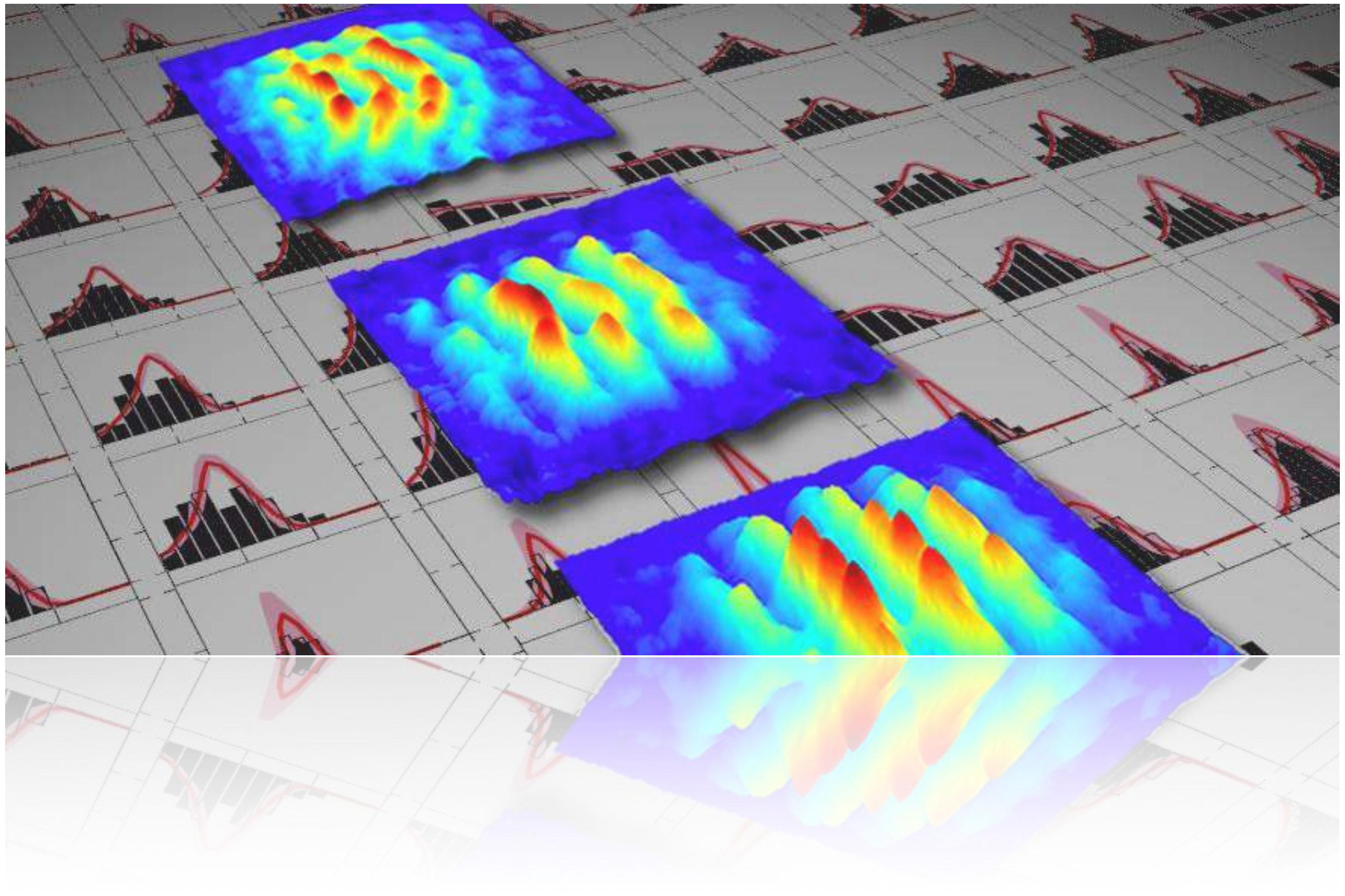
Hydrodynamics for systems with extensive memory



Hydrodynamics can describe nearly **anything.**

We need only:

1. Local Equilibrium
2. Few Conservation laws

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Do we actually need these conditions?

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Not quite! (stay posted)

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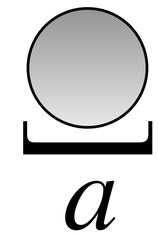
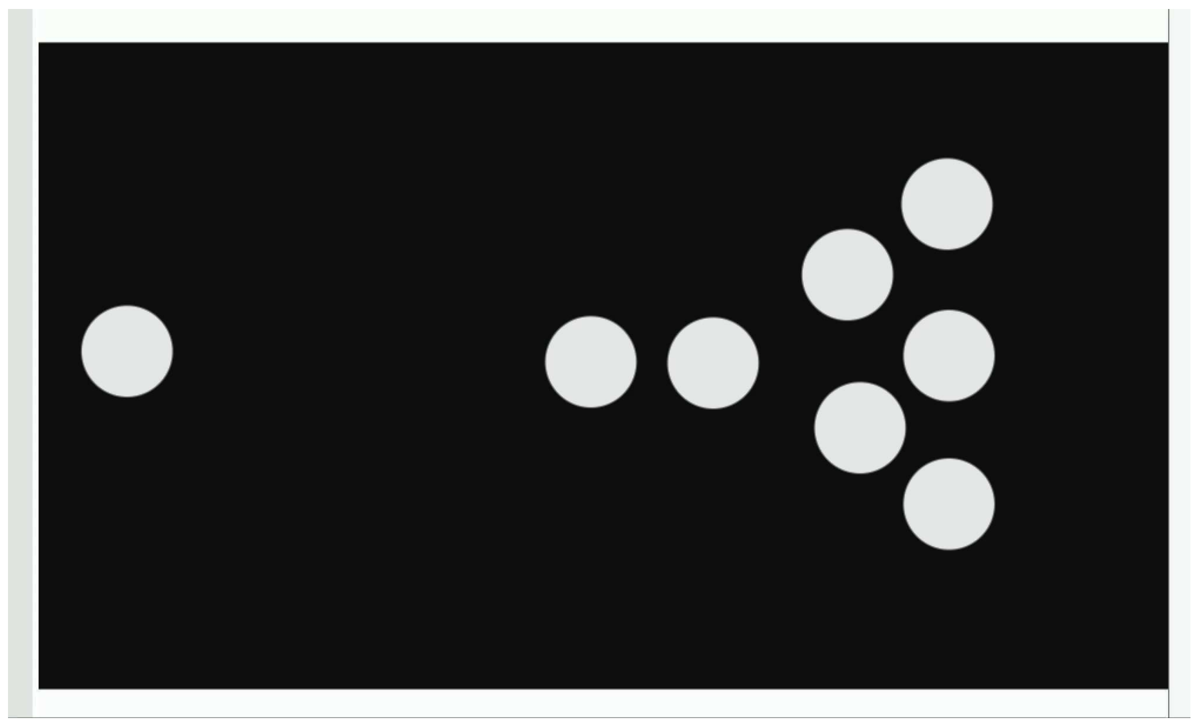
Do we actually need these conditions?

