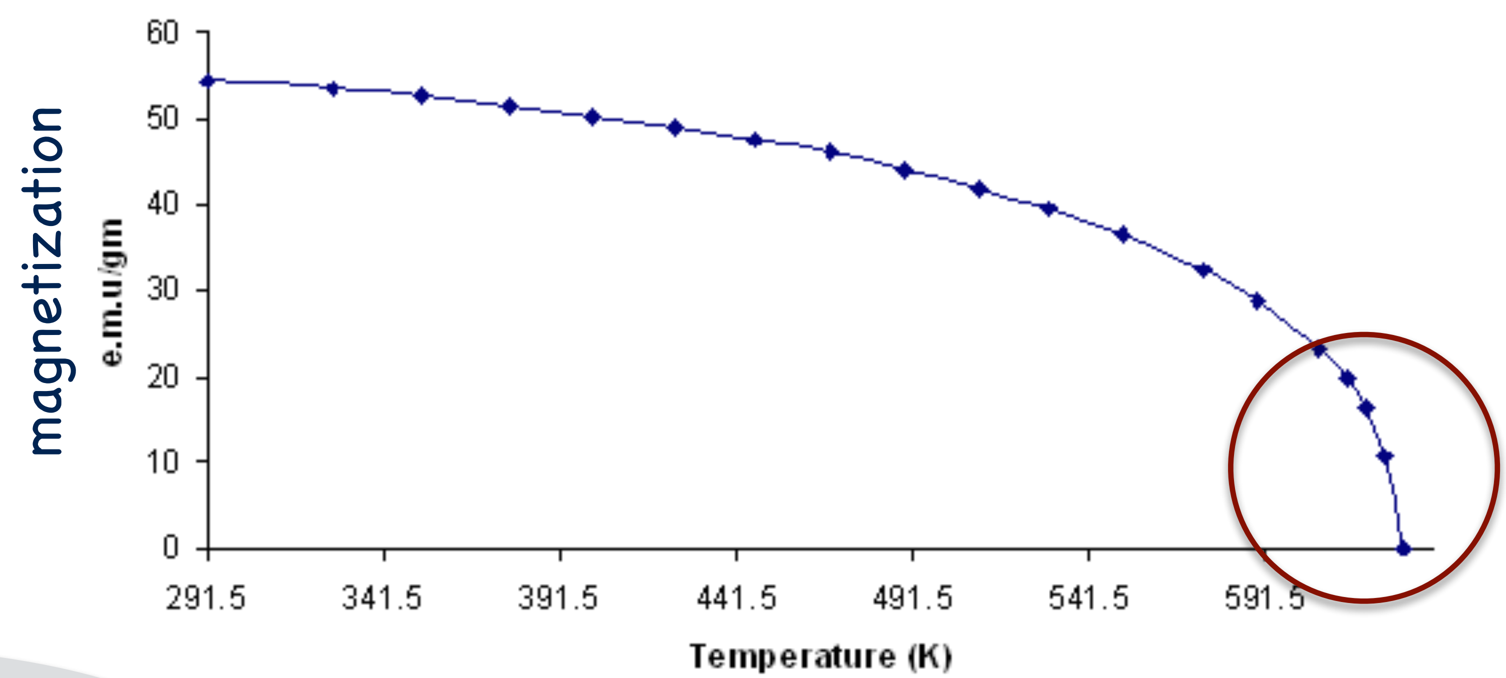
State of Matter	Crystals	Magnets	Liquid Crystals	Standard Model vacuum	Super conductors
Symmetry	translations	spin rotations	spatial rotations	gauge symmetry	gauge symmetry

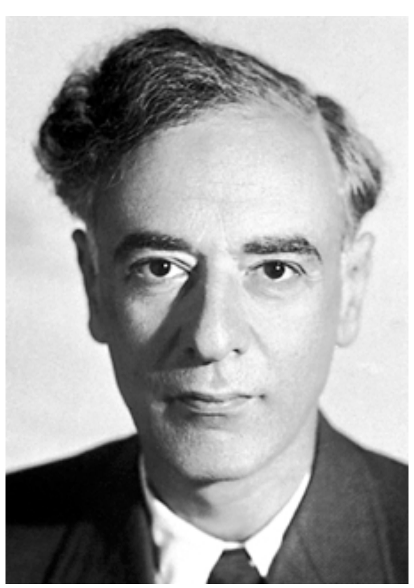
Spontaneous Magnetization

In our case minimizing F gives

$$m_0 \propto \left[\frac{T_c - T}{T_c}\right]^{\frac{1}{2}}$$

Agrees well with experiment:





Я рок

Landau theory in its full glory works with a **spatially varying, fluctuating** order parameter

$$F = \text{const} + \int d\mathbf{r}\left[\alpha_1|\nabla m(\mathbf{r})|^2 + \alpha_2 m^2(\mathbf{r}) + \alpha_4 m^4(\mathbf{r}) + \ldots\right]$$

⇒fully fledged Quantum Field Theory!

