Identical Particles - from One to Many

John Chalker

Electrical resistivity

Copper	$1.7 \times 10^{-8}\,\Omega\mathrm{m}$
Diamond	$1.0 \times 10^{12}\,\Omega\mathrm{m}$

More is Different

P. W. Anderson (1972)

“The reductionist hypothesis does not by any means imply a ‘constructionist’ one: The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe.”

“The behaviour of large aggregates of particles, it turns out, is not to be understood in terms of simple extrapolation. Instead, at each level of complexity entirely new properties appear.”

2016 Nobel Prize in Physics

For theoretical discoveries of topological phase transitions and topological phases of matter

D. J. Thouless, F. D. M. Haldane and J. M Kosterlitz

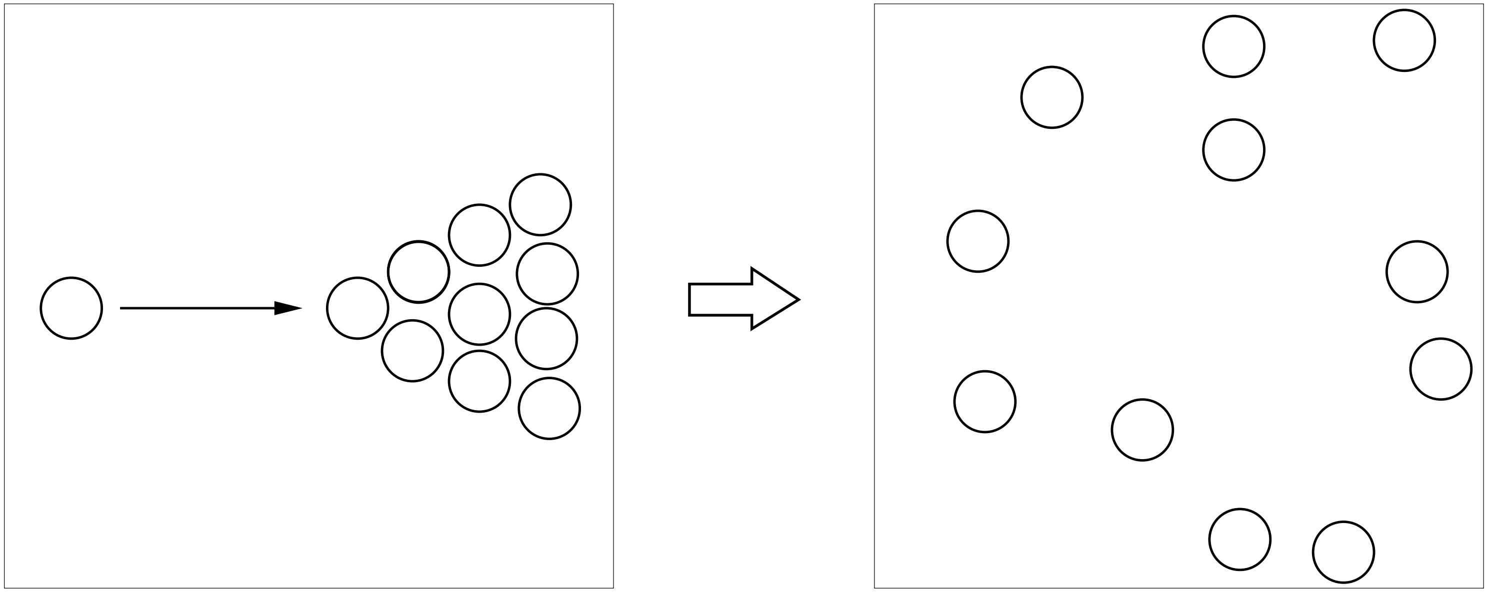
Outline

Identical particles in quantum mechanics

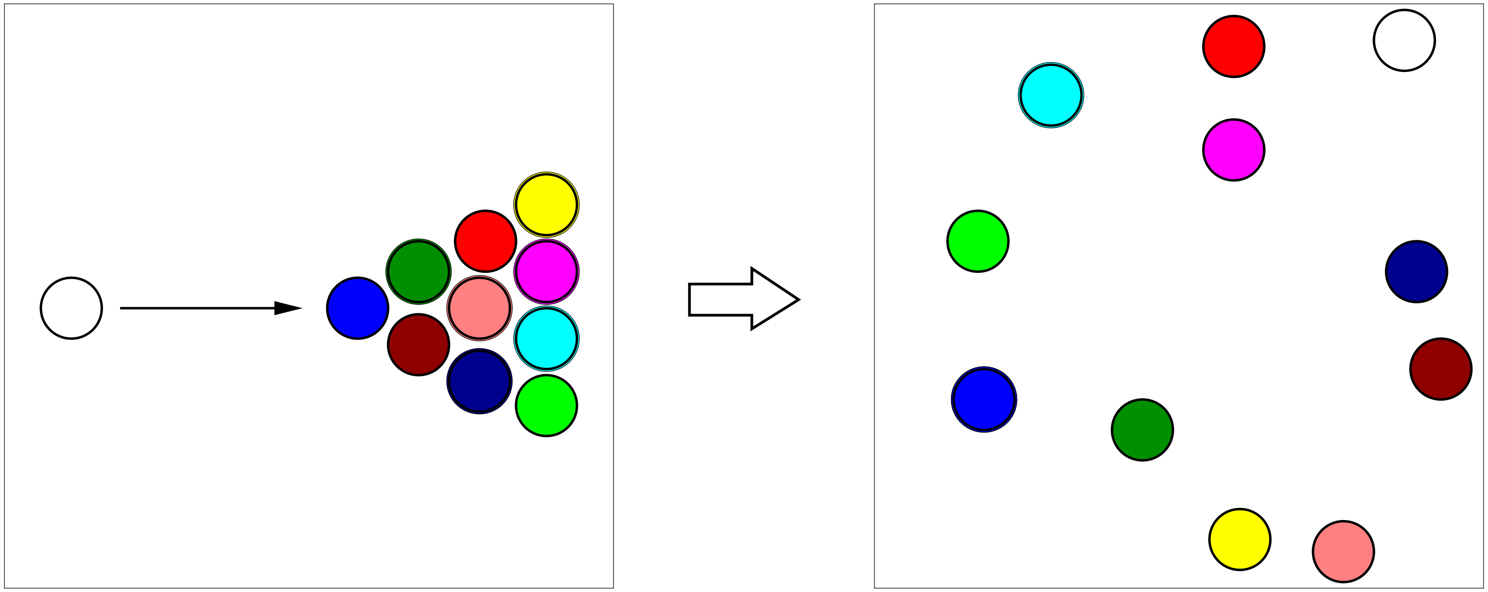
Metals and insulators

The integer quantum Hall effect

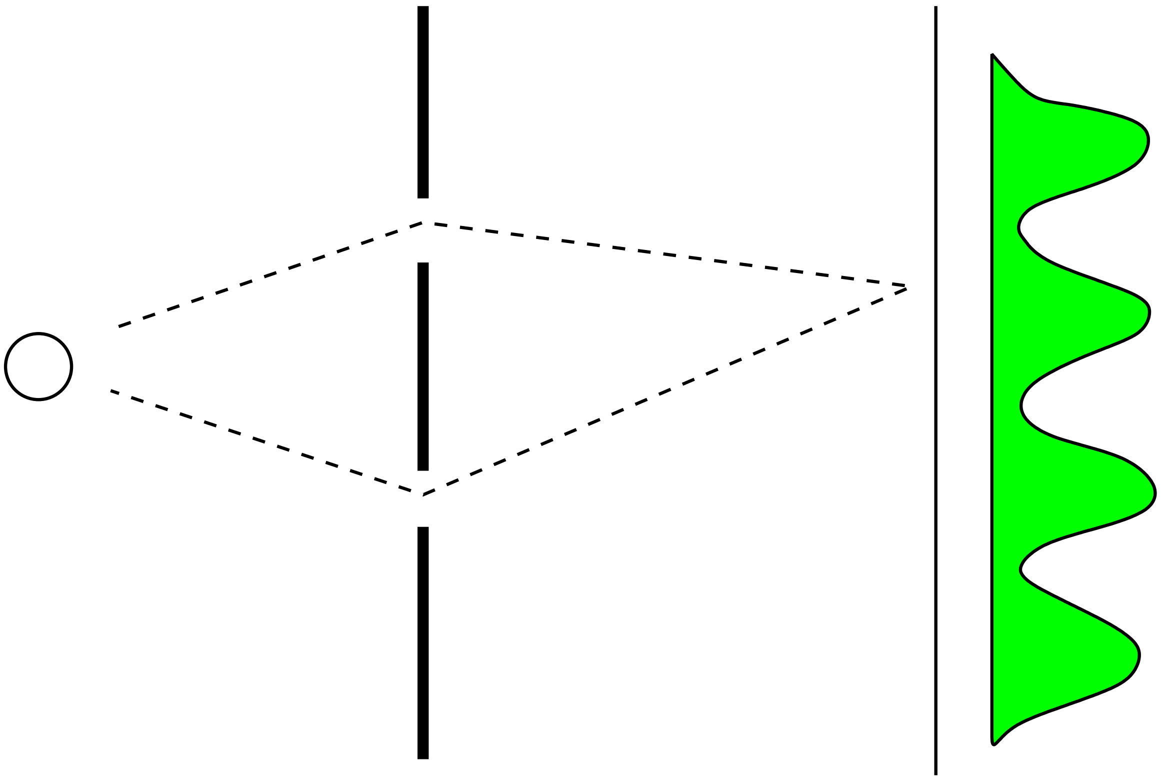
Dynamics of identical particles



Dynamics of identical particles



Unobservability of quantum trajectories



Quantum indistinguishability

N-particle system described by wavefunction