Few Conservation laws $\Leftrightarrow$ "loss of memory" of the initial condition

Consider **classical hard spheres**



2d



$\Rightarrow$ Only conserve Number, Energy, Momentum

$\Rightarrow$ Specify the local equilibrium state with $\{n(x,t),\ e(x,t),\ \mathbf{g}(x,t)\}$

$\Rightarrow$ Few hydrodynamic equations $\{\frac{Dn}{Dt}=0,\quad \frac{De}{Dt}=0,\quad \frac{D\mathbf{g}}{Dt}=0\}$

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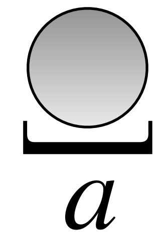

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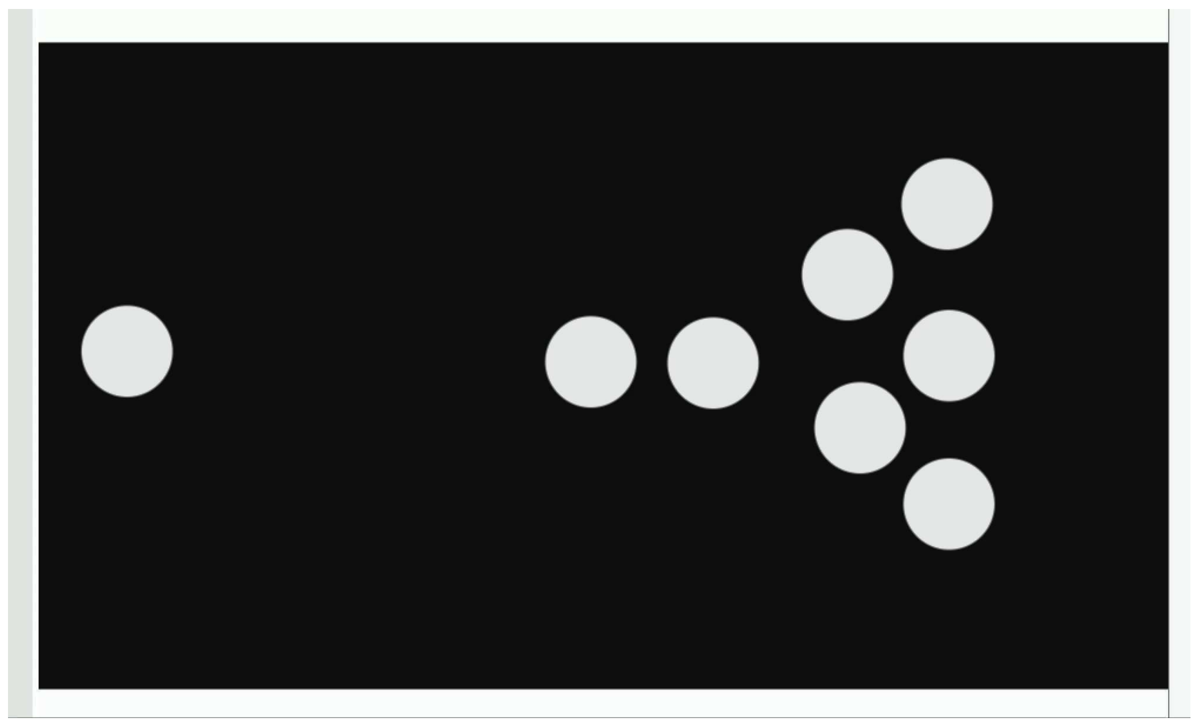

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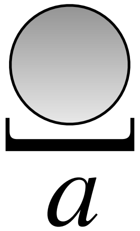

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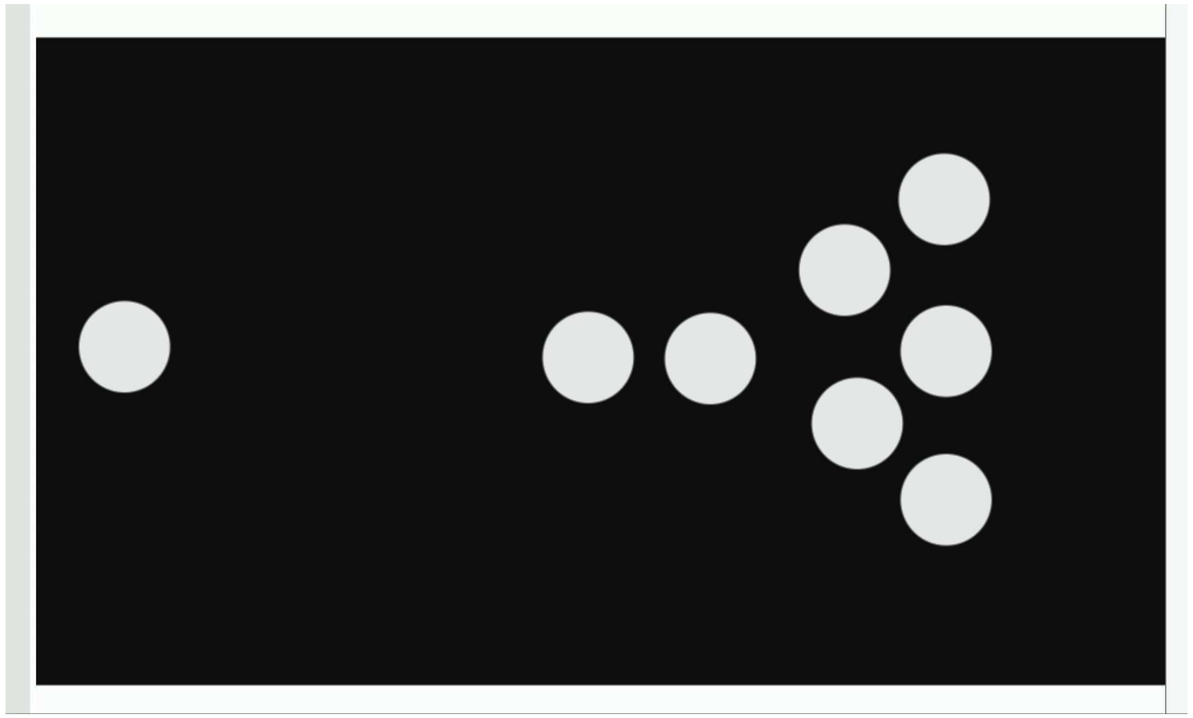

- Velocities "randomize"

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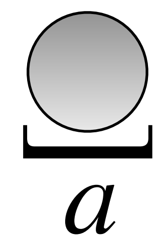

