Fasten Your Safety Belts: Turbulent Flows in Nature

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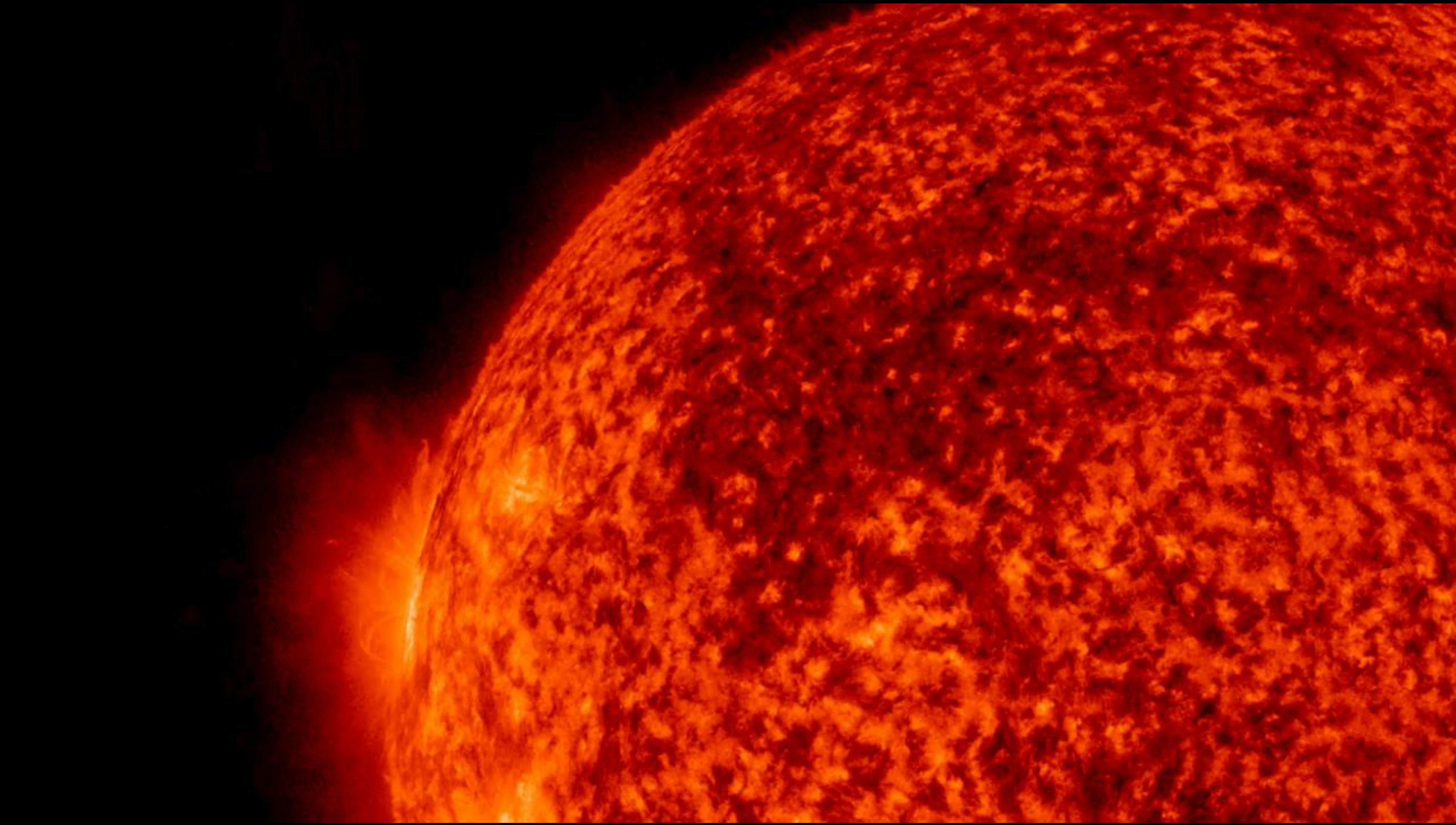
Thanks to Alex Schekochihin

Turbulence is ubiquitous

Carina Nebula

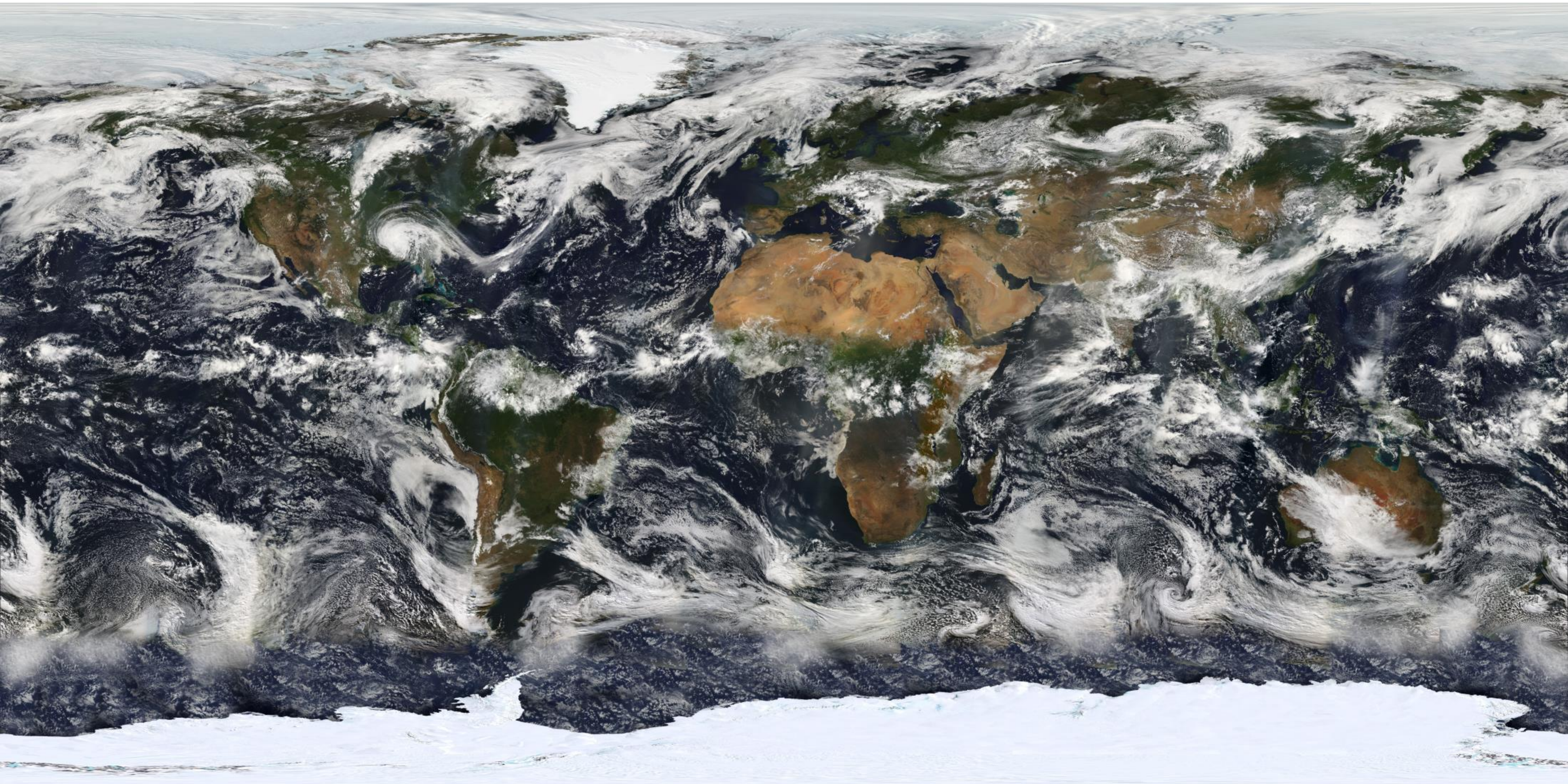
Credit: NASA, ESA, N. Smith (UC-Berkeley) and the Hubble Heritage Team

Turbulence is ubiquitous



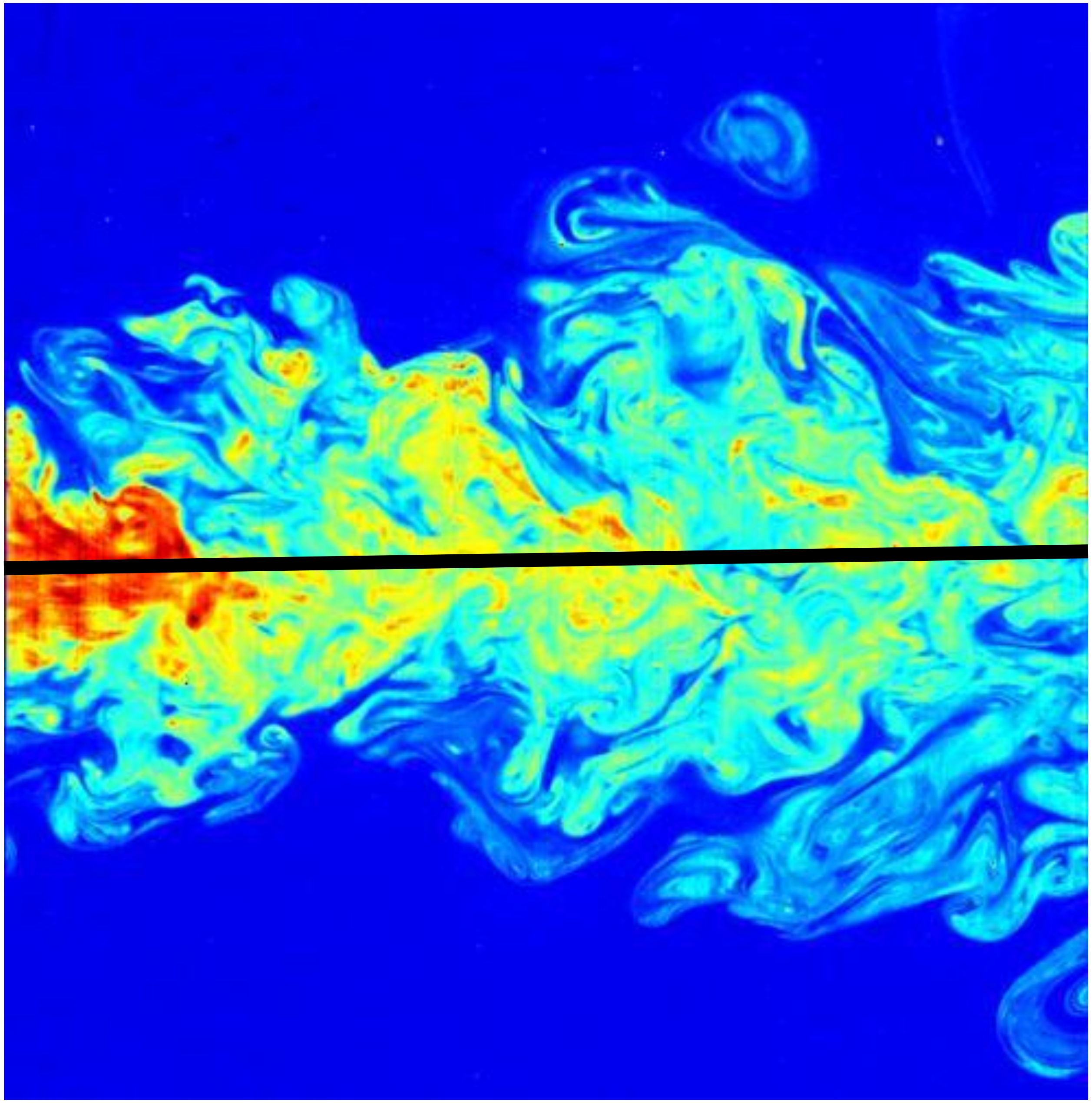
Surface of the sun
Credit: NASA/SDO/Goddard Space Flight Center/Joy Ng