- fields live on a D-dimensional "Euclidean" space rather than 3+1 dimensional Minkowski space.
- fluctuations are thermal rather than quantum
- tools for analysis are the same

Order parameter vs wave function

Probability to find a particular particle with spin σ at position r averaged over the behaviours of the other N-1 particles.

$$n_\sigma(\mathbf{r}) = \sum_{\sigma_2,\ldots,\sigma_N} \int d\mathbf{r}_2 \ldots \int d\mathbf{r}_N |\Psi_{\sigma\sigma_2\ldots\sigma_N}(\mathbf{r},\mathbf{r}_2,\ldots,\mathbf{r}_N)|^2$$

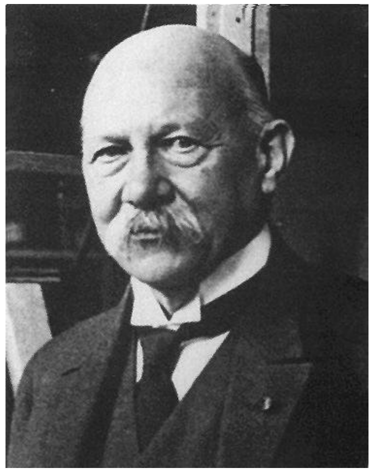
order parameter:

$$m(\mathbf{r}) \propto n_\uparrow(\mathbf{r}) - n_\downarrow(\mathbf{r})$$

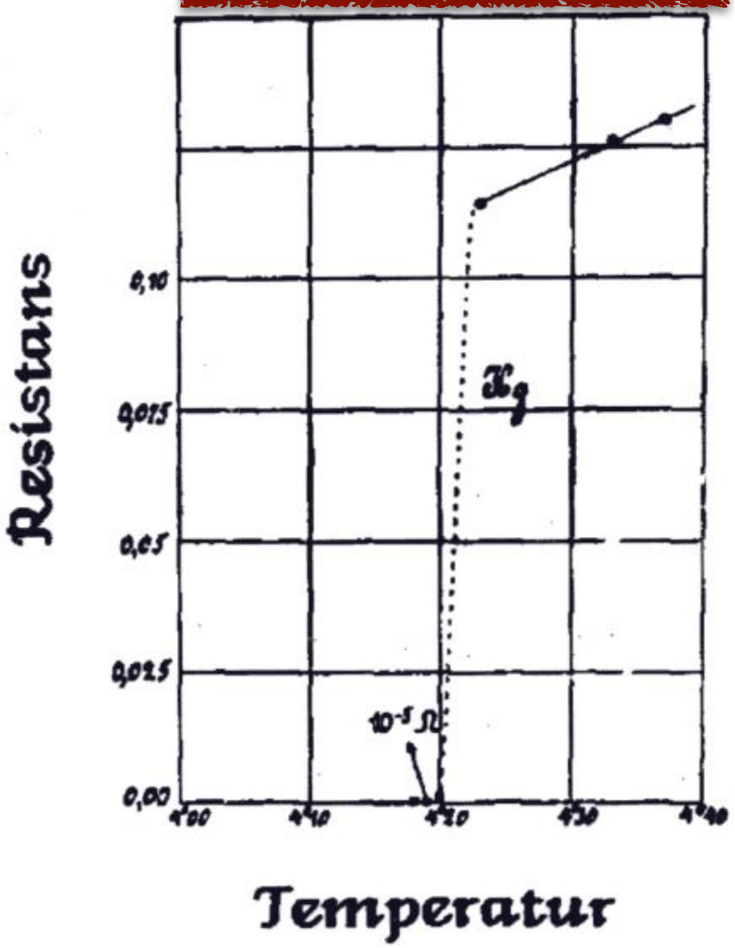
More complicated states of matter: **Superconductivity**