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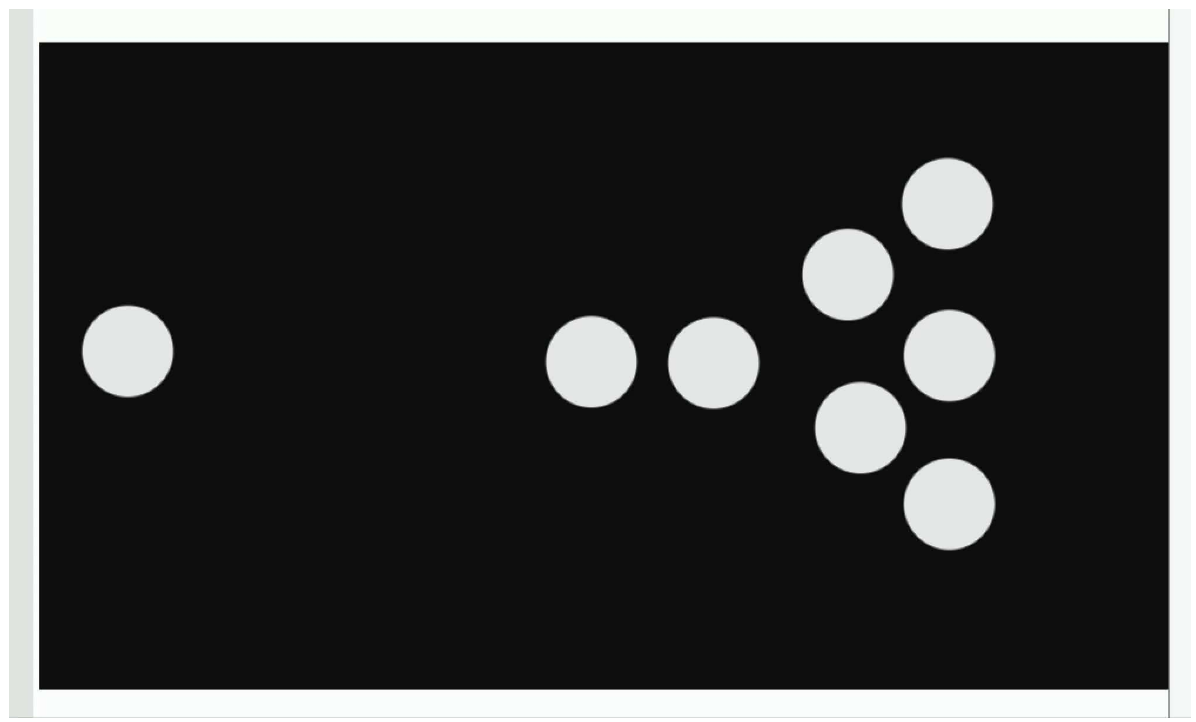

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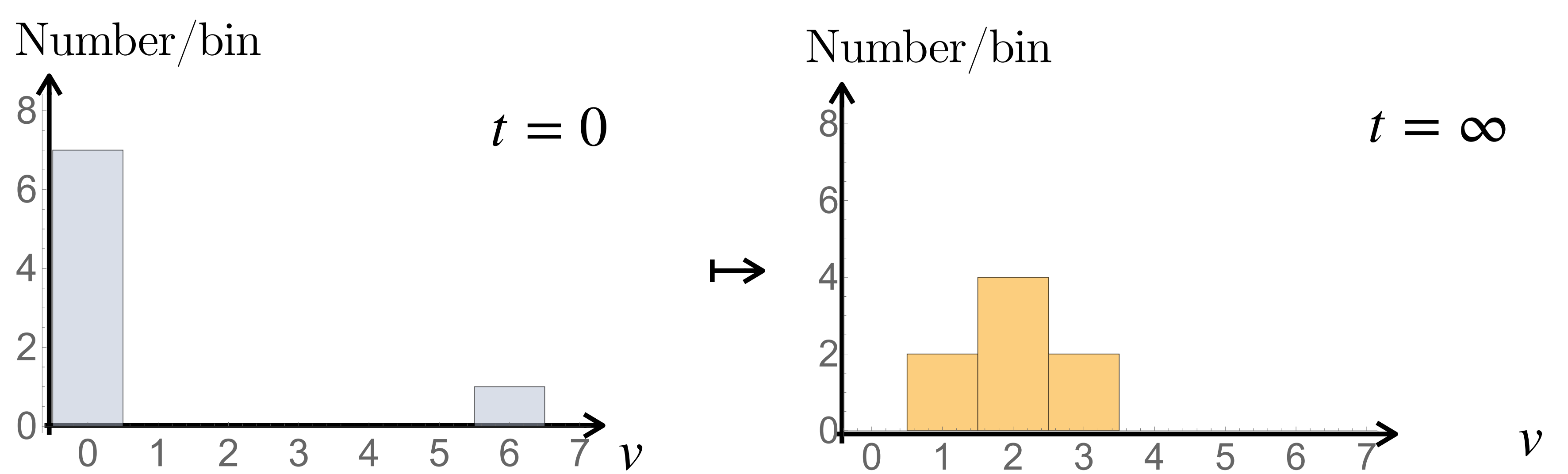

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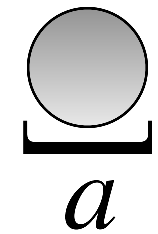
Consider **classical hard spheres** 

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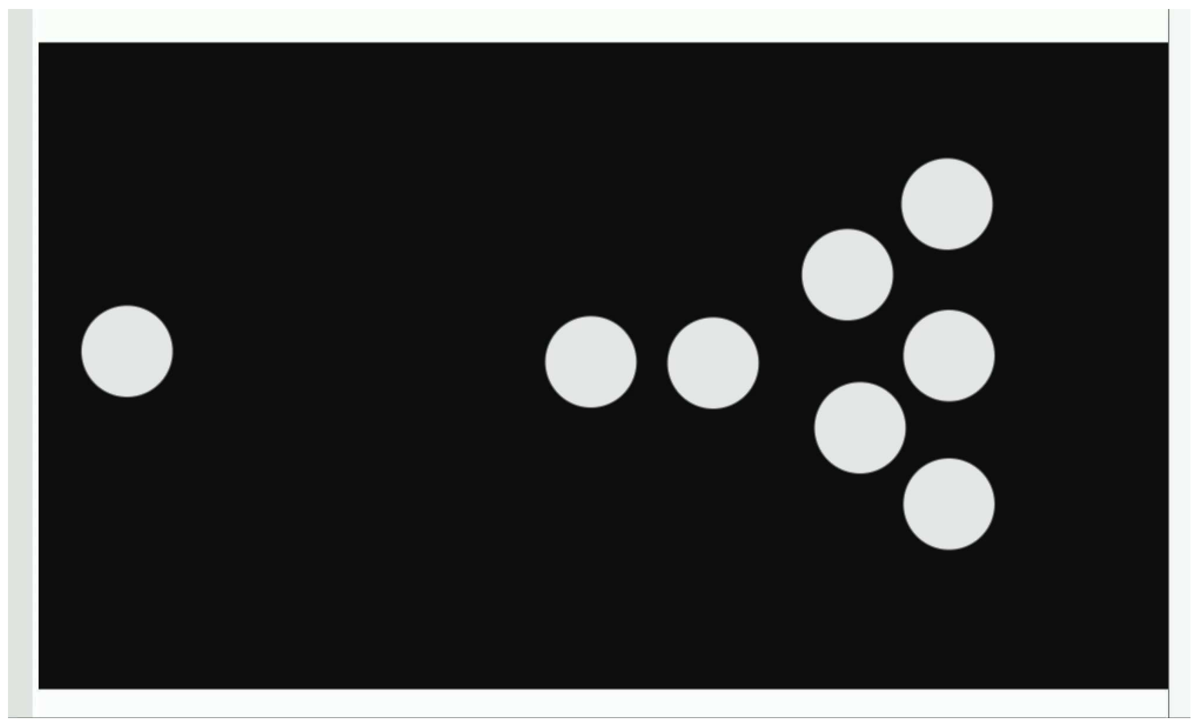