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_N)$$

Probability density unchanged by particle exchange

$$|\Psi(\ldots \mathbf{r}_i \ldots \mathbf{r}_j \ldots)|^2 = |\Psi(\ldots \mathbf{r}_j \ldots \mathbf{r}_i \ldots)|^2$$

Consequence of exchange is (at most) a phase

$$\Psi(\ldots \mathbf{r}_i \ldots \mathbf{r}_j \ldots) = e^{i\chi} \times \Psi(\ldots \mathbf{r}_j \ldots \mathbf{r}_i \ldots)$$

Quantum indistinguishability

Consequence of exchange is (at most) a phase

$$\Psi(\ldots \mathbf{r}_i \ldots \mathbf{r}_j \ldots) = e^{i\chi}\Psi(\ldots \mathbf{r}_j \ldots \mathbf{r}_i \ldots)$$

Exchanging twice returns to initial configuration, so

$$(e^{i\chi})^2 = 1$$

Hence two possibilities

$$e^{i\chi} = \begin{cases} +1 & \textbf{bosons} \\ -1 & \textbf{fermions} \end{cases}$$

The importance of quantum statistics

^{4}He 2 electrons + 2 protons + 2 neutrons

boson

Superfluid below 2.17 kelvin

^{3}He 2 electrons + 2 protons + 1 neutron

fermion

Superfluid below 0.00249 kelvin

The Pauli exclusion principle

For two independent fermions, we might expect

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \varphi_A(\mathbf{r}_1)\varphi_B(\mathbf{r}_2)$$