Turbulence is ubiquitous

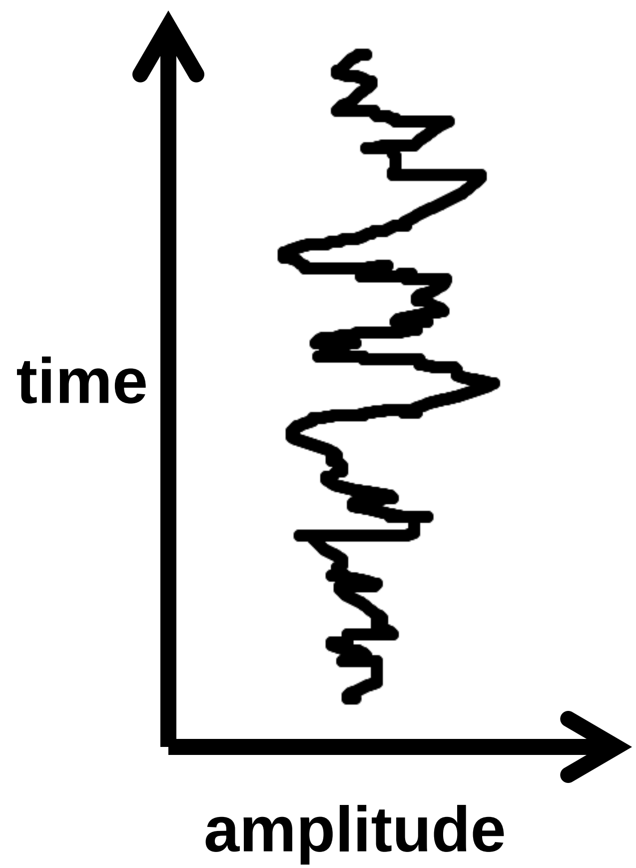


Earth's atmosphere
Credit: NASA/Goddard Space Flight Center Scientific Visualization Studio

Turbulent flow is irregular



False-color image of a submerged turbulent jet



Credit: C. Fukushima and J. Westerweel, Technical University of Delft, Netherlands

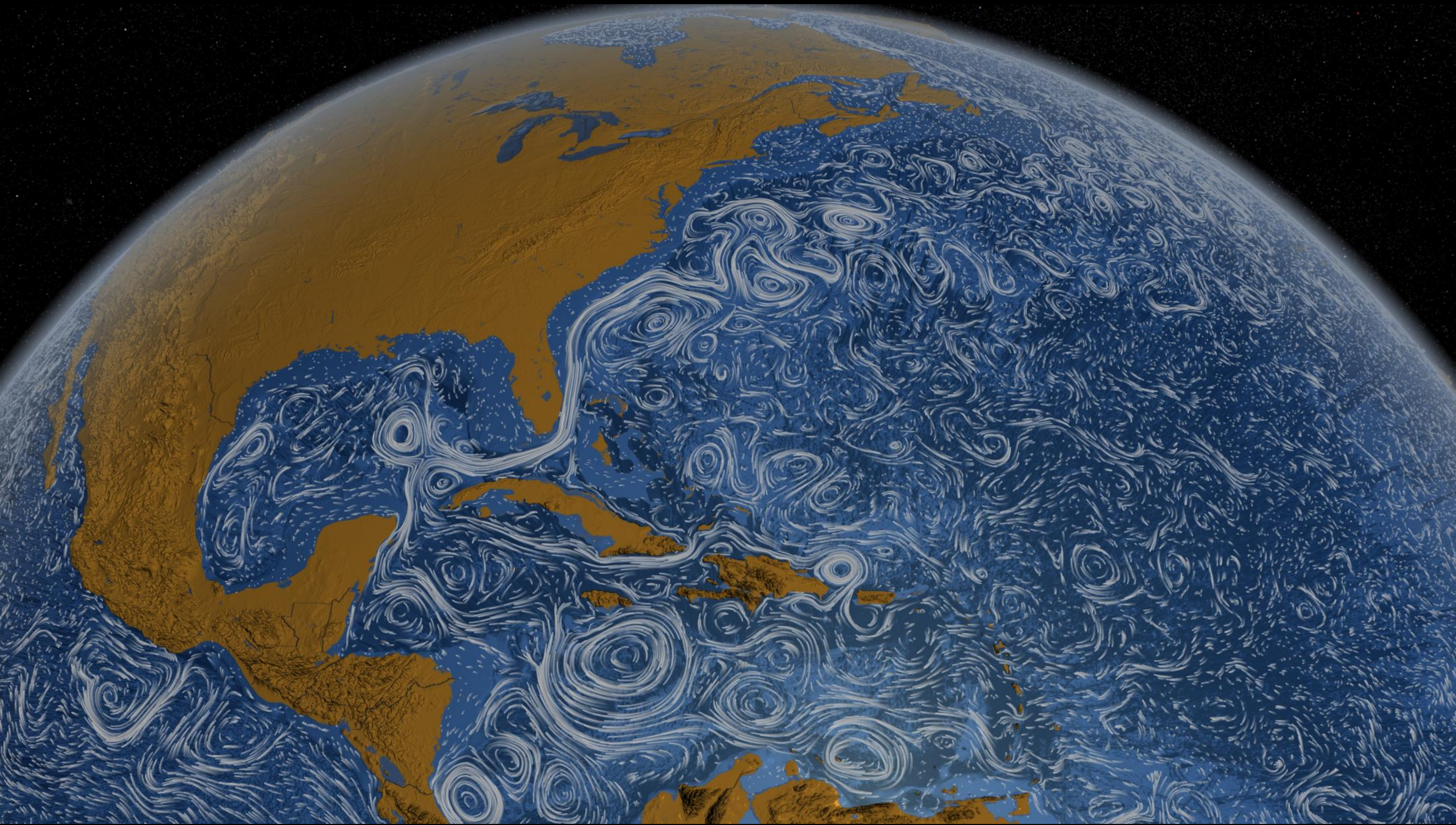
Turbulent flow enhances mixing

Credit: Connie Ma via flickr

Turbulence is rotational

Credit: NASA Langley Research Center

Turbulence spans many space-time scales



Ocean currents

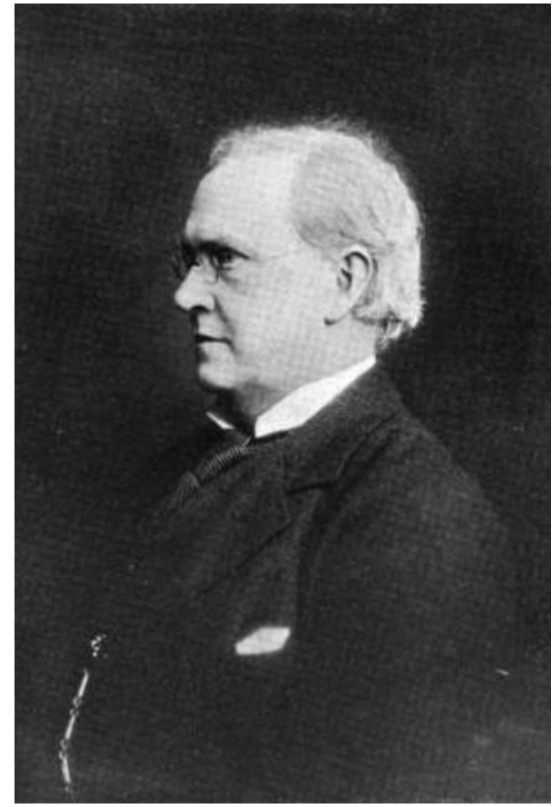
Credit: NASA/Goddard Space Flight Center Scientific Visualization Studio

Leonardo da Vinci and 'la turbolenza'

Leonardo da Vinci and 'la turbolenza'

"Observe the motion of the surface of the water, how it resembles that of hair, which has two motions – one depends on the weight of the hair, the other on the direction of the curls; thus the water forms whirling eddies, one part following the impetus of the chief current, and the other following the incidental motion and return flow."