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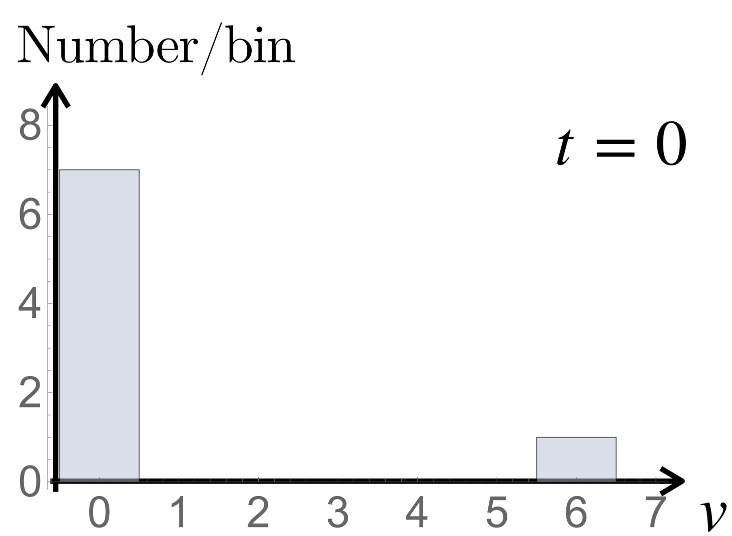
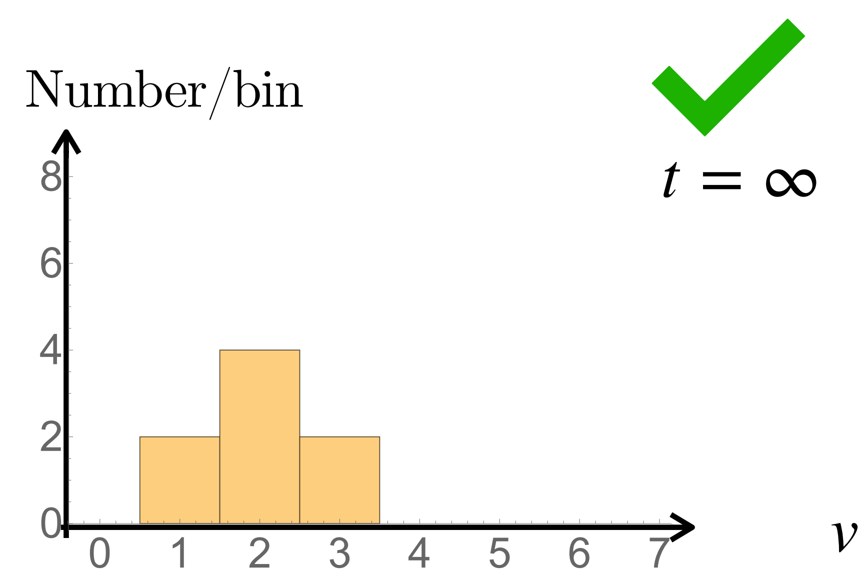


$\mapsto$



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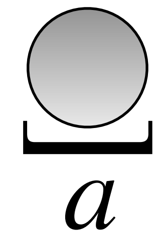

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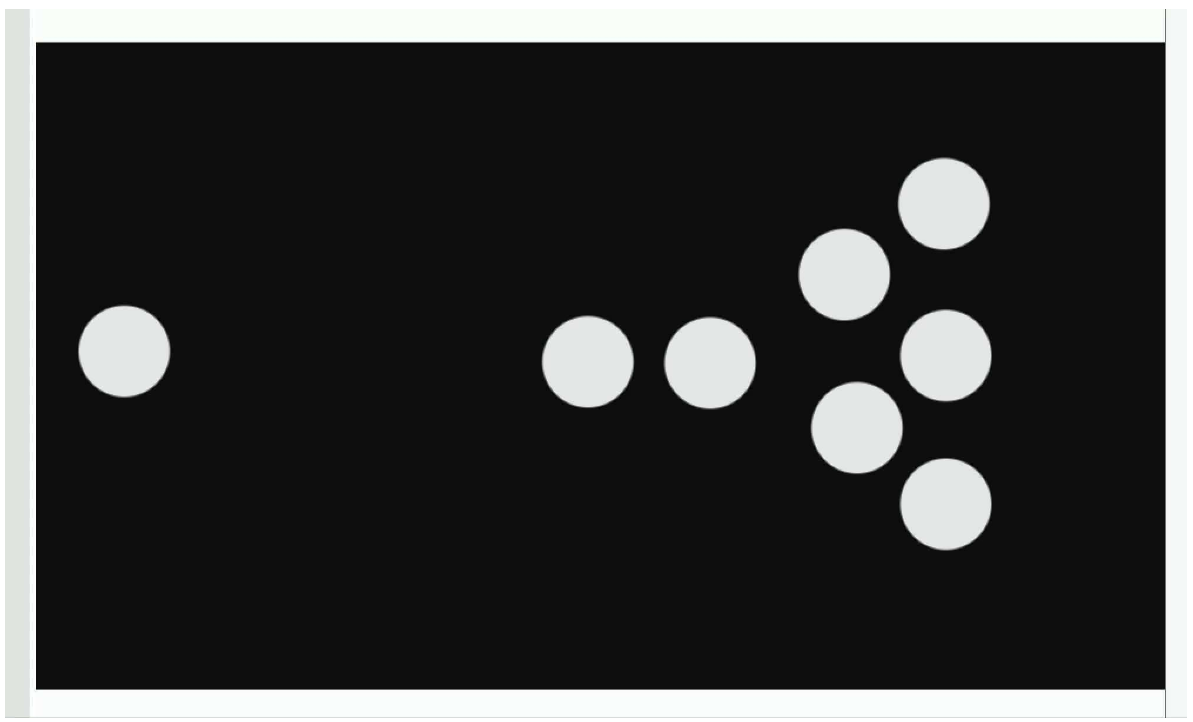
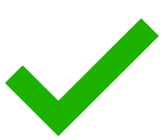


• Velocities “random

What if we squash the system?