1911

H. Kamerlingh Onnes



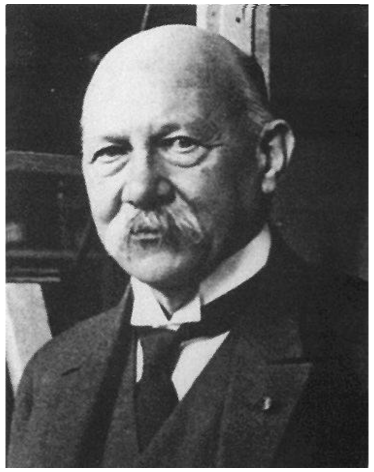
Zero resistance



More complicated states of matter: **Superconductivity**

1911

H. Kamerlingh Onnes



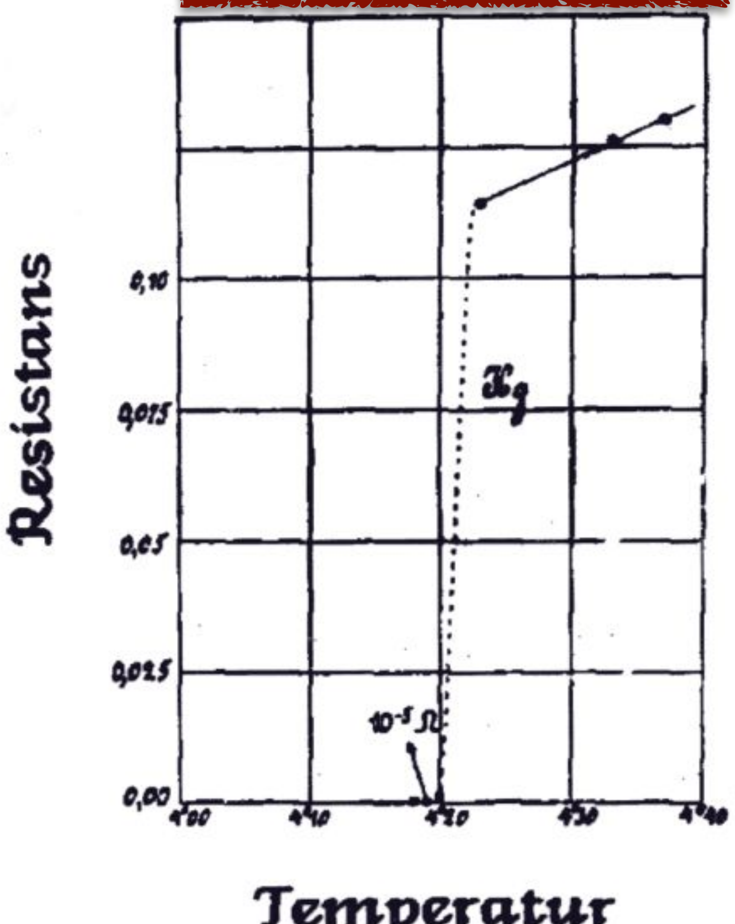
1933

Walter Meissner

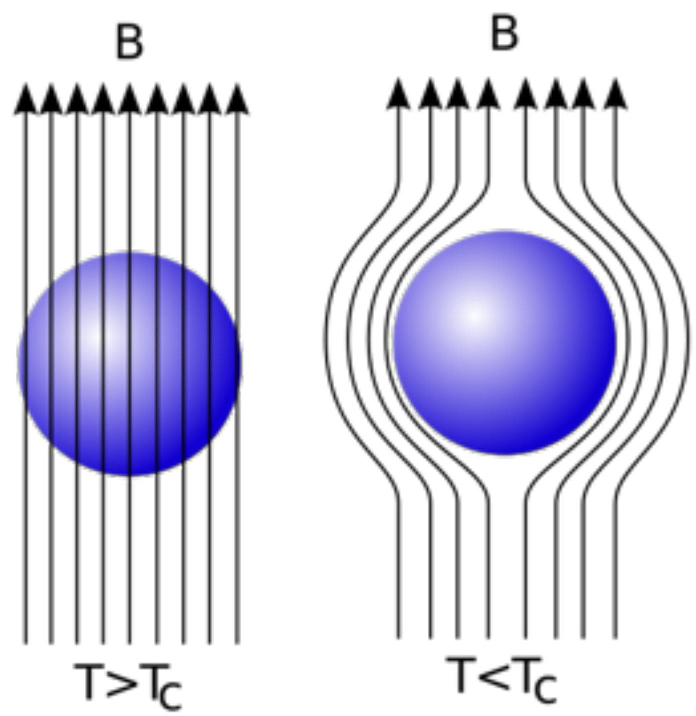
Robert Ochsenfeld

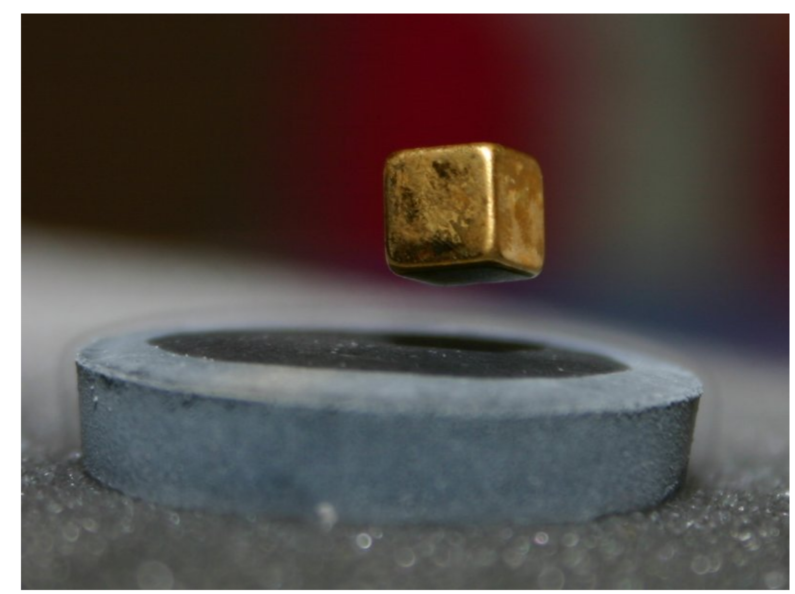
Descubrieron que los superconductores expulsan los campos magnéticos, popularmente conocido como efecto Meissner

Zero resistance



Meissner effect:





Landau-Ginzburg theory of superconductivity

Recall that for magnets we had

$$F = \text{const} + \int d\mathbf{r}\,\left[\alpha_1|\nabla m(\mathbf{r})|^2 + \alpha_2 m^2(\mathbf{r}) + \alpha_4 m^4(\mathbf{r}) + \dots\right]$$