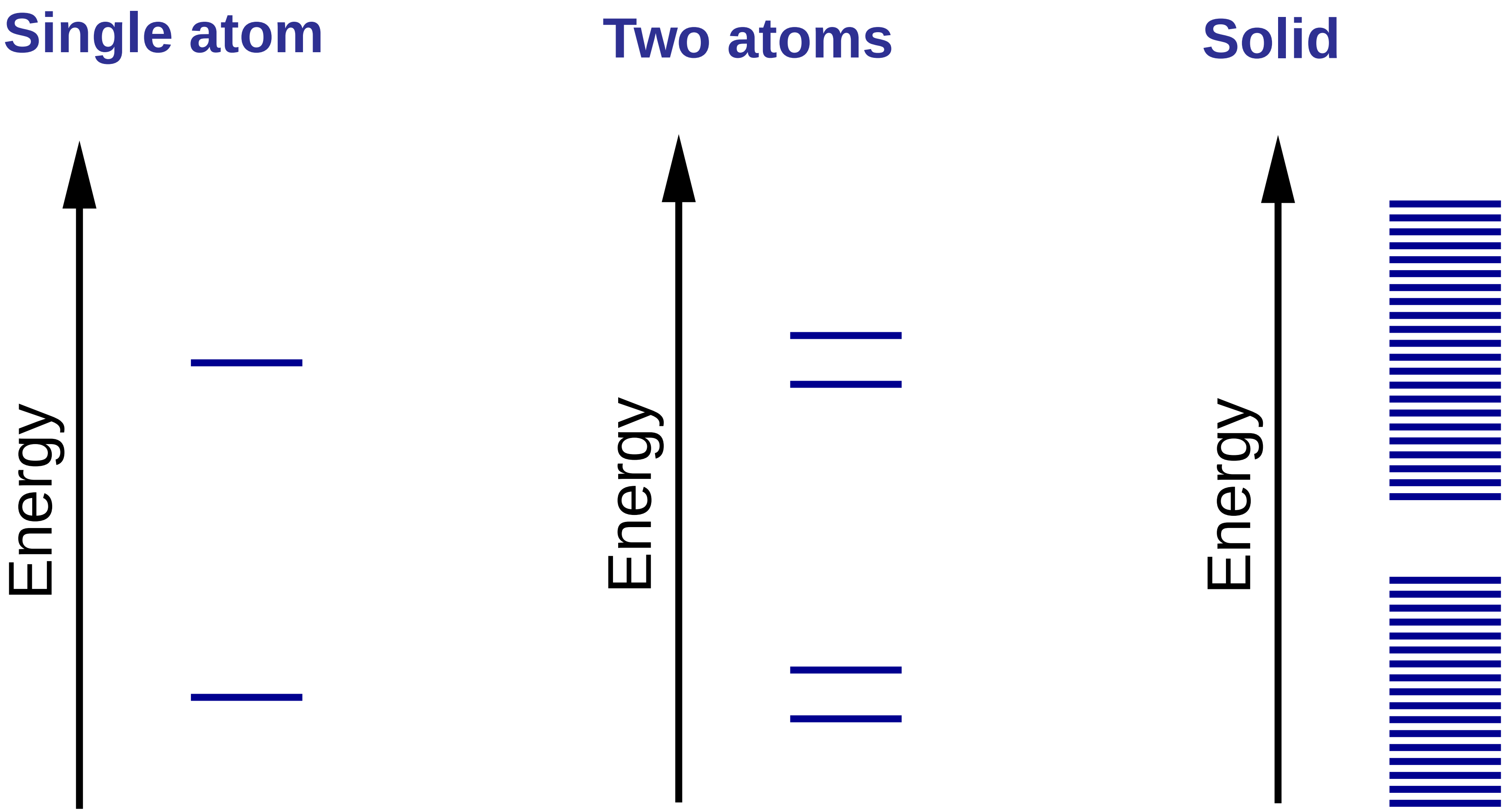
Antisymmetry under exchange requires instead

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \propto [\varphi_A(\mathbf{r}_1)\varphi_B(\mathbf{r}_2) - \varphi_A(\mathbf{r}_2)\varphi_B(\mathbf{r}_1)]$$

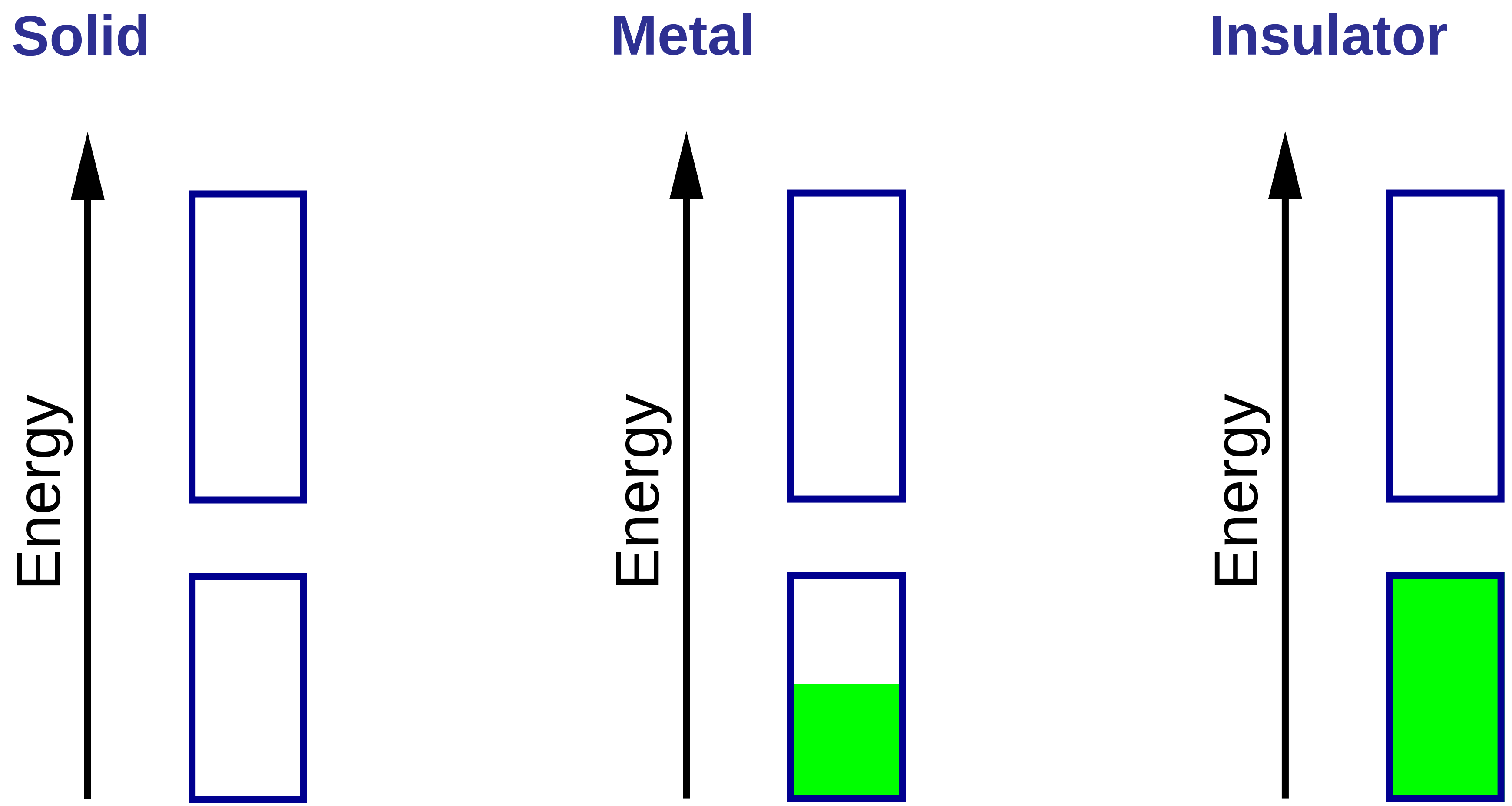
What if $\varphi_A(\mathbf{r}) = \varphi_B(\mathbf{r})$?

NO STATE!