Words of encouragement

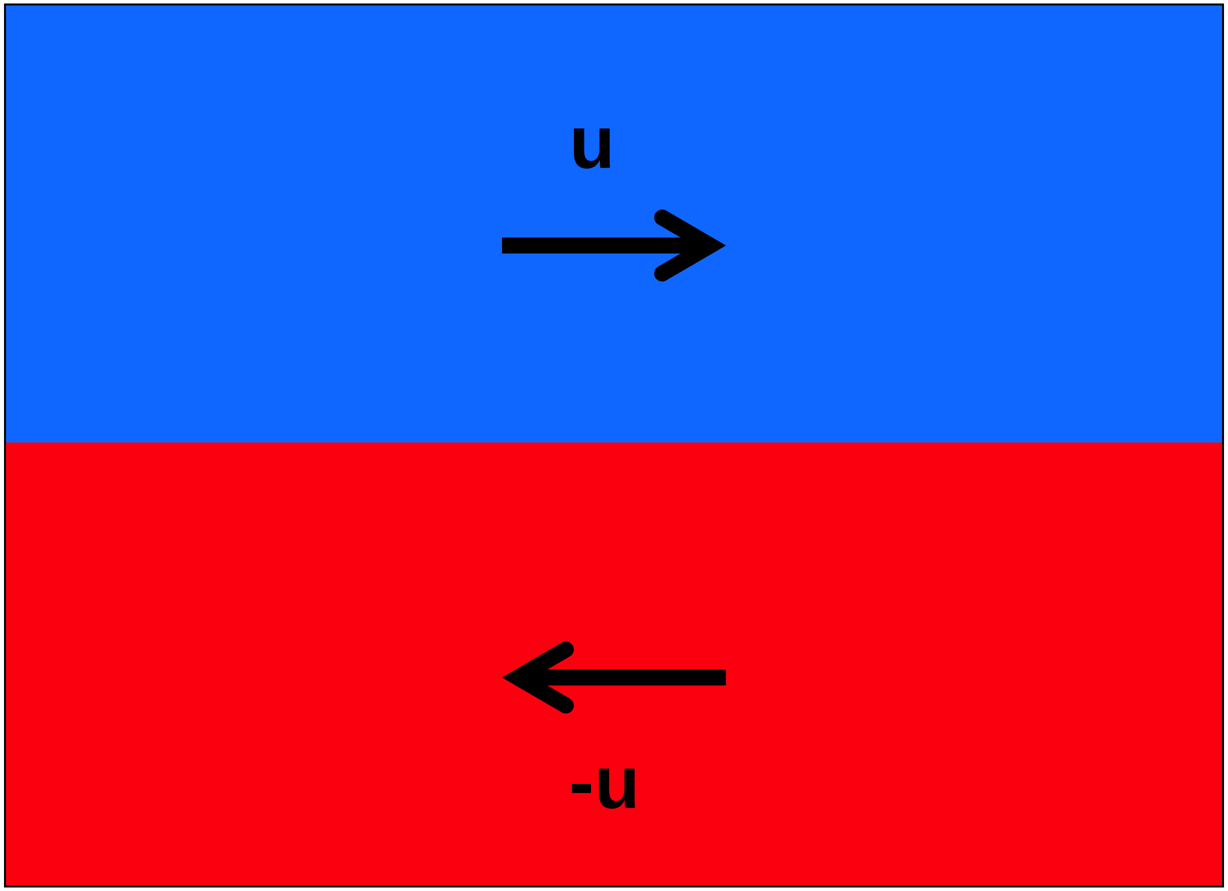


Sir Horace Lamb
(1904)

"When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first."

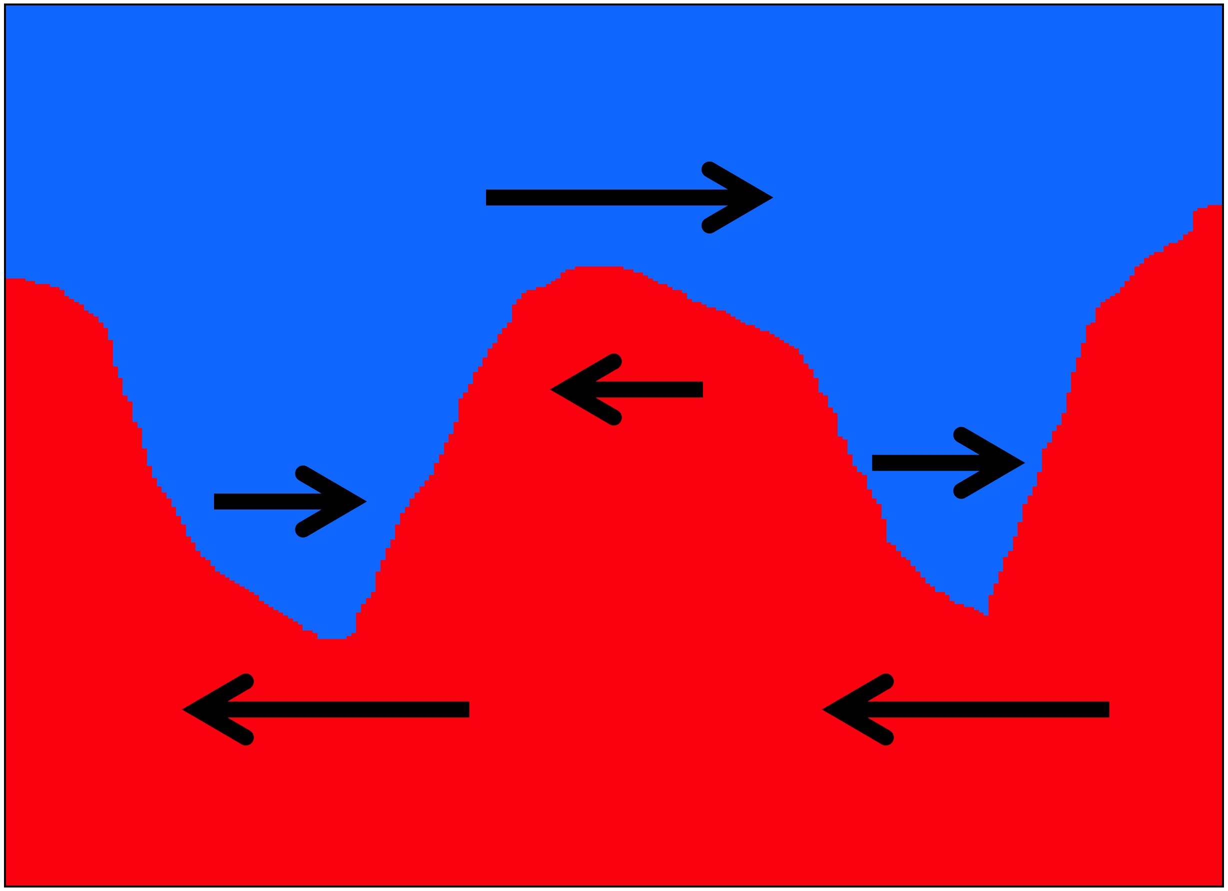
Werner Heisenberg (1933),
German Federal Archives

(In)stability of sheared flow



Two fluids with a sharp change in velocity at their interface

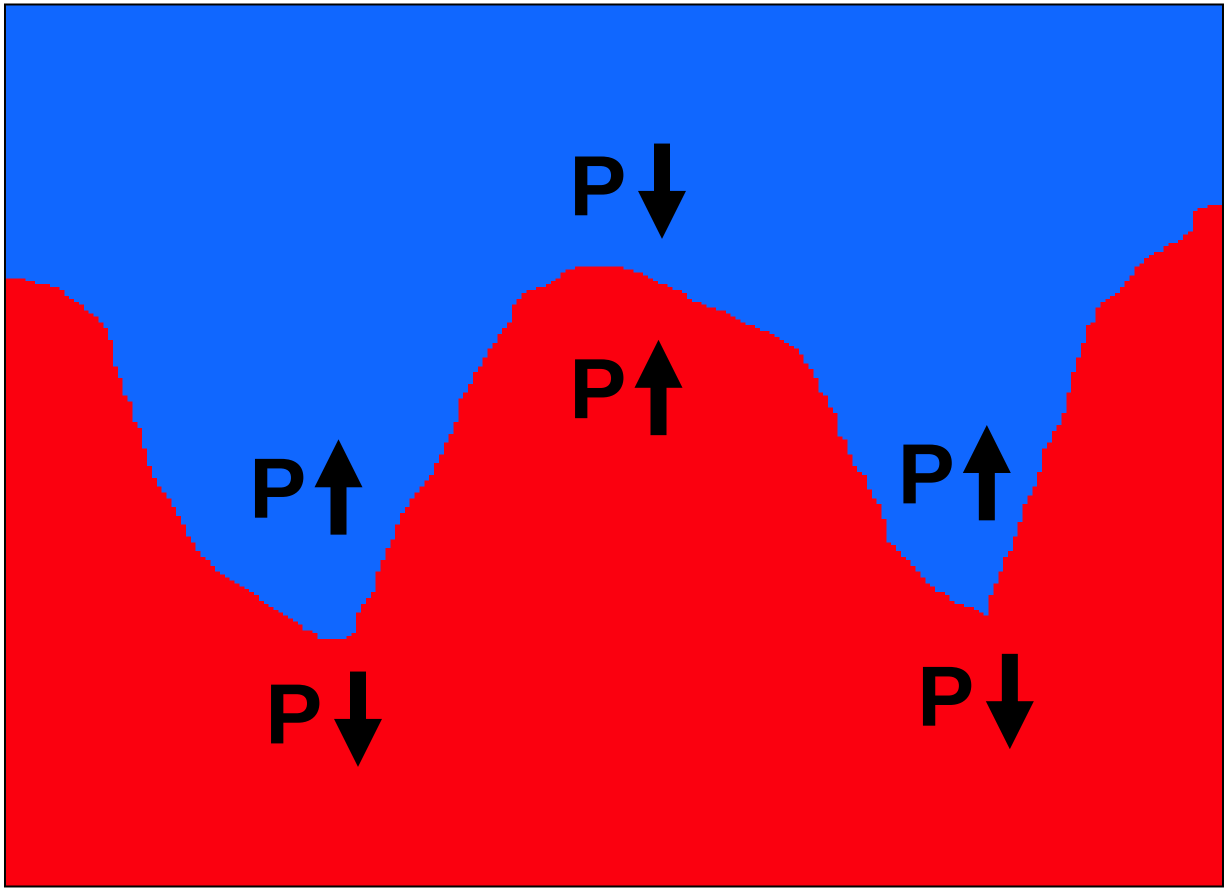
(In)stability of sheared flow



Conservation of mass → mass flow rate constant

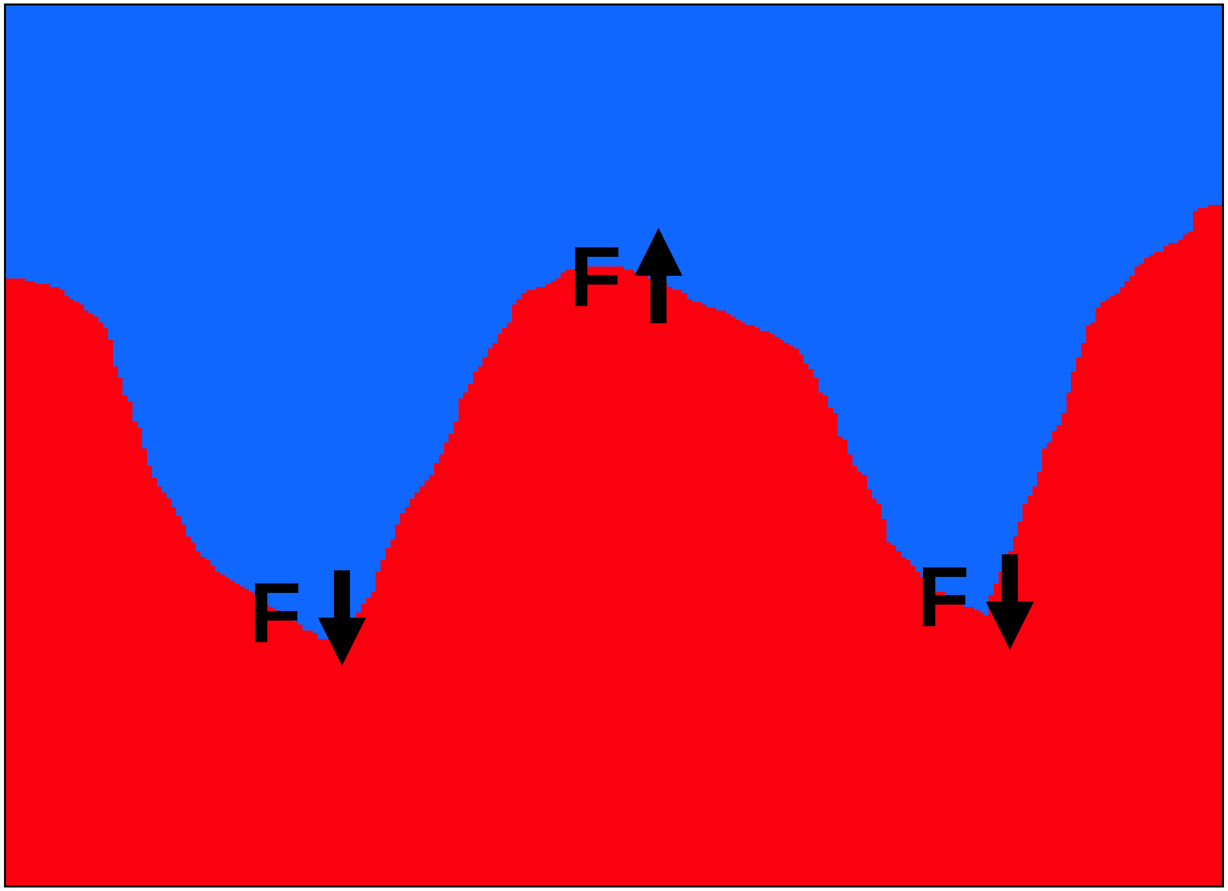
→ Flow speed inversely proportional to flow area