$t = \infty$

Few Co… of the initial condition

$2d$

- Velocities “random…

$t = \infty$

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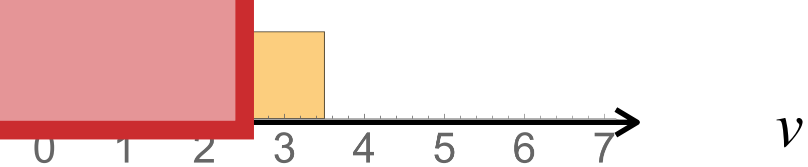

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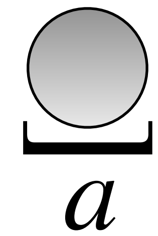
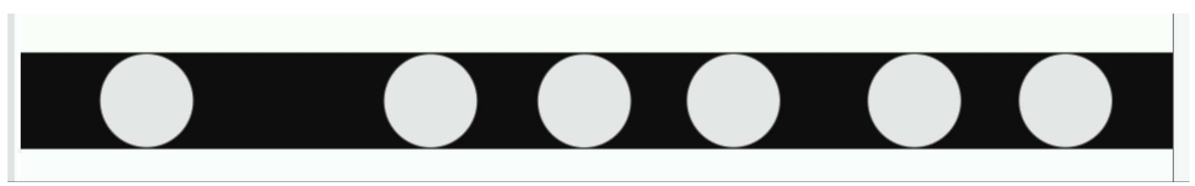


~~$2d$~~

$1d$



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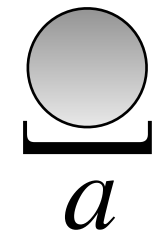

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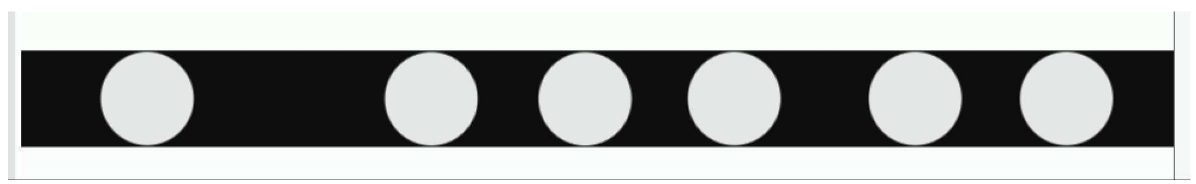

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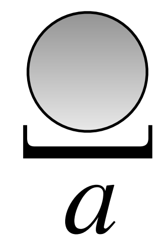

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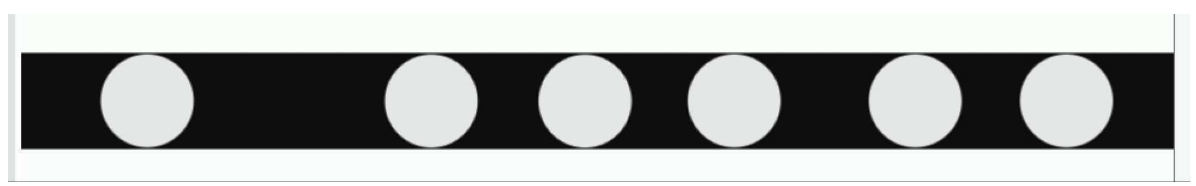
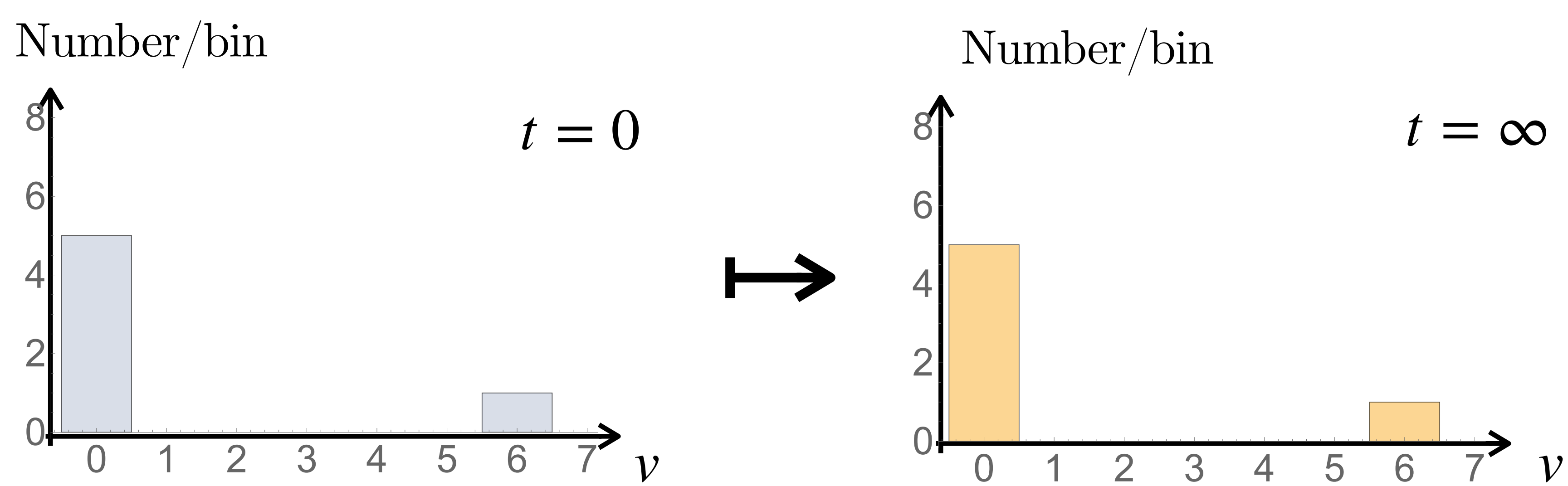


• Initial distribution of velocities is conserved



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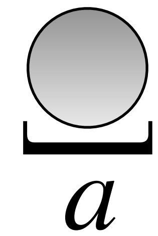

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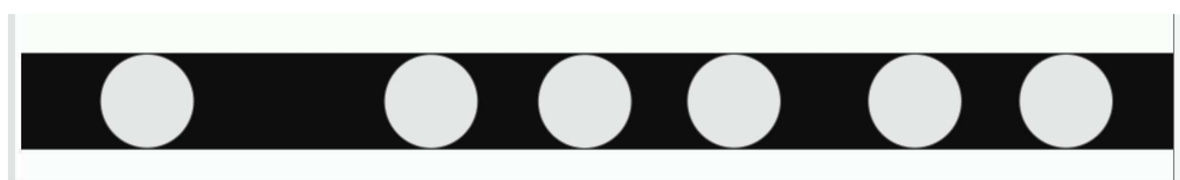

- Initial distribution of velocities is conserved

$N_j :=$ number of particles with velocity v_j

$v_j :=$ initial velocity of the j-th sphere

$\Longrightarrow$ N_j is conserved for $j = 1, \ldots, N_{\mathrm{sp}}$

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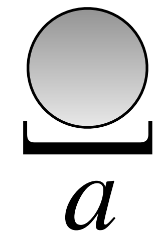

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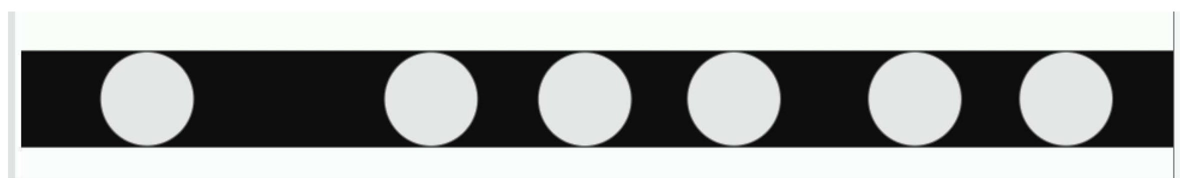

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$\Rightarrow$ N_j is conserved for $j = 1, \ldots, N_{sp}$

$\Rightarrow$ to specify local equilibrium state I need $\{n_j(x,t)\}_{j=1}^{N_{sp}}$

$\Rightarrow$ N_{sp} hydrodynamic equations

Infinitely many in the thermodynamic limit!