1950 Landau & Ginzburg apply Landau theory to superconductors

What is the order parameter?

LG postulate that it is a **complex scalar $\psi(\mathbf{r}) = |\psi(\mathbf{r})|e^{i\theta(\mathbf{r})}$**

$$F = \text{const} + \int d\mathbf{r}\left[-\frac{\hbar^2}{2m^*}|\nabla\Psi(\mathbf{r})|^2 + a|\Psi(\mathbf{r})|^2 + b|\Psi(\mathbf{r})|^4 + \dots\right]$$

Coupling to EM field?

do "the usual" $\nabla \rightarrow \nabla - \frac{ie*}{\hbar}\mathbf{A}$

& add $\frac{1}{2\mu_0}\int d\mathbf{r}\mathbf{B}^2(\mathbf{r})$

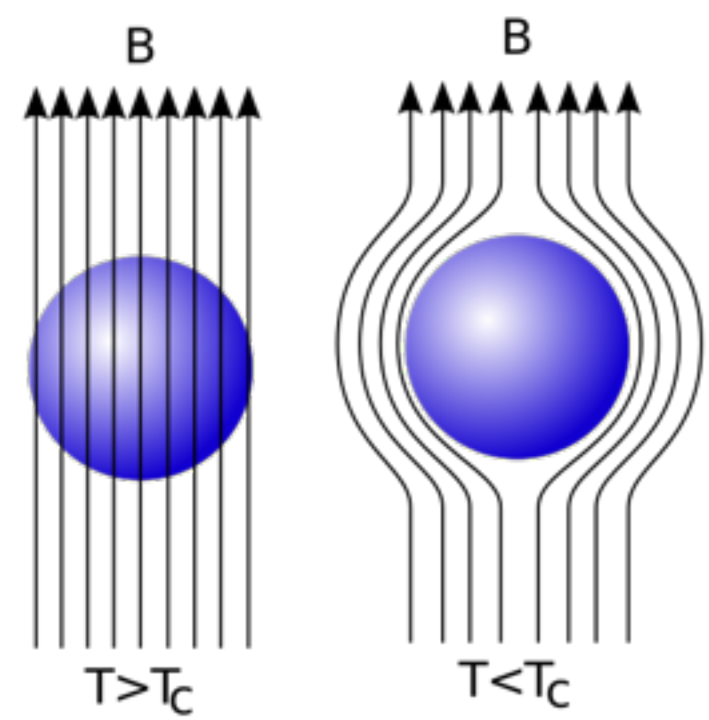
Minimize F $\Rightarrow$ "Landau-Ginzburg equations"