(In)stability of sheared flow



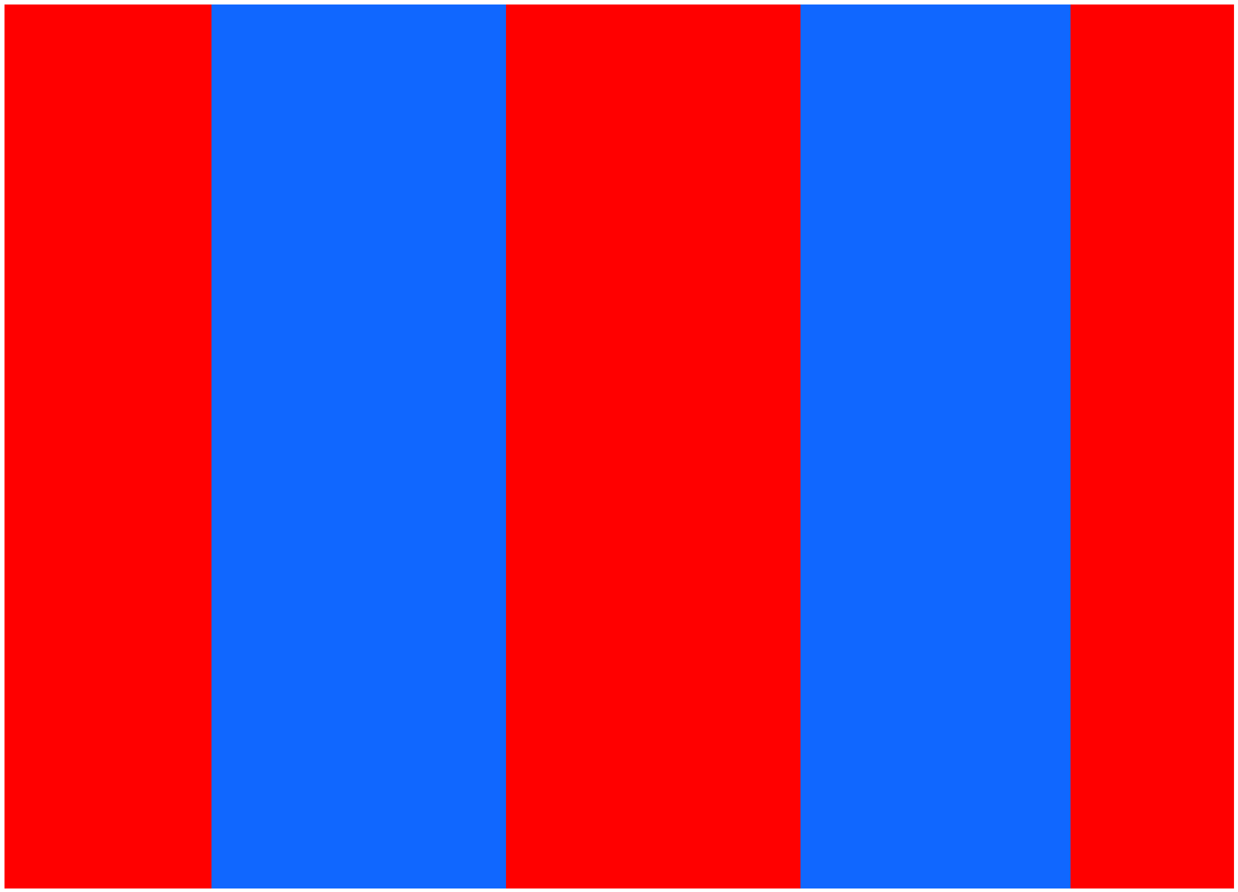
Conservation of energy a.k.a. Bernoulli's Principle → change in mean kinetic energy offset by change in thermal energy (pressure)

(In)stability of sheared flow



Pressure difference leads to net force that reinforces the initial perturbation → instability (Kelvin-Helmholtz)

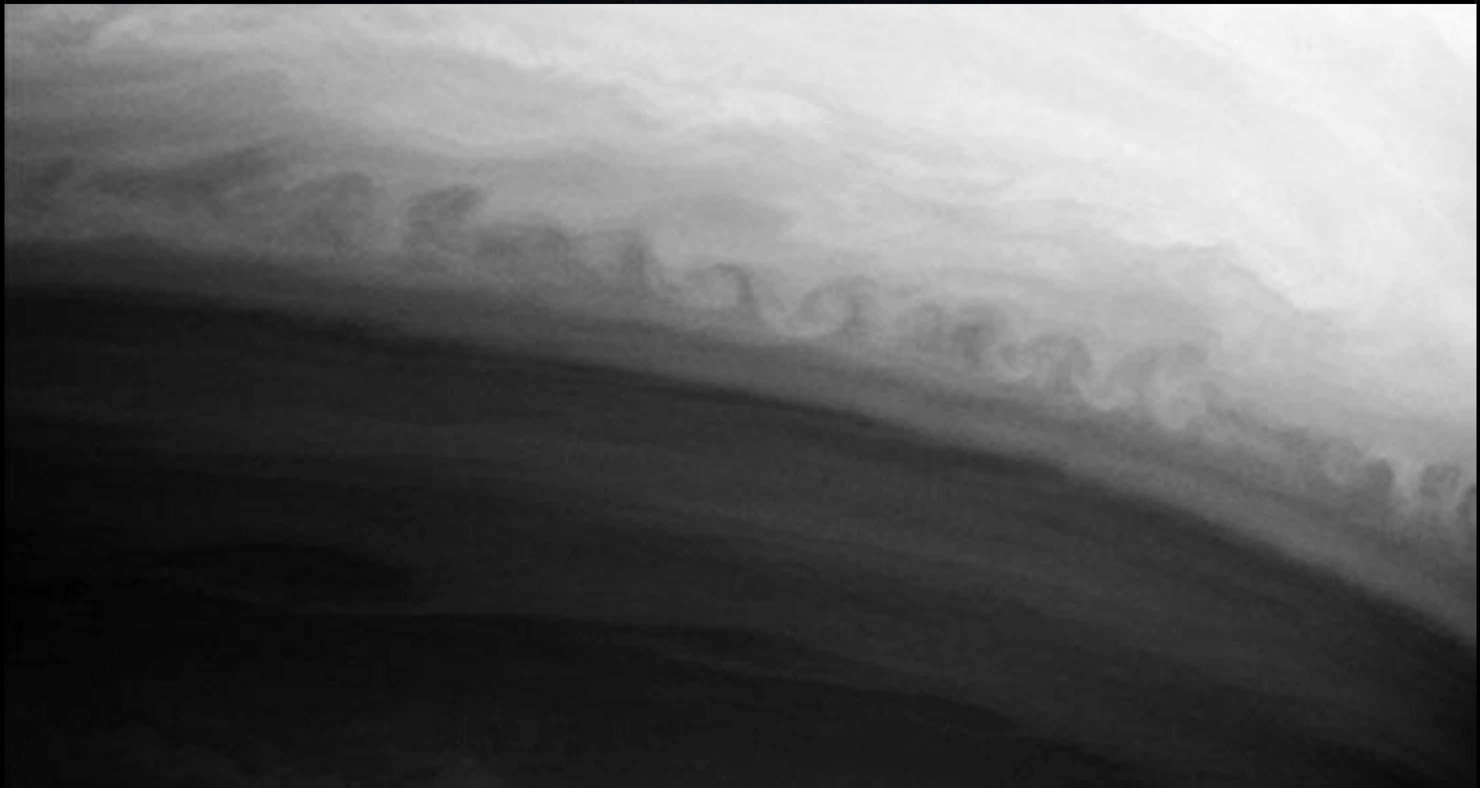
(In)stability of sheared flow



Instability leads to new ‘equilibrium’ that itself is unstable, which leads to new ‘equilibrium’…