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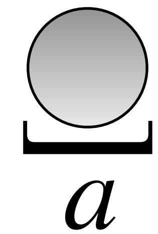

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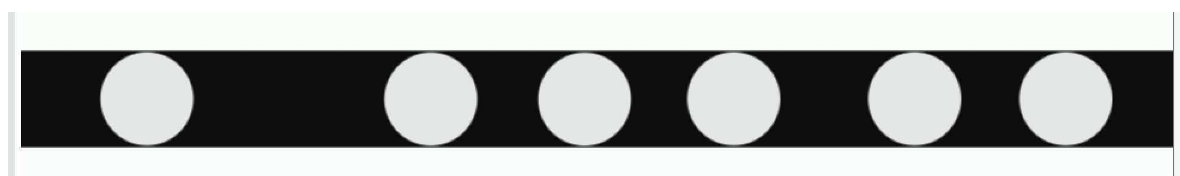

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Quantum systems with extensive memory

Are there **quantum systems** with this property?

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Crash course on QM

- Wavefunction $\psi(\mathbf{r}_1, \ldots, \mathbf{r}_N, t)$

- Schrödinger Equation $i\hbar\partial_t\psi(\mathbf{r}_1, \ldots, \mathbf{r}_N, t) = \hat{H}\psi(\mathbf{r}_1, \ldots, \mathbf{r}_N, t)$

- Conserved Charges $[\hat{H}, \hat{Q}] = 0$

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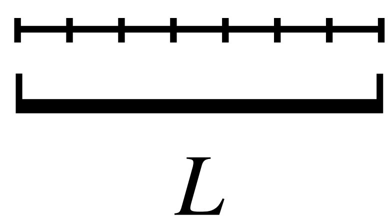
1d lattice of length L



the problem becomes linear algebra!

$$N = 1 \quad \begin{cases} \psi(r_1, t) & L\text{-dimensional vector} \\ \hat{H} & L \times L \text{ matrix} \end{cases}$$

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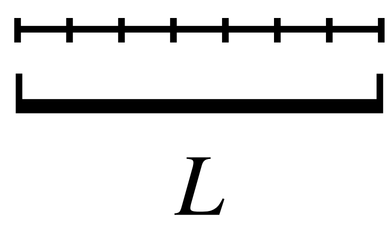

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$$N = 2 \quad \begin{cases} \psi(r_1, r_2, t) & L^2\text{-dimensional vector} \\ \hat{H} & L^2 \times L^2 \text{ matrix} \end{cases}$$

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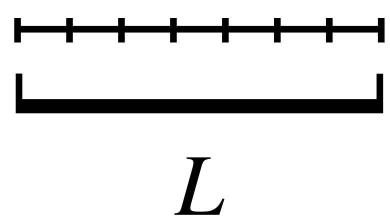

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at least L^N independent matrices commute with $\hat{H}$ (diagonal in the same basis)

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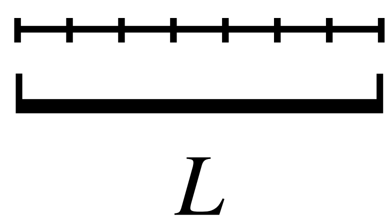

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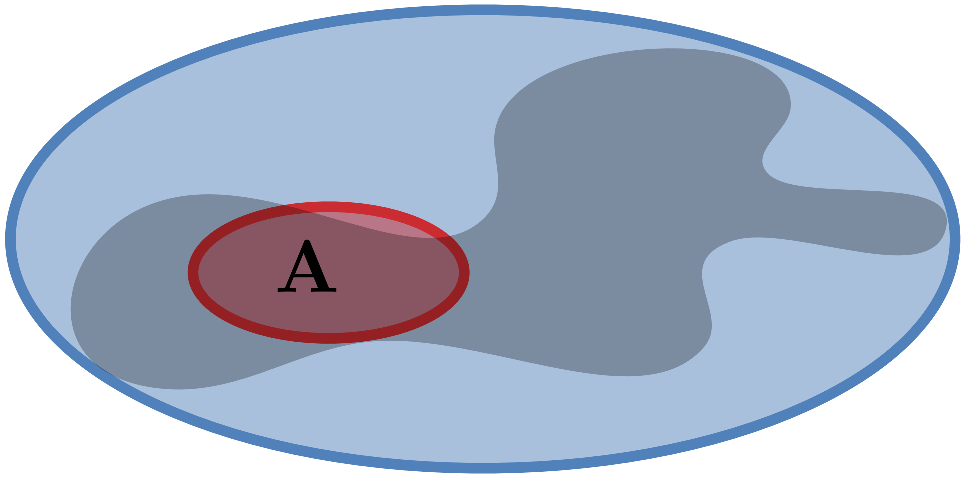
Most of them don't have local densities $\Rightarrow$ Not relevant for local physics



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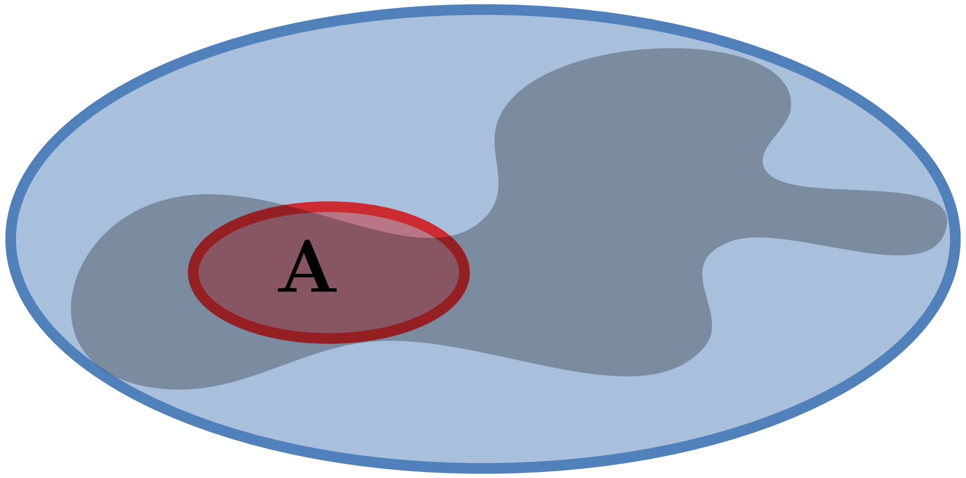