Electron energy bands in solids



Metals vs insulators



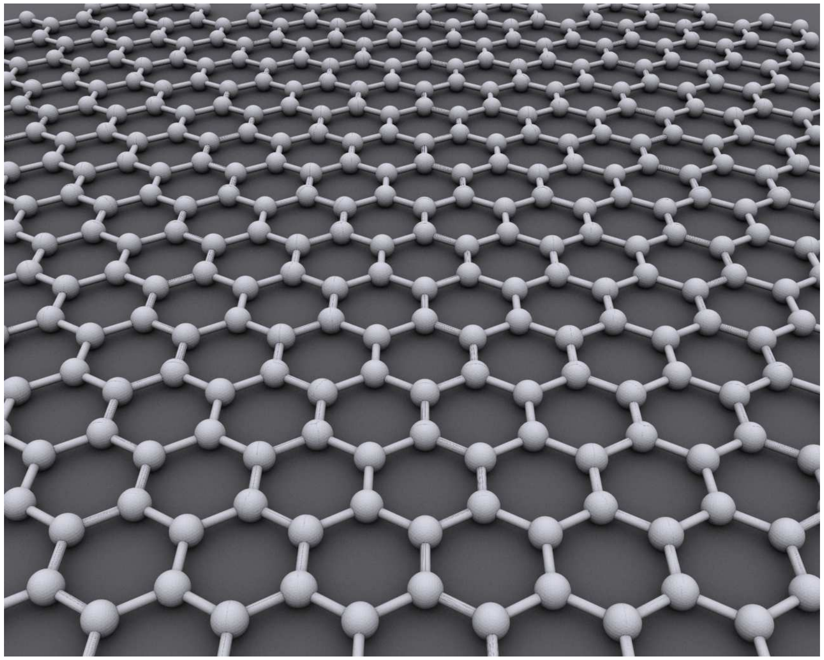
Solid

Metal

Insulator

The quantum Hall effect

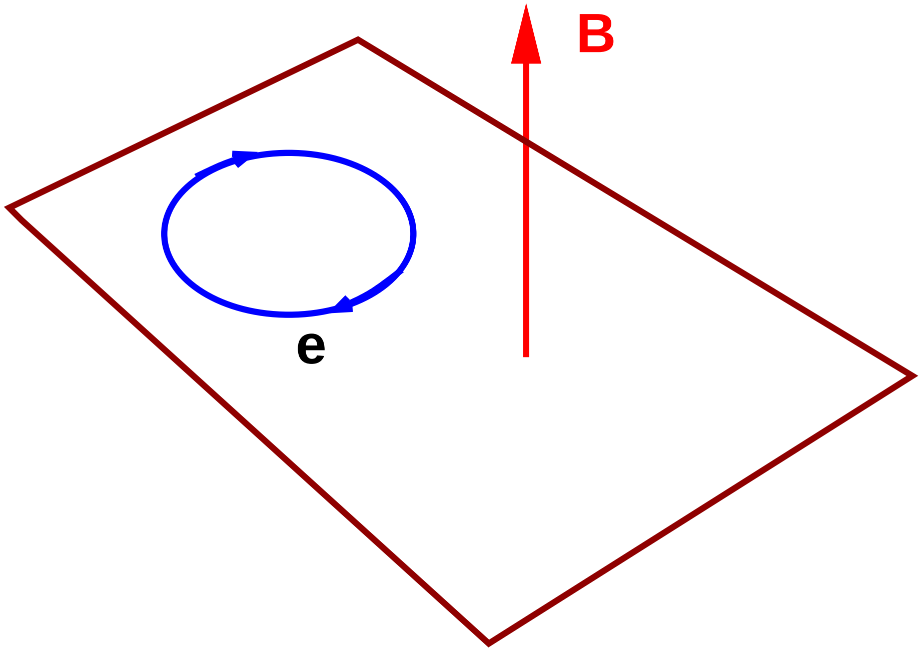
Two-dimensional electron system + magnetic field



For example: in graphene — Geim and Novoselov (2004)

The quantum Hall effect

Controlling energy levels in a (two-dimensional) solid



Cyclotron frequency

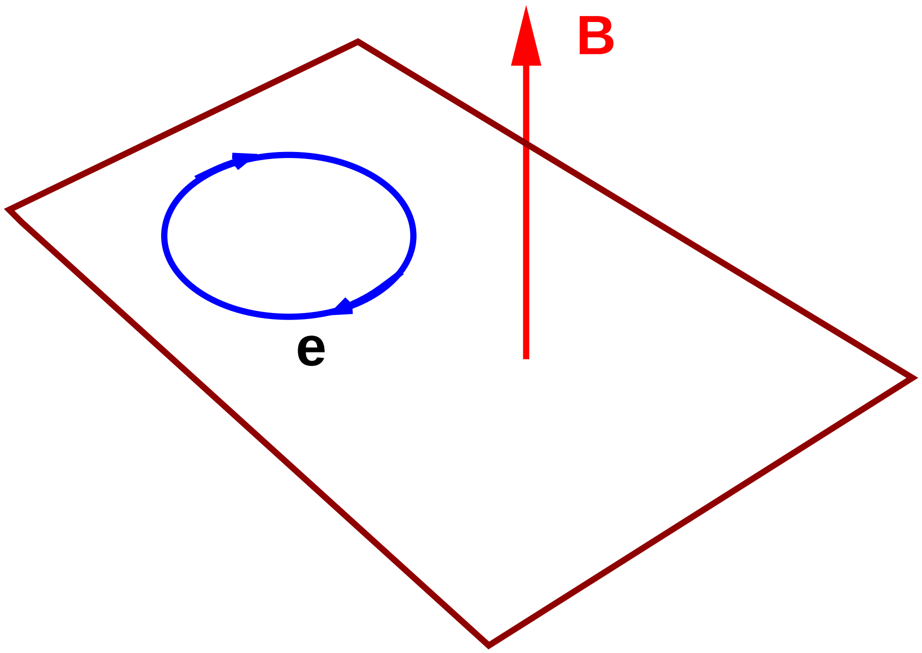
$$\omega_c = \frac{eB}{m}$$

Spectrum

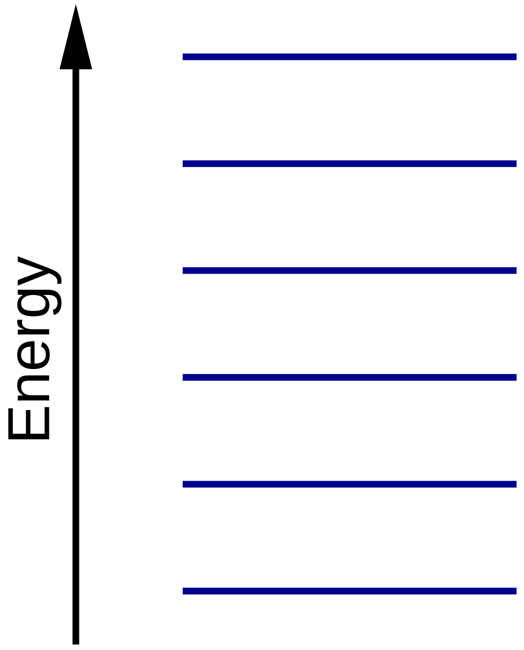
$$E_n = \hbar\omega_c(n + \frac{1}{2})$$