Transition to turbulence

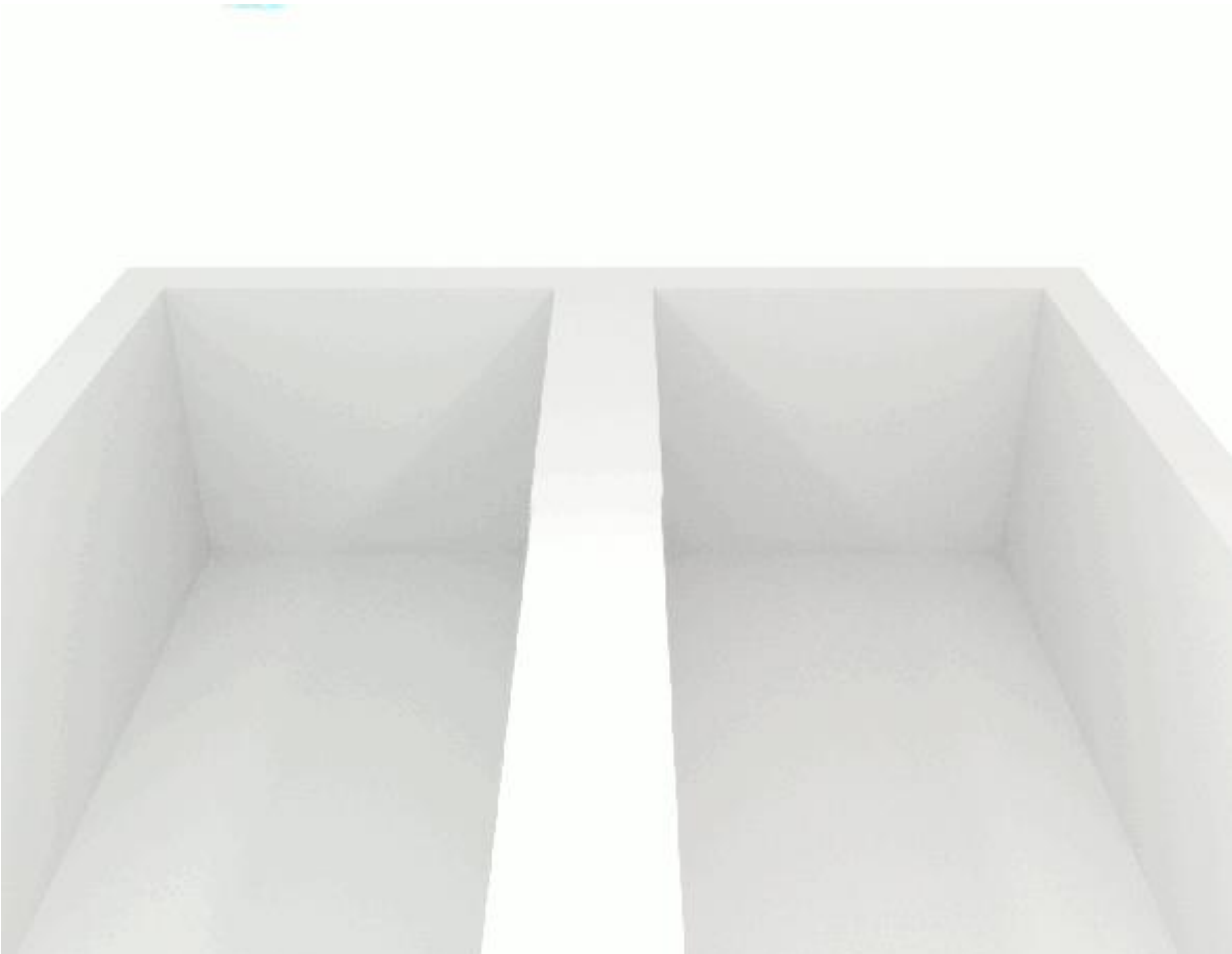
K-H clouds
Credit: Brocken Inaglory



K-H Saturn
Credit: NASA

V. Van Gogh, *The Starry Night (*MoMa, NY)

Viscosity and the Reynolds number



Credit: Synapcticrelay via commons.wikimedia.org

$$\mathrm{Re} \doteq \frac{UL}{\nu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$

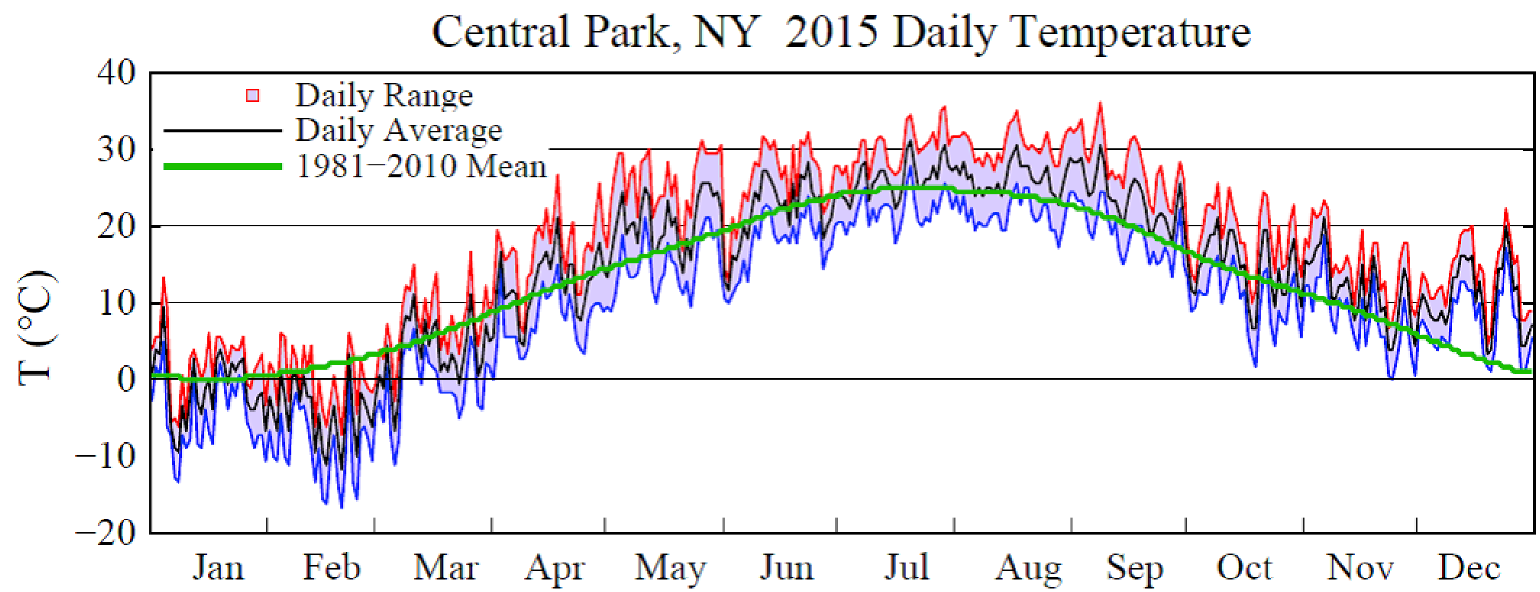
What we can say exactly about turbulence

What we can say exactly about turbulence

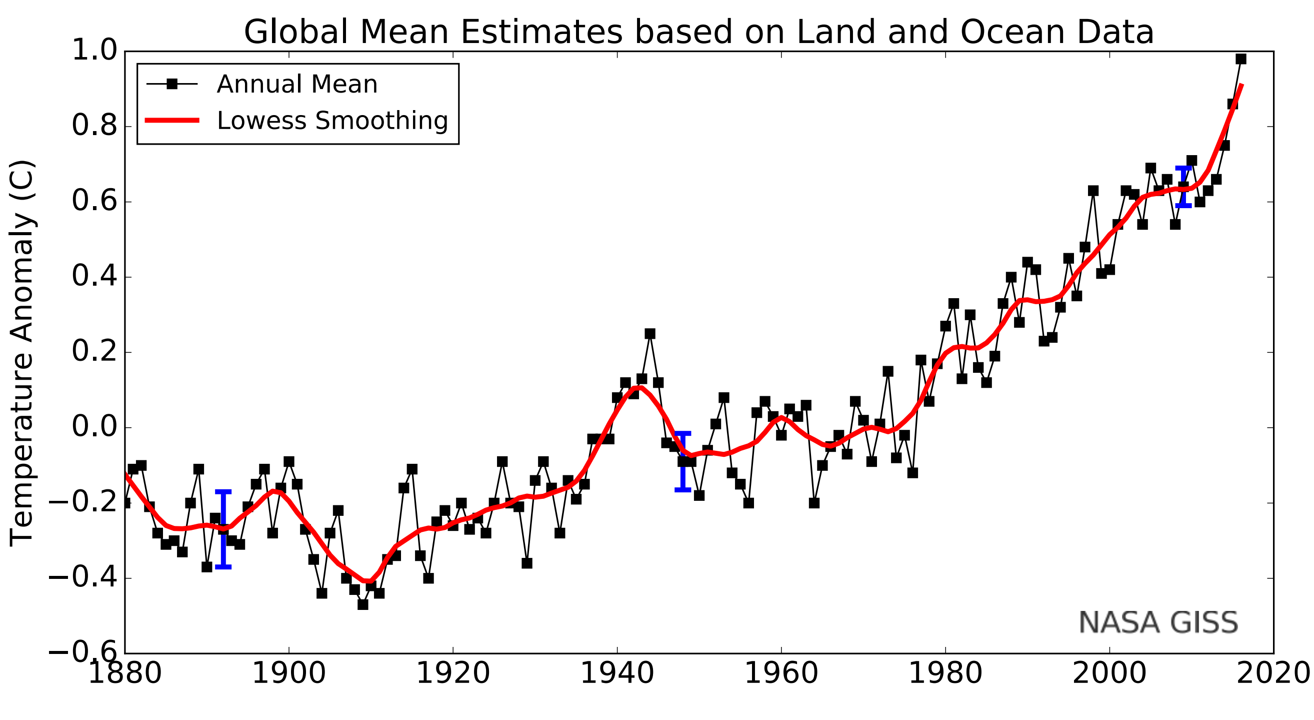
Turbulence is irregular and multiscale

☹

Statistical description of turbulence



Credit: Earth Institute, Columbia University. Data source NOAA



Credit: NASA GISS

Statistical description of turbulence

$$\frac{d\mathbf{u}}{dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\mathbf{u} \doteq \overline{\mathbf{u}} + \delta\mathbf{u} \qquad \overline{\delta\mathbf{u}} = 0$$

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}$$

$$\overline{\frac{d\mathbf{u}}{dt}} = \frac{\partial \overline{\mathbf{u}}}{\partial t} + \overline{\mathbf{u}} \cdot \nabla \overline{\mathbf{u}} + \overline{\delta\mathbf{u} \cdot \nabla \delta\mathbf{u}}$$

Need $\overline{\delta\mathbf{u}\delta\mathbf{u}}$ to get $\overline{\mathbf{u}}$. But $\overline{\delta\mathbf{u}\delta\mathbf{u}}$ requires $\overline{\delta\mathbf{u}\delta\mathbf{u}\delta\mathbf{u}}$.

Energy balance in turbulence

$$\mathbf{u} \cdot \left(\frac{d\mathbf{u}}{dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f} \right)$$

$$\Rightarrow \frac{d\varepsilon}{dt} = P_{\mathrm{inj}} - P_{\mathrm{diss}}$$

$$\varepsilon = \frac{1}{2} \int \frac{d^3\mathbf{r}}{V} \rho u^2 \qquad P_{\mathrm{inj}} = \int \frac{d^3\mathbf{r}}{V} \rho \mathbf{u} \cdot \mathbf{f}$$

$$P_{\mathrm{diss}} = \int \frac{d^3\mathbf{r}}{V} \rho \nu \left| \nabla \mathbf{u} \right|^2$$

Energy balance in steady-state turbulence

$$\frac{d\varepsilon}{dt} = P_{\mathrm{inj}} - P_{\mathrm{diss}} = 0$$

$$P_{\mathrm{inj}} = \int \frac{d^3\mathbf{r}}{V} \rho \mathbf{u} \cdot \mathbf{f} \sim \frac{\rho U^3}{L}$$

$$P_{\mathrm{diss}} = \int \frac{d^3\mathbf{r}}{V} \rho \nu \left|\nabla \mathbf{u}\right|^2 \sim \frac{\rho \nu U^2}{L^2}$$

$$\Rightarrow \frac{P_{\mathrm{inj}}}{P_{\mathrm{diss}}} \sim \frac{UL}{\nu} = \mathrm{Re} \gg 1$$ ← Imbalance!