$$-\frac{\hbar^2}{2m^*}\left(\boldsymbol{\nabla}+i\frac{e^*}{\hbar}\mathbf{A}\right)^2\Psi(\mathbf{r})+(a+2b|\Psi(\mathbf{r})|^2)\Psi(\mathbf{r})=0$$

$$\nabla\times\mathbf{B}=\mu_0\left[-i\frac{\hbar e^*}{2m^*}\Big(\Psi^*(\mathbf{r})\nabla\Psi(\mathbf{r})-\Psi(\mathbf{r})\nabla\Psi^*(\mathbf{r})\Big)\right]-\frac{(e^*)^2}{m^*}|\Psi(\mathbf{r})|^2\mathbf{A}(\mathbf{r})$$

These are a mere **two** partial differential equations; solve them

→ Meissner effect



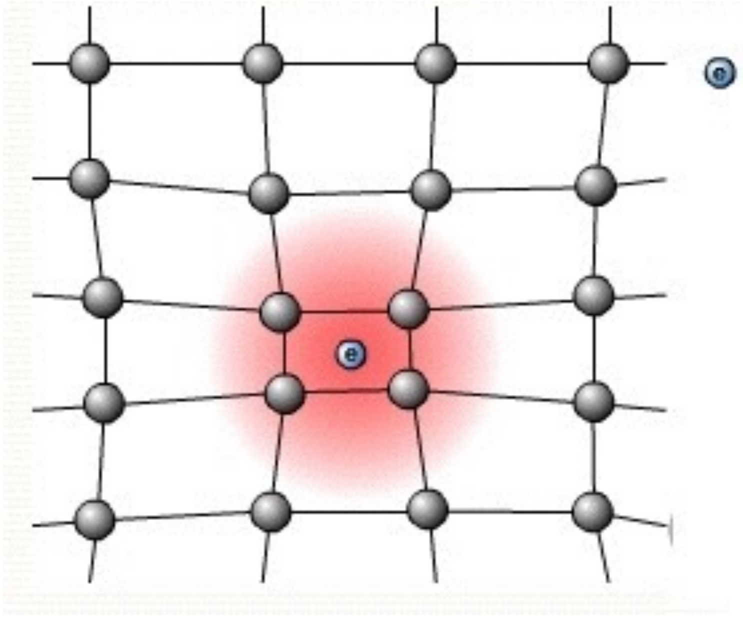
That's all very nice, but doesn't really explain how superconductivity works....

→ Go back to microscopics

Microscopic theory of "classic" superconductivity

1957

"Electron-phonon" interaction leads to attraction between electrons



$\Rightarrow$ Formation of "Cooper pairs" 1956

BCS (variational) wave function

$$\Psi_{\sigma_1,\ldots\sigma_N}(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \mathcal{N}\ \mathcal{A}\Big[\phi_{\sigma_1\sigma_2}(\mathbf{r}_1,\mathbf{r}_2)\ldots\phi_{\sigma_{N-1}\sigma_N}(\mathbf{r}_{N-1},\mathbf{r}_N)\Big]$$

$$\phi_{\sigma_1\sigma_2}(\mathbf{r}_1,\mathbf{r}_2) = \phi(\mathbf{r}_1-\mathbf{r}_2)\ (\uparrow_1\downarrow_2 - \downarrow_2\uparrow_1)$$

describe Cooper pairs

Huge reduction in complexity:

$$\Psi_{\sigma_1,\ldots\sigma_N}(\mathbf{r}_1,\ldots,\mathbf{r}_N) \longrightarrow \phi(\mathbf{r})$$

$\phi(\mathbf{r})$ can be found by minimising the energy for a given Hamiltonian (variational principle)

$$H = \sum_{j=1}^{N} -\frac{\hbar^2}{2m}\nabla_j^2 + \sum_{k\neq j} V_{\text{int}}(\mathbf{r}_j-\mathbf{r}_k)$$