Are there QM systems with extensively many conserved charges **with local density?**

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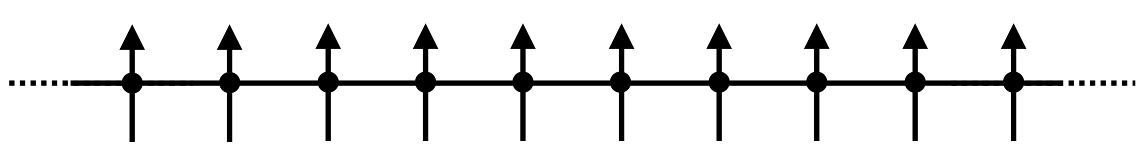
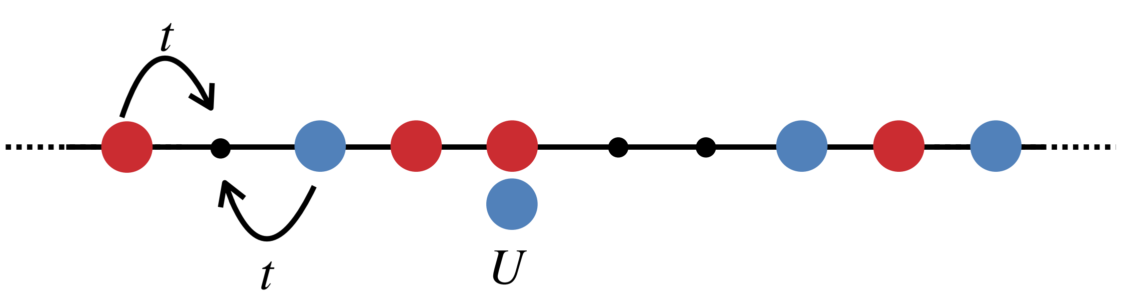
Integrable "spin-chains"



Integrable quantum many-body systems



Integrable quantum field theories

???

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Ultracold Quantum Matter Group, Christmas Dinner 2018

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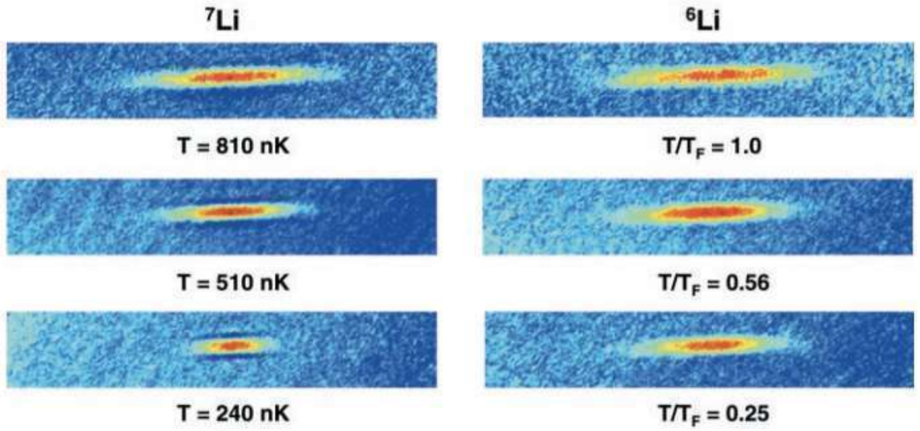
Clouds of atoms at low T



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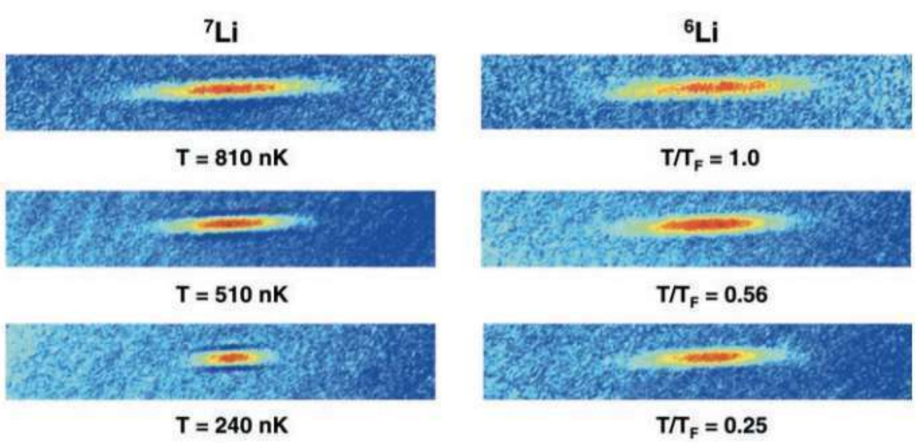

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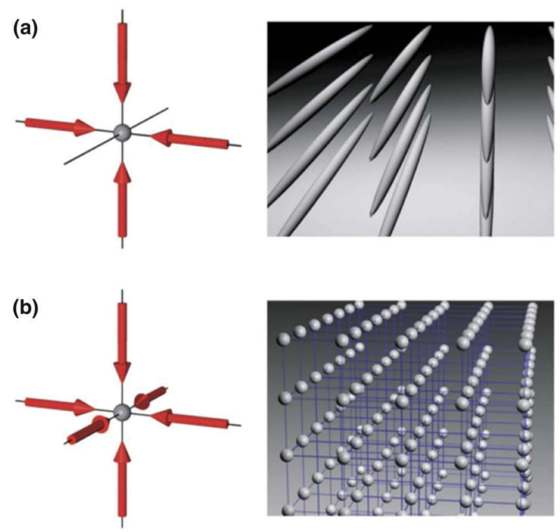
Lattices of laser beams



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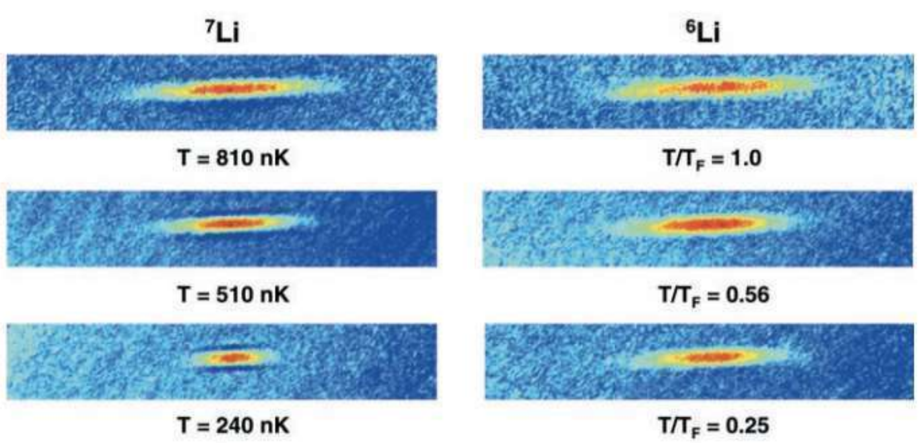