Energy balance in steady-state turbulence

$$\frac{d\varepsilon}{dt} = P_{\mathrm{inj}} - P_{\mathrm{diss}} = 0$$

$$P_{\mathrm{inj}} = \int \frac{d^3\mathbf{r}}{V} \rho \mathbf{u} \cdot \mathbf{f} \sim \frac{\rho U^3}{L}$$

$$P_{\mathrm{diss}} = \int \frac{d^3\mathbf{r}}{V} \rho \nu \left| \nabla \mathbf{u} \right|^2 \sim \frac{\rho \nu U^2}{L^2}$$

Turbulence must create small scales to overcome imbalance!

Energy balance in steady-state turbulence

$$\frac{d\varepsilon}{dt} = P_{\rm inj} - P_{\rm diss} = 0$$

At what scale is balance achieved?

Dimensional analysis:

$$\ell_\nu = \ell_\nu(\nu, P_{\rm inj}) \quad \ell_\nu \sim \left(\frac{\rho\nu^3}{P_{\rm inj}}\right)^{1/4} \sim L\mathrm{Re}^{-3/4}$$

$$\mathrm{Re} \gg 1 \Rightarrow L \gg \ell \gg \ell_\nu$$

outer scale

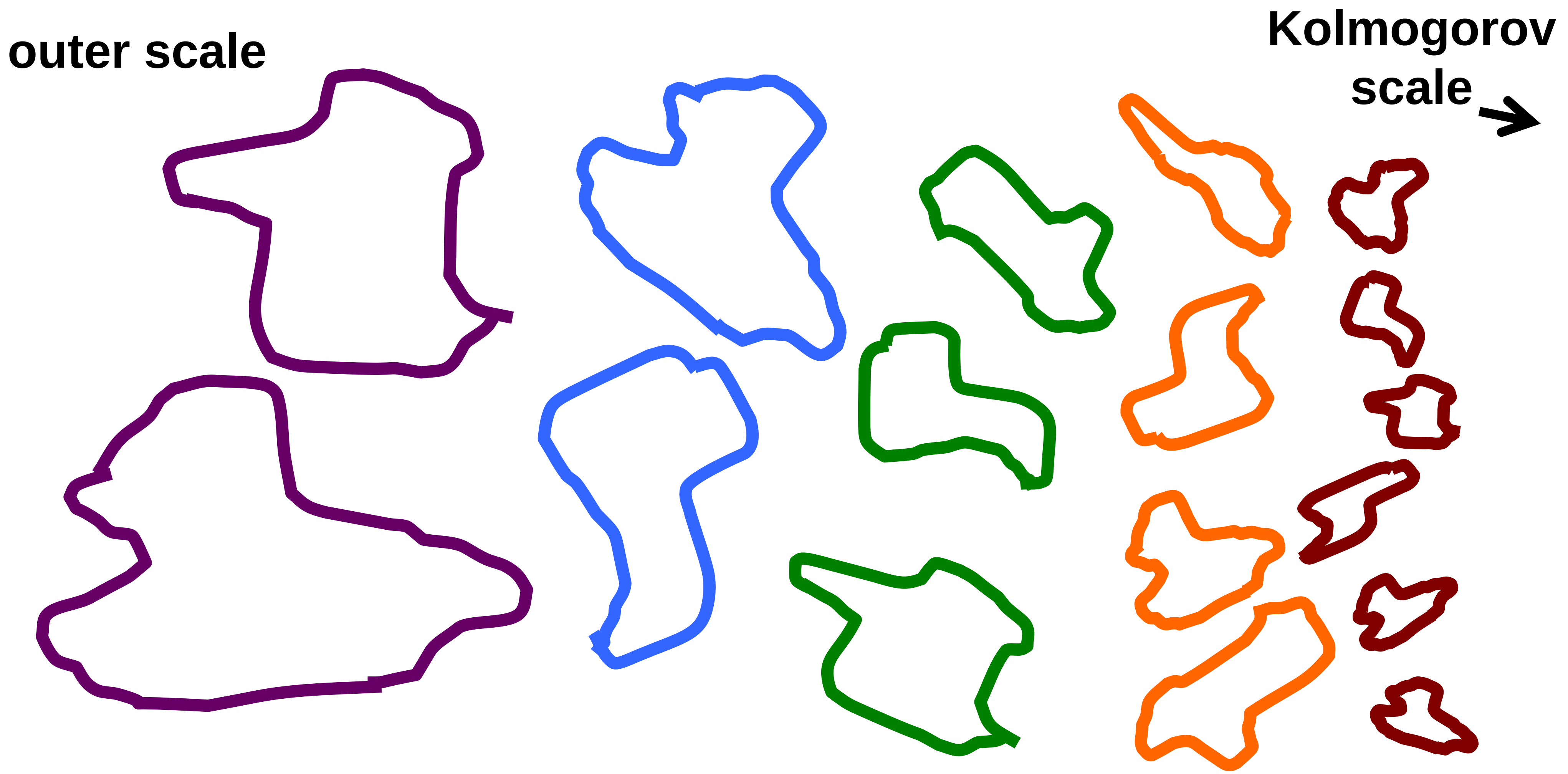
inertial range

Kolmogorov scale

Richardson’s turbulence cascade

Big whorls have little whorls
which feed on their velocity,
And little whorls have lesser whorls
and so on to viscosity.

L. F. Richardson
(1922)



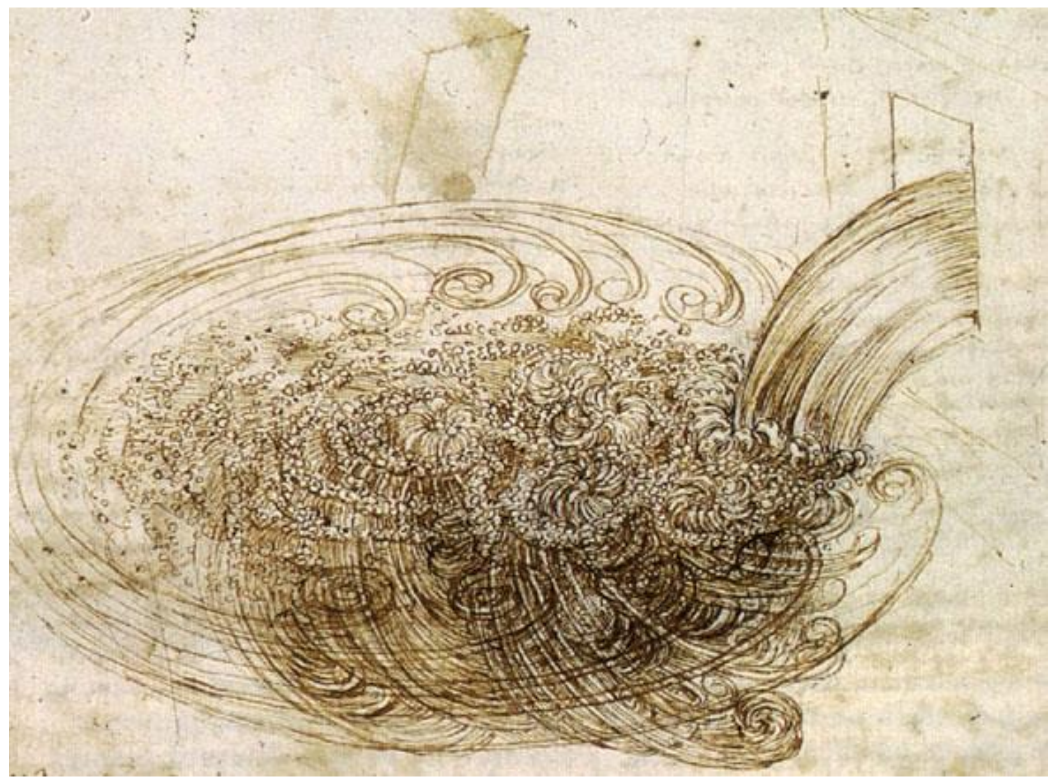
Kolmogorov’s turbulence cascade

To proceed, make assumptions about turbulence properties:

- Universality (no special systems)

Universality

Credit: Gary Settles



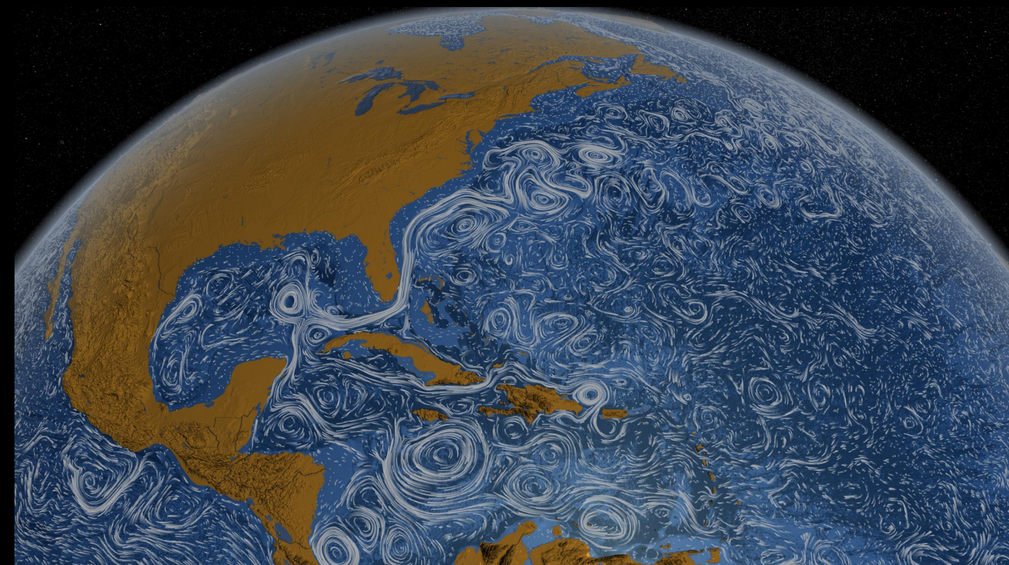
Kolmogorov's turbulence cascade

To proceed, make assumptions about turbulence properties:

- Universality (no special systems)
- Homogeneity (no special locations)
- Isotropy (no special directions)
- Locality (no special scales)

Any broken symmetries due to details of outer scale are restored in the inertial range