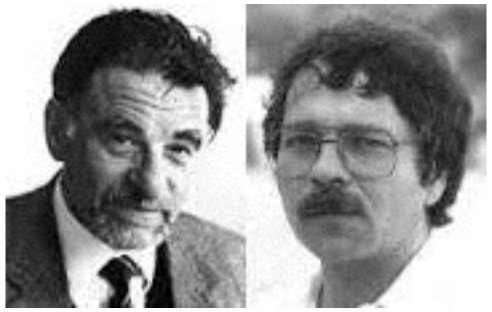
→ … → essentially complete microscopic understanding

$\phi(\mathbf{r})$ is closely related to LG order parameter...

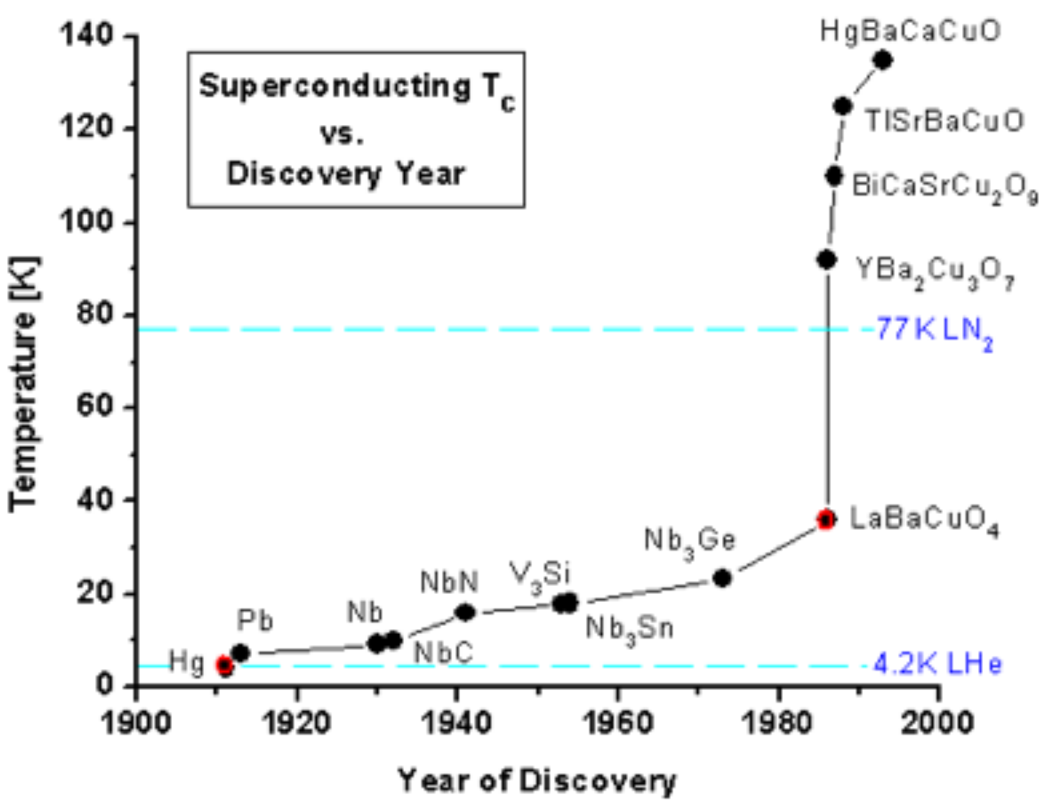
BCS and Landau theory have been **hugely** successful, but modern "quantum materials" often defy these classic approaches...