The quantum Hall effect

Controlling energy levels in a (two-dimensional) solid

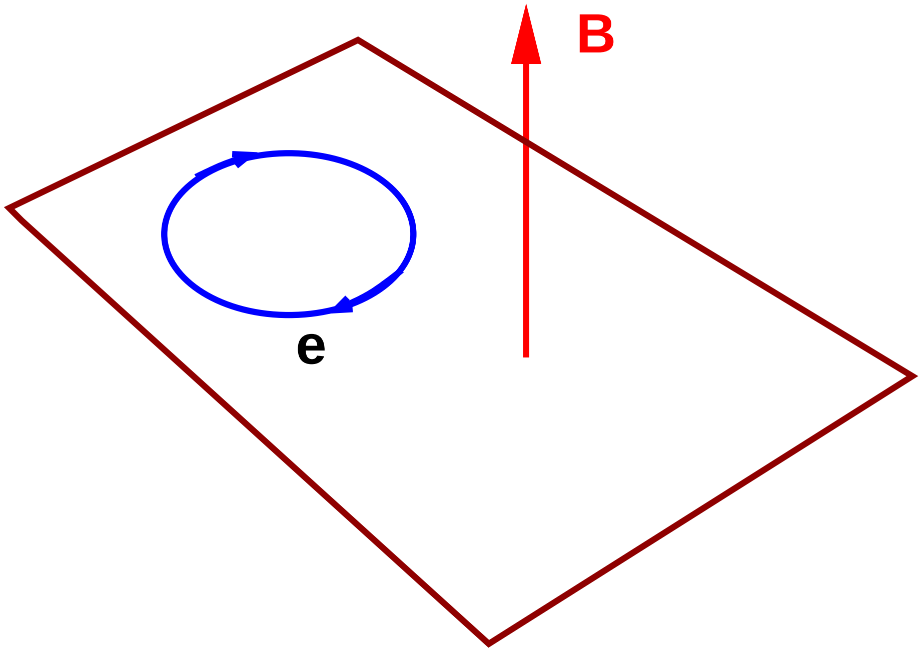


QH system

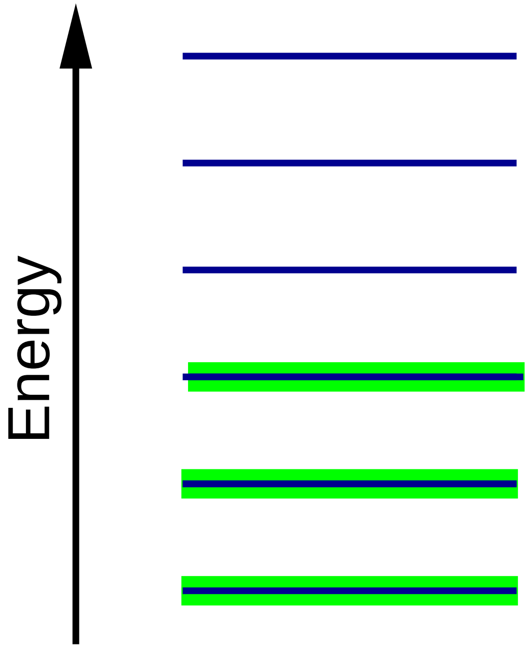


The quantum Hall effect

Controlling energy levels in a (two-dimensional) solid

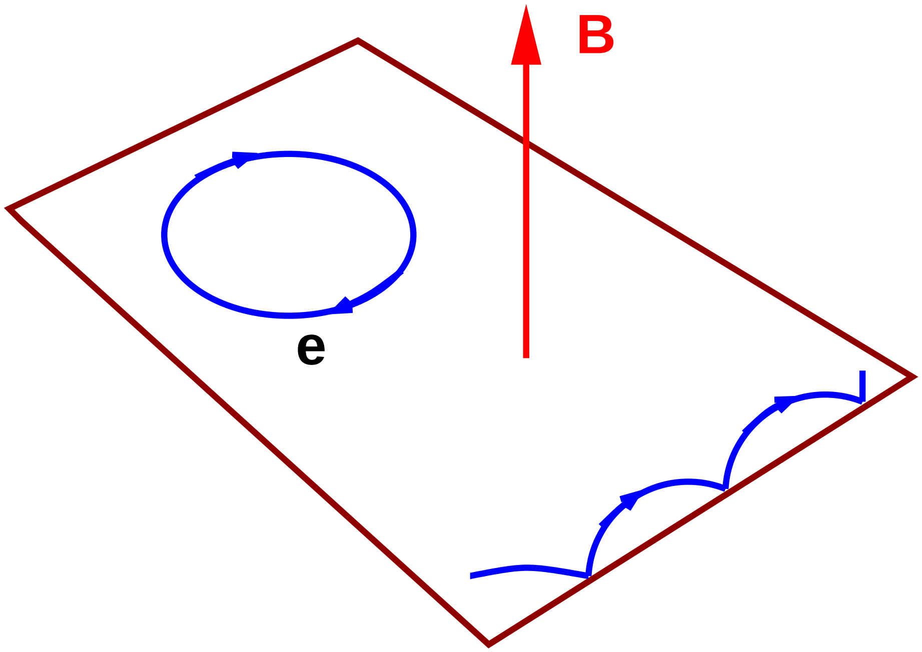


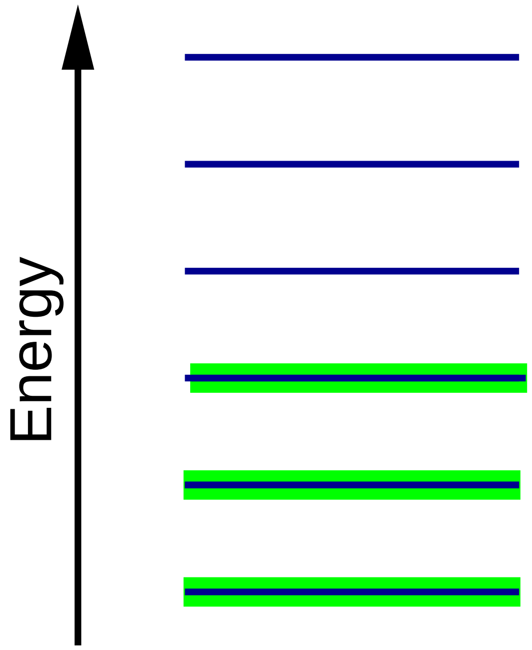
Insulator?