Homogeneous, isotropic, local

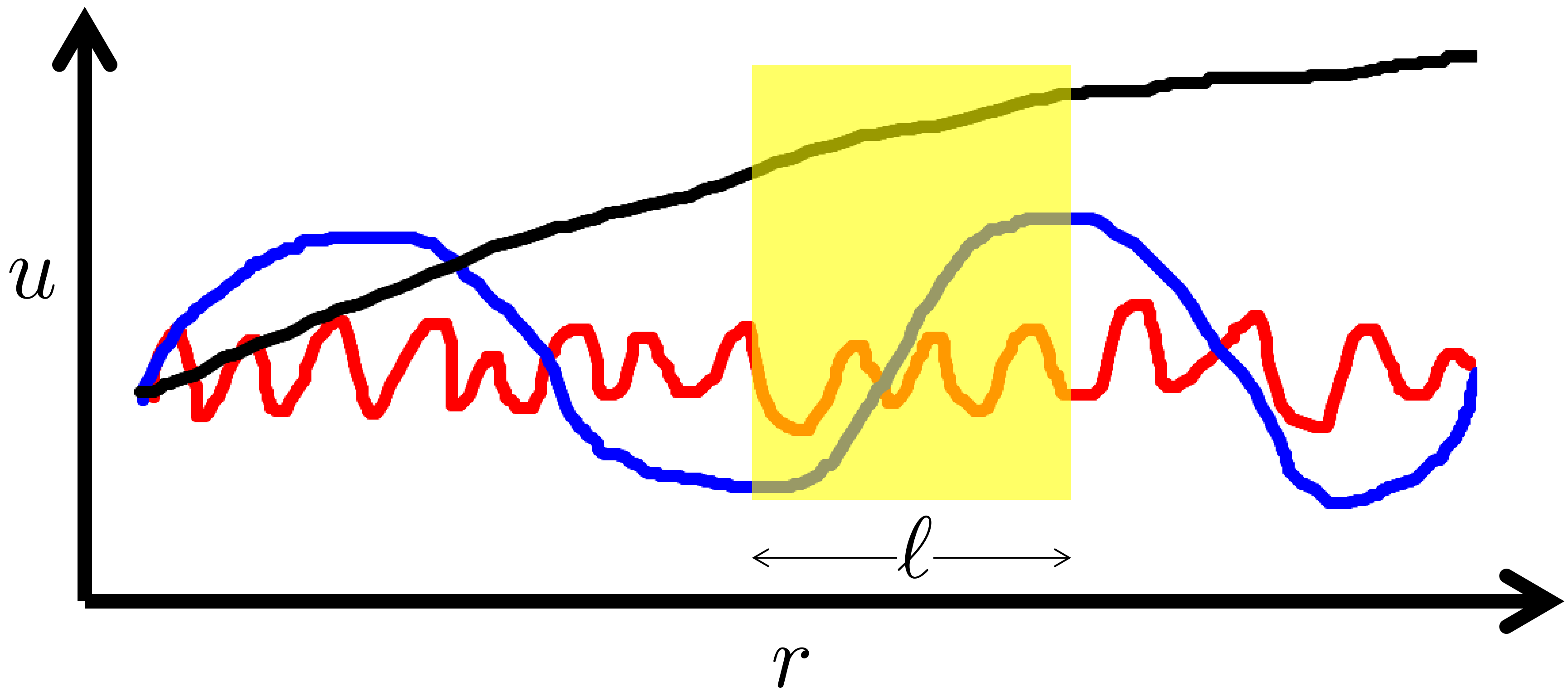


Credit: Steven Mathey

Kolmogorov's turbulence cascade

Want to know $\delta \mathbf{u}_{\boldsymbol{\ell}}(\mathbf{r}) = \mathbf{u}(\mathbf{r} + \boldsymbol{\ell}) - \mathbf{u}(\mathbf{r})$

= velocity 'increment'



Kolmogorov's turbulence cascade

Want to know $\delta \mathbf{u}_{\boldsymbol{\ell}}(\mathbf{r}) = \mathbf{u}(\mathbf{r} + \boldsymbol{\ell}) - \mathbf{u}(\mathbf{r})$

Homogeneous + isotropic: $\delta u_\ell = u(r + \ell) - u(r)$

In statistical equilbrium, energy flux through each scale is scale-independent:

$$\mathrm{const} = P_{\mathrm{inj}} \sim P_{\mathrm{cascade}} \sim \frac{\rho \delta u_\ell^2}{\tau_\ell}$$

Dimensional analysis: $\tau_\ell \sim \dfrac{\ell}{\delta u_\ell}$

$$\Rightarrow \delta u_\ell \propto \ell^{1/3}$$

Kolmogorov's energy spectrum

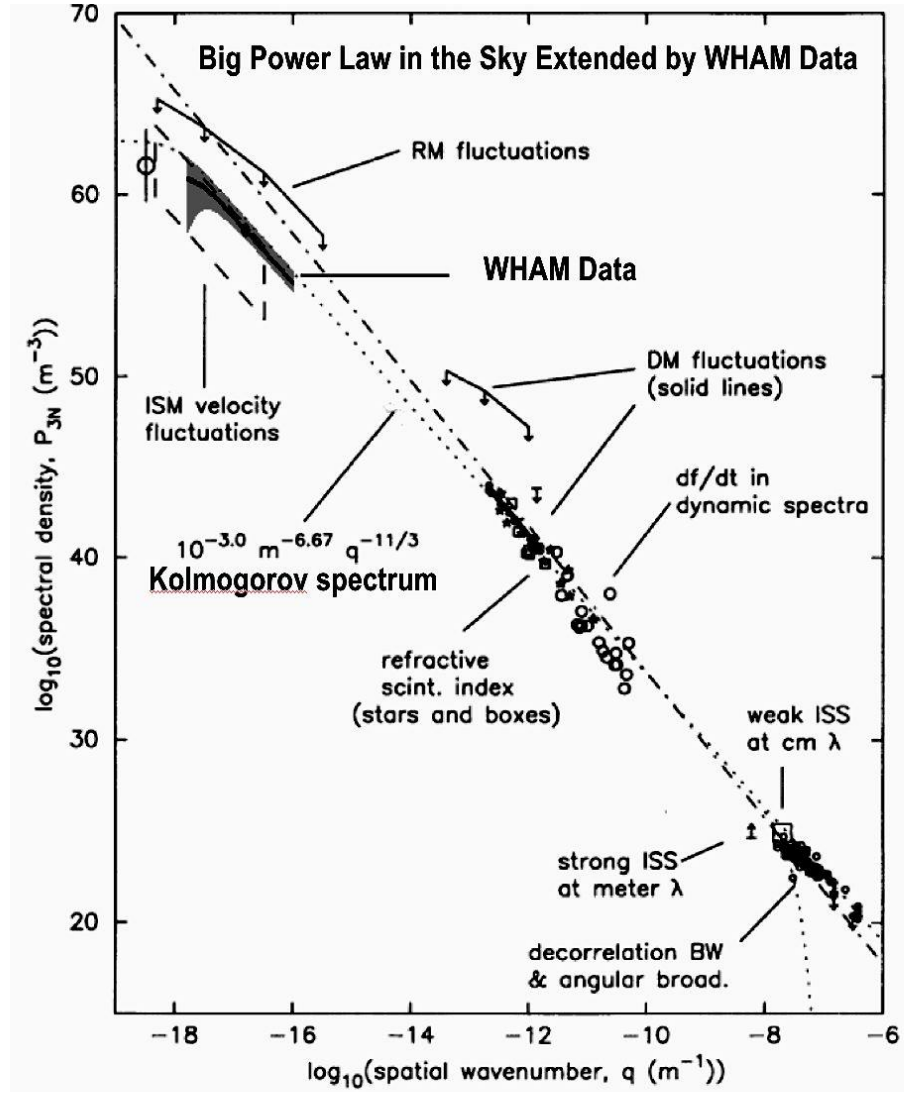
$$\delta u_\ell \propto \ell^{1/3}$$

$$\delta u_\ell^2 \sim \int_{1/\ell}^{\infty} dk E(k) \Rightarrow E(k) \propto k^{-5/3}$$

Kolmogorov's energy spectrum

$$E(k) \propto k^{-5/3}$$

"Big power law in the sky"



Armstrong, Cordes, & Rickett, Nature (1981) + mods by Lazarian, et al., Space Sci. Rev (2012)

A small sample of active research areas

- Coherent structures
- Intermittency of dissipation
- Transition to turbulence
- Closure models and numerical simulations
- Multiphysics turbulence (chemistry + fluid dynamics + ...)
- Magnetized fluids (plasma)
- Beyond fluids: kinetic turbulence
- Much more...

Turbulence in magnetized plasma and ‘critical balance’