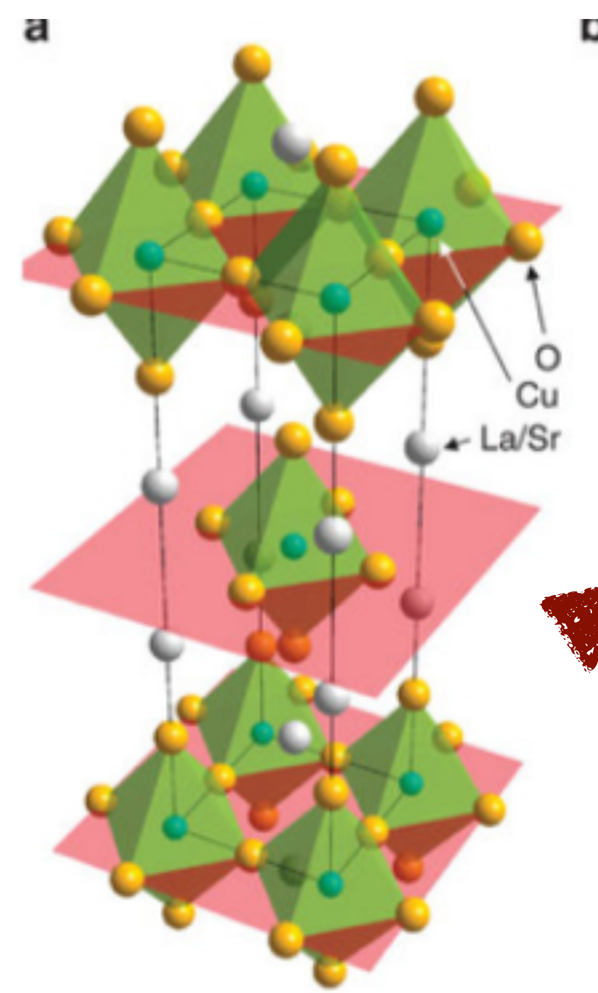
High-Temperature Superconductivity

1986



superconductivity in ceramic materials.