The quantum Hall effect

Controlling energy levels in a (two-dimensional) solid

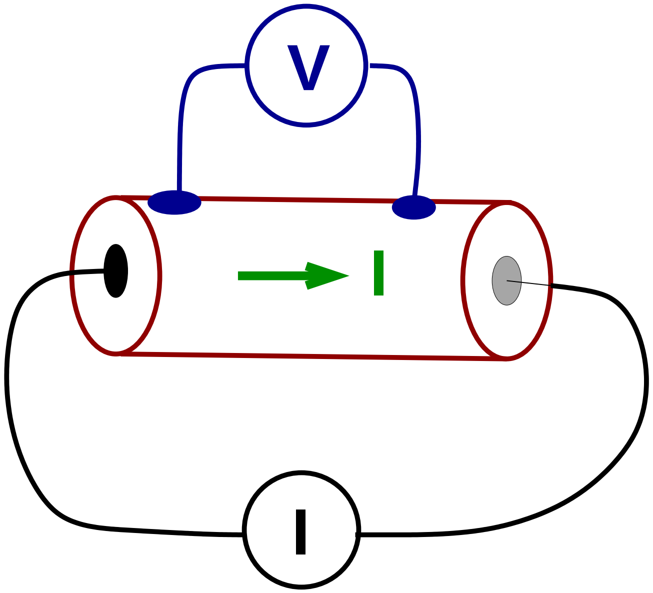




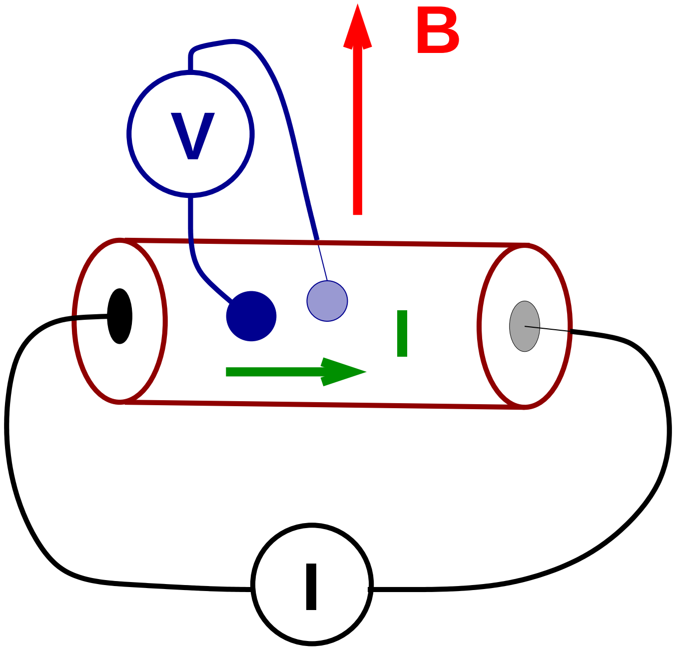
Conduction via edge states

Generic resistance measurements

. . . every detail matters



resistance



Hall resistance

Value of Hall resistance?

Faraday's law

$$\oint \vec{E}(t)\mathrm{d}\vec{\ell} = -\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t}$$

Hall effect

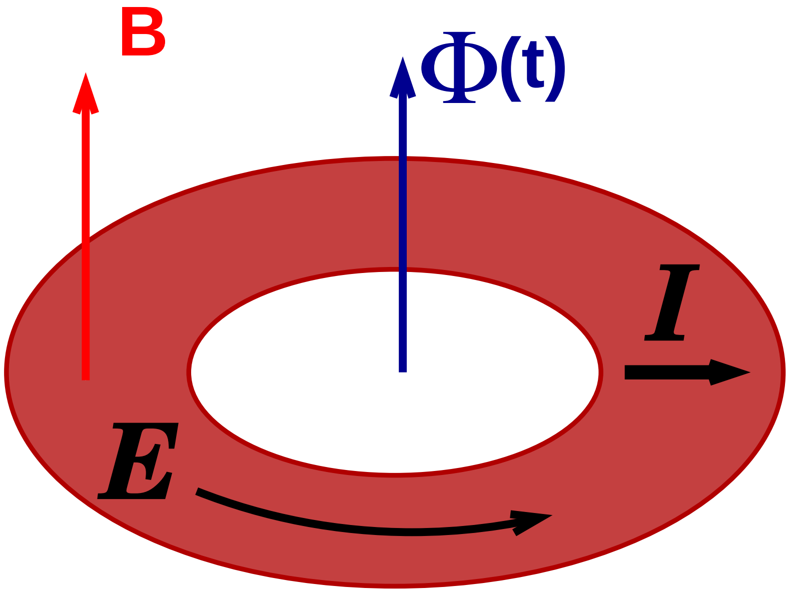
$$I(t) = \frac{1}{R_{\mathrm{H}}} \oint \vec{E}(t)\mathrm{d}\vec{\ell}$$