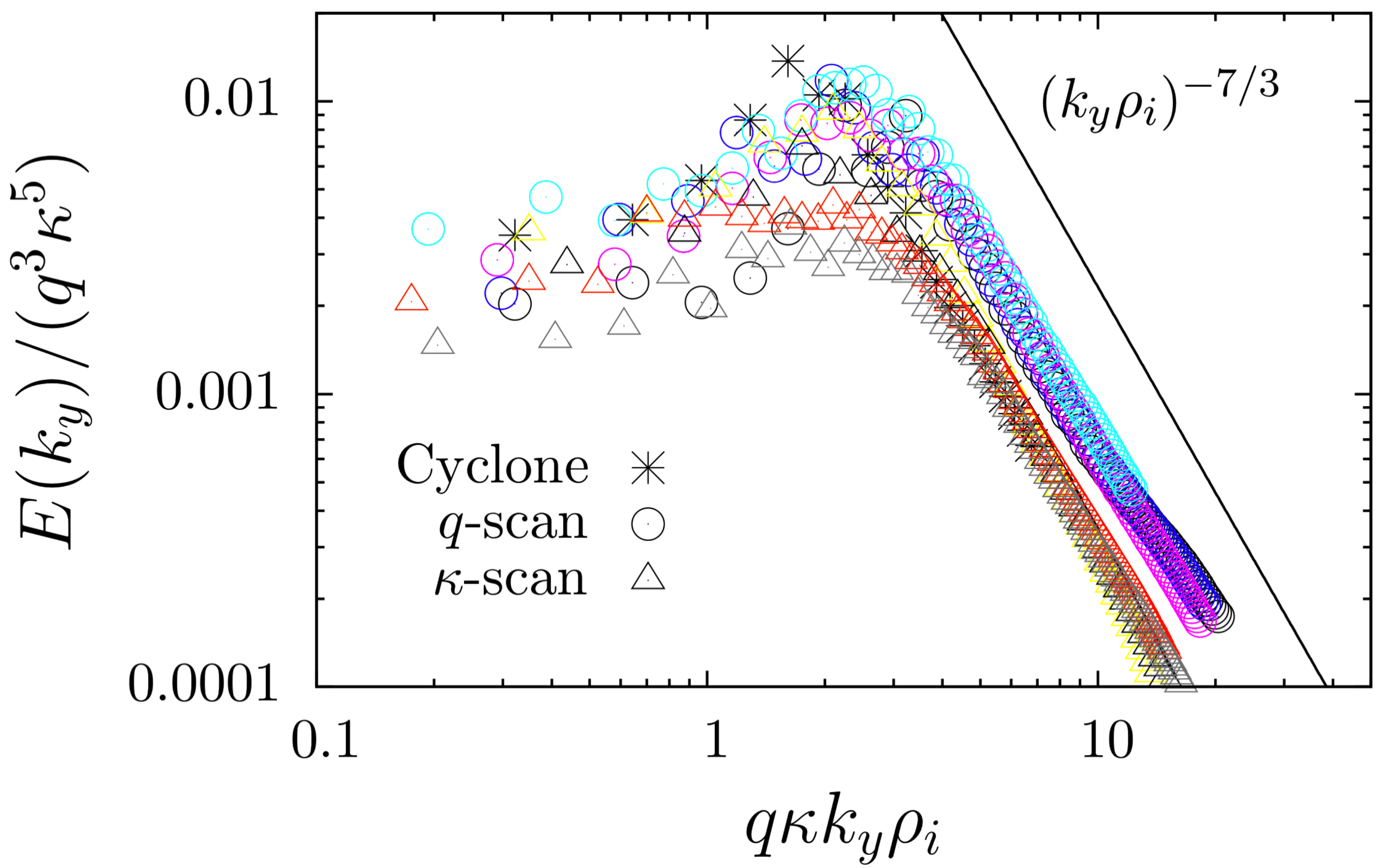
DIII-D Shot 121717

GYRO Simulation

Cray X1E, 256 MSPs

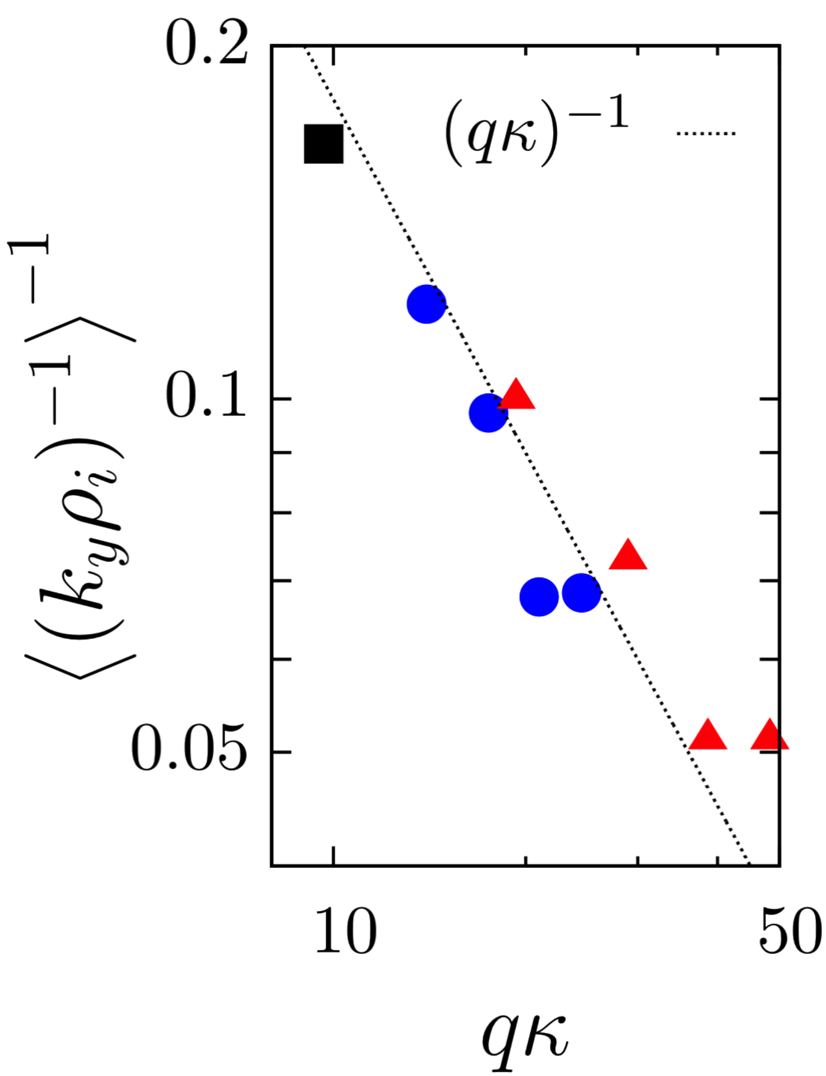
Credit: J. Candy
General Atomics

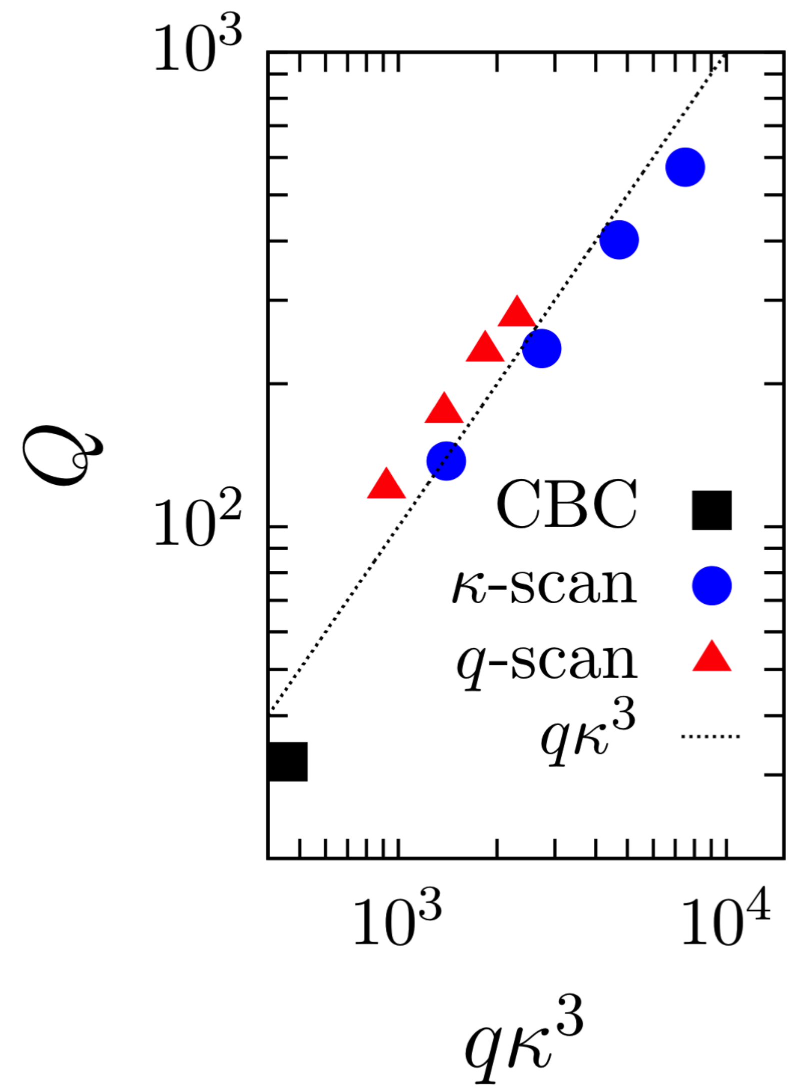
Energy spectrum for critically balanced, kinetic turbulence



Barnes, Parra, & Schekochihin, Phys Rev Lett 2011

Energy confinement scalings

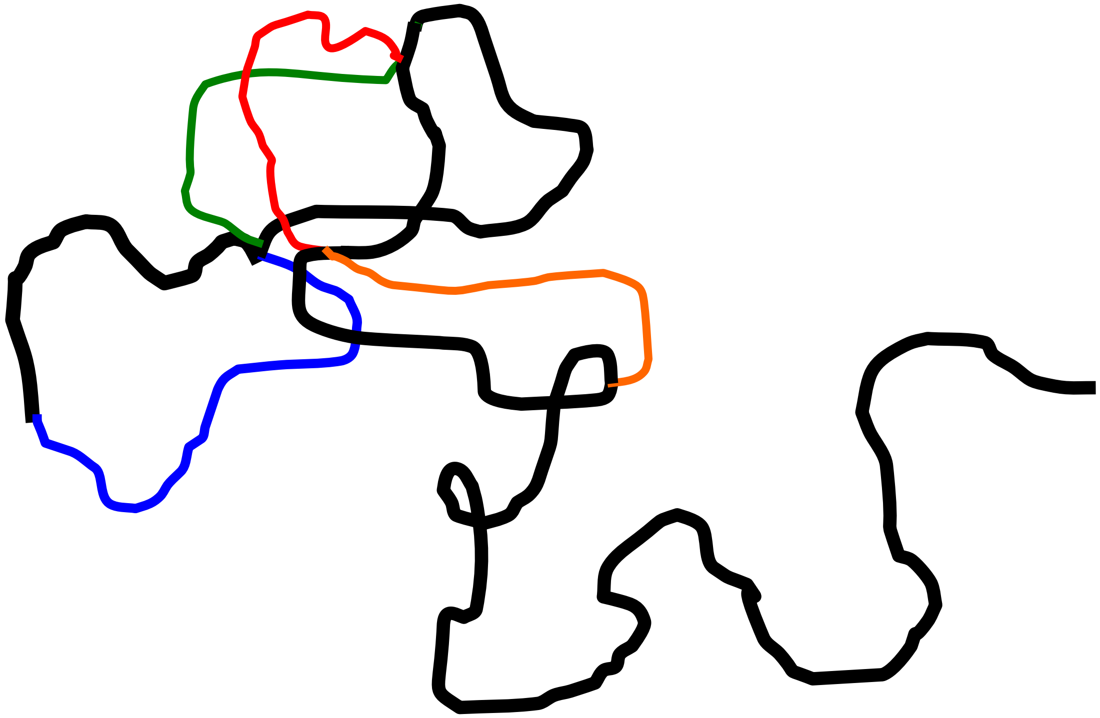




$\kappa = R/L_T$

Barnes et al., PRL 2011

Turbulence and random walks



Random walk: (time to move move distance L) = (time per step) x $(L/d)^2$ steps

L = system size, d = eddy size, time per step = turbulence time

Viscosity and the Reynolds number

$$\frac{d\mathbf{u}}{dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Non-dimensionalize: Multiply Navier-Stokes by $\frac{L}{U^2}$ **and define**

$$\tilde{\mathbf{u}} \doteq \frac{\mathbf{u}}{U},\ \tilde{p} \doteq \frac{p}{\rho U^2},\ \tilde{\mathbf{f}} \doteq \frac{\mathbf{f}L}{U^2},\ \tilde{t} \doteq \frac{tL}{U},\ \tilde{\nabla} \doteq L\nabla$$

$$\frac{d\tilde{\mathbf{u}}}{d\tilde{t}} = -\tilde{\nabla}\tilde{p} + \frac{1}{\mathrm{Re}}\tilde{\nabla}^2\tilde{\mathbf{u}} + \tilde{\mathbf{f}}$$

$$\mathrm{Re} \doteq \frac{UL}{\nu} = \frac{\text{inertial forces}}{\text{viscous forces}}$$