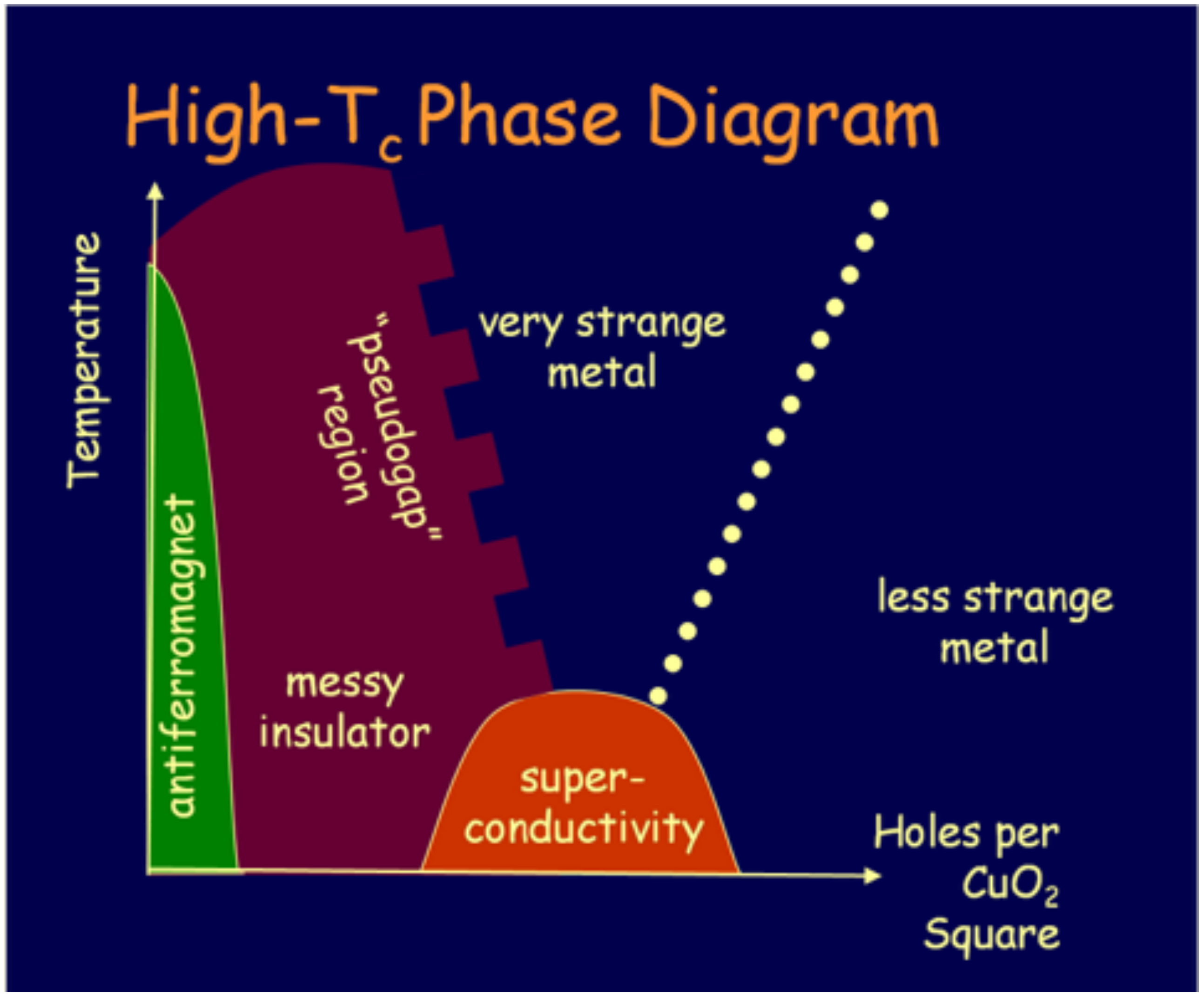




$La_{2-x}Sr_xCuO_4$

Cu-O planes

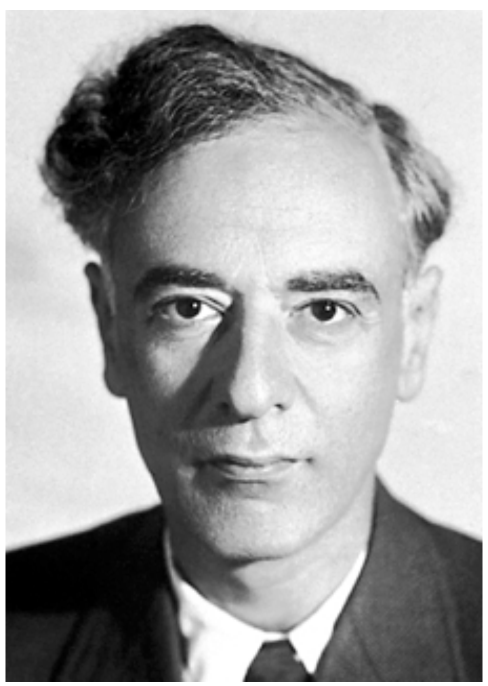
30 years later these still remain a mystery....



- **Competing phases:** order parameter theories difficult to formulate.
- There is (exotic) pairing of electrons
- Pairing is not mediated by lattice vibrations- **what is the glue** ?

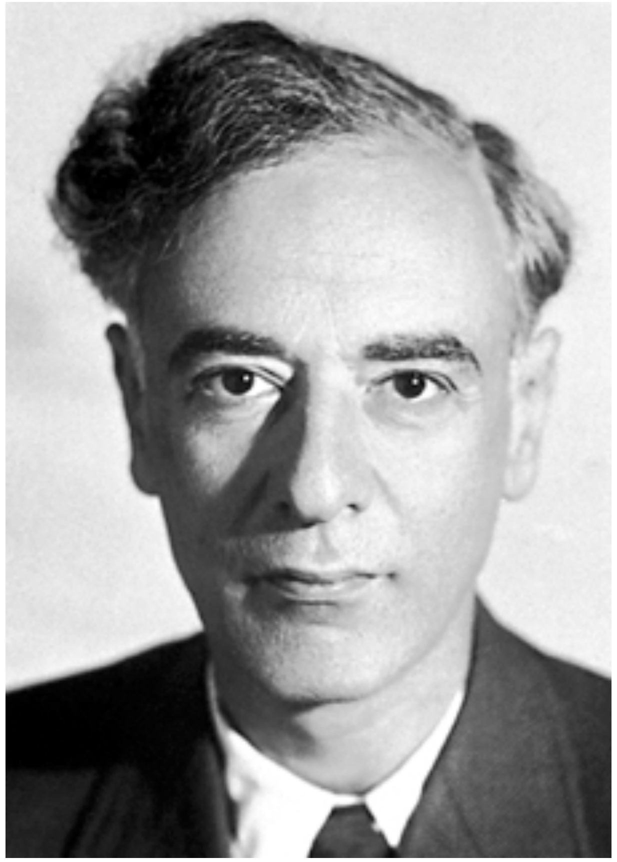
Summary

- More is surprising, fascinating and of great practical importance.
- More is difficult.
- Symmetry breaking is a powerful tool for understanding More.



Landau covers superfluids, superconductors, magnetic order (ferro or other), liquid crystals, crystals, ...

but not everything....



"This work contains many things which are new and interesting. Unfortunately, everything that is new is not interesting, and everything which is interesting, is not new."