Charge transfer

$$Q = \int I(t)\mathrm{d}t = \frac{\Delta\Phi}{R_{\mathrm{H}}}$$

Flux quantum $\Delta\Phi = h/e$

Charge transfer $Q = ne$

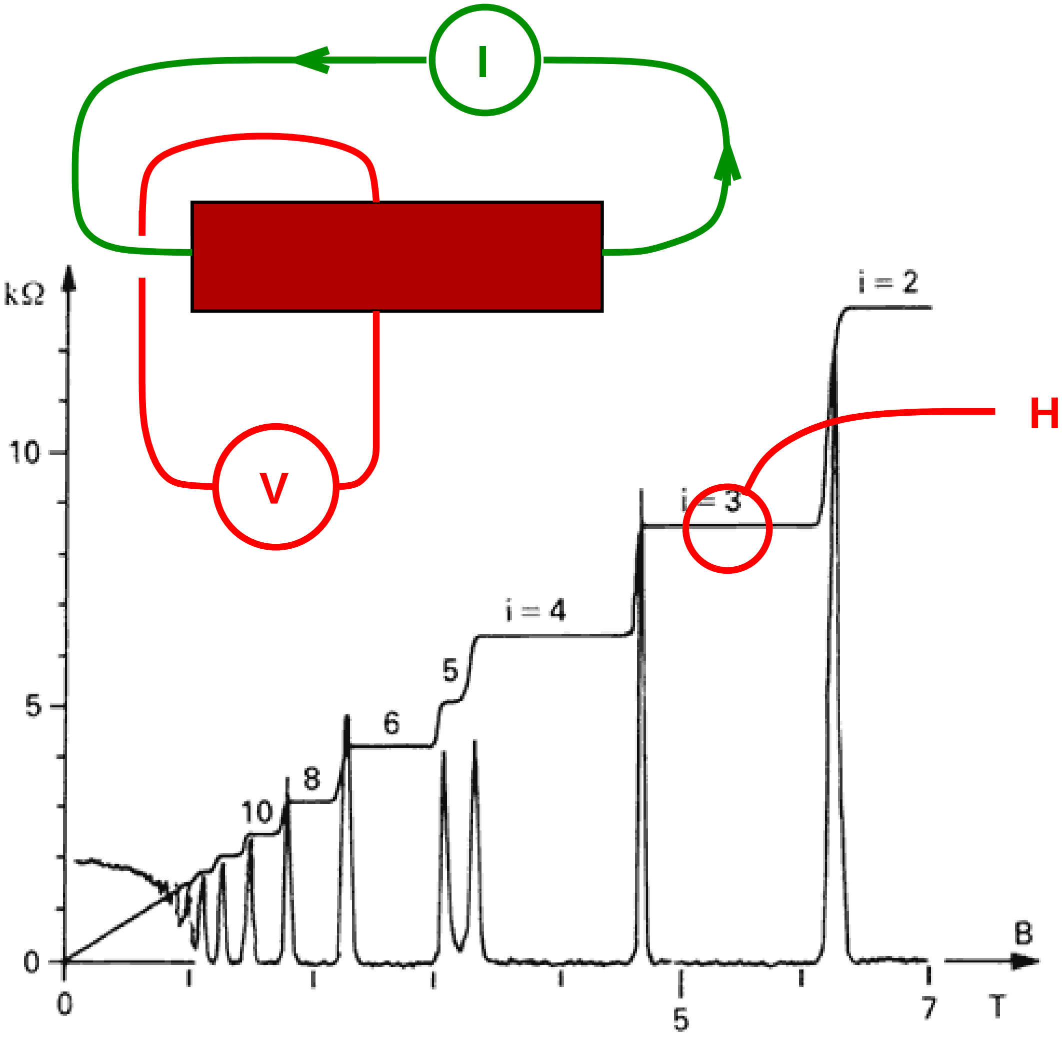


Value of Hall resistance

$$R_{\mathrm{H}} = \frac{\Delta\Phi}{Q}$$

$$\boxed{R_{\mathrm{H}} = \frac{h}{ne^2}}$$

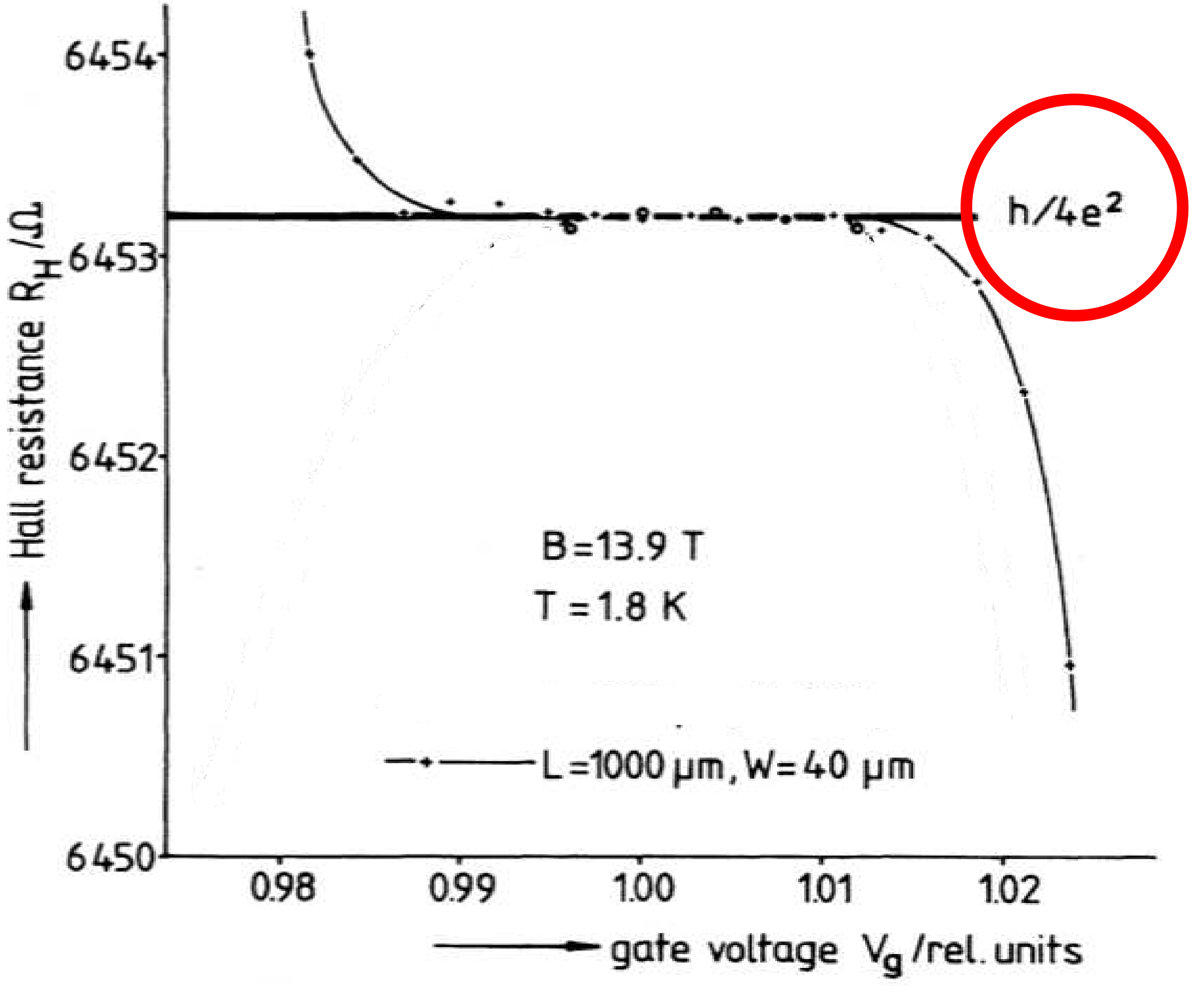
Experiment



Hall resistance

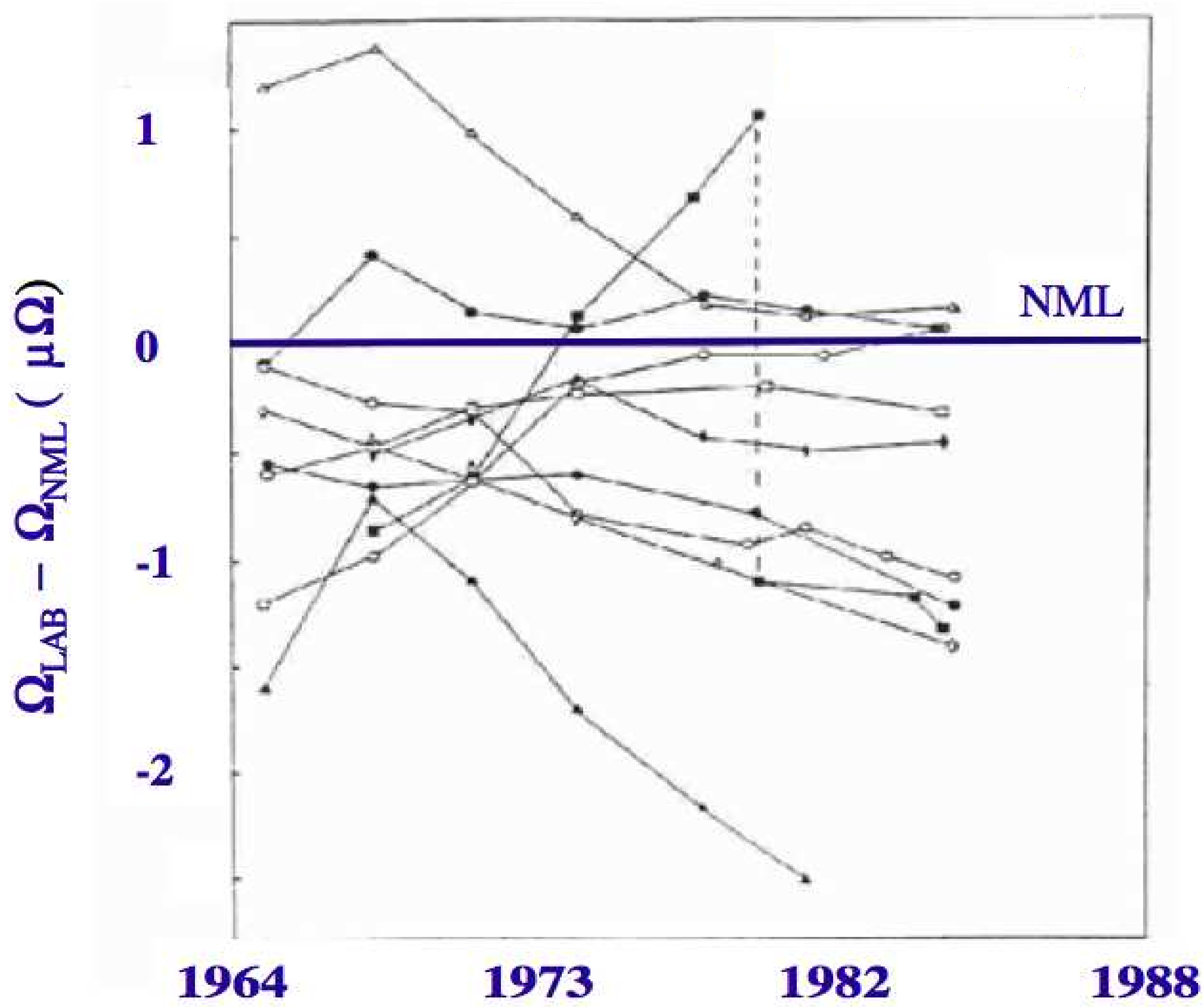
von Klitzing, Dorda, and Pepper (1980).

Experiment

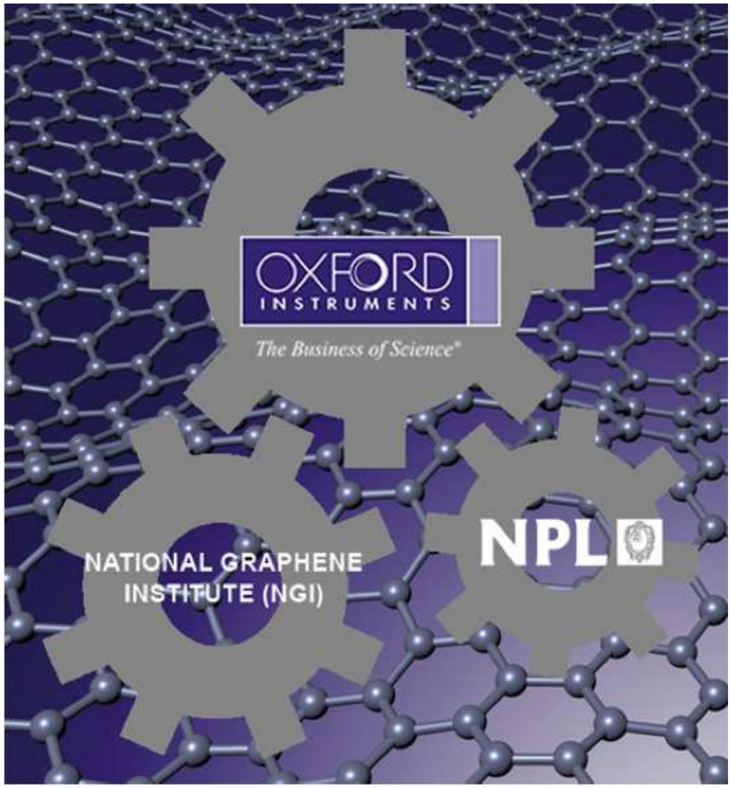


von Klitzing, Dorda, and Pepper (1980).

Re-defining the ohm



Re-defining the ohm



“. . . the graphene-enabled quantum resistance system will provide the high-end electronics instrumentation industry with a primary resistance standard . . . ”

Summary

“More is different”

— **reductionism vs emergence**

Identical particles in quantum mechanics

— *really, really* **identical**

Metals & insulators

— **macroscopic consequences of the exclusion principle**

The integer quantum Hall effect

— **resistance standard from fundamental constants**