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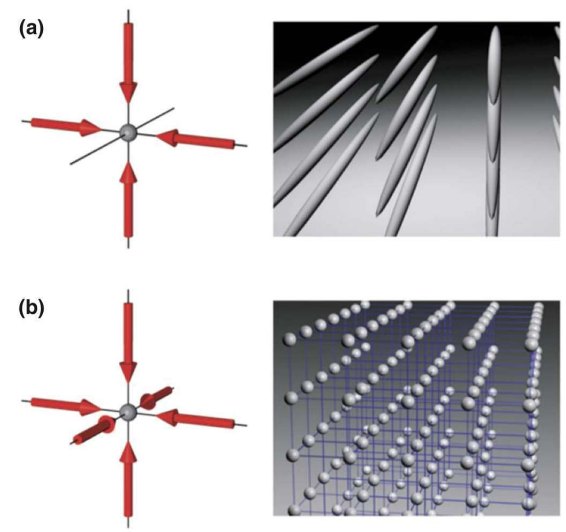

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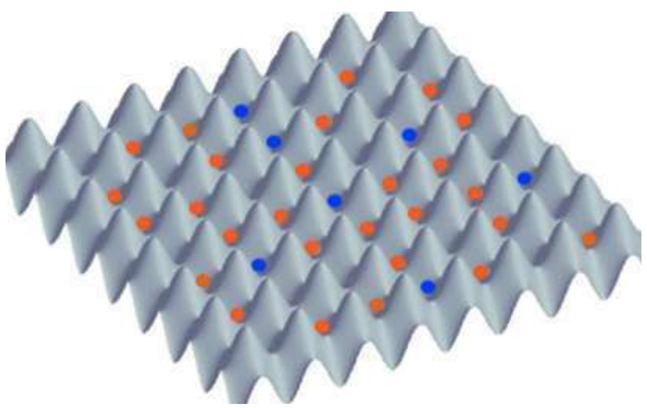

$$d = 2$$

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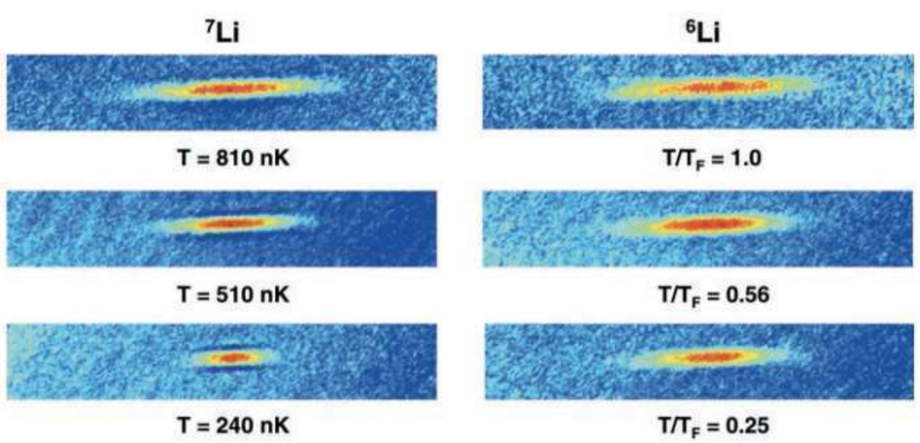

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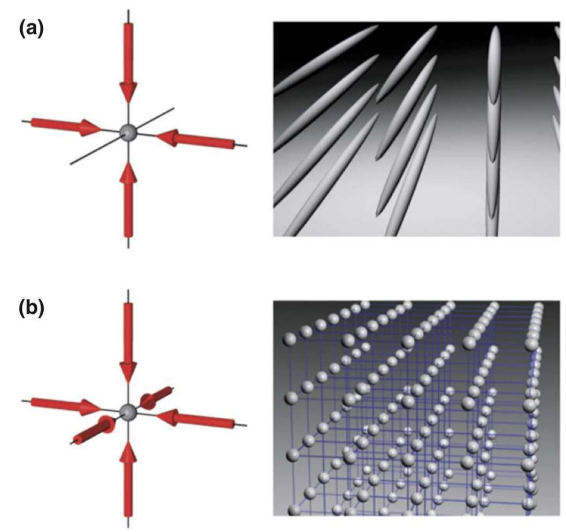

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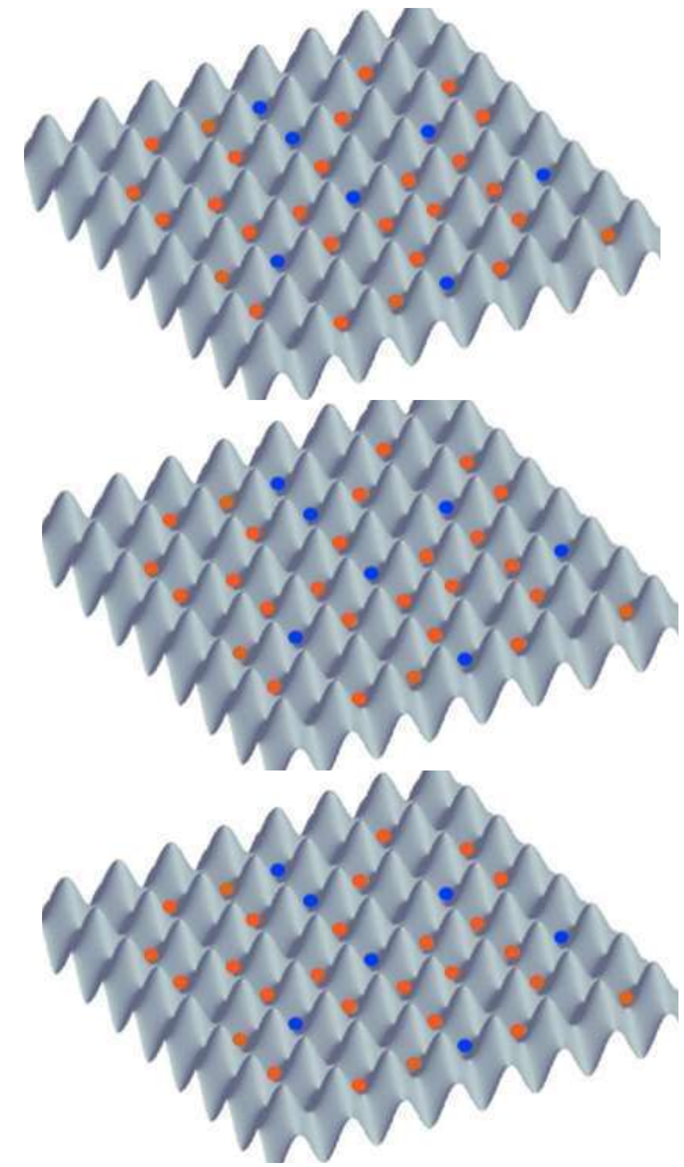

$d = 3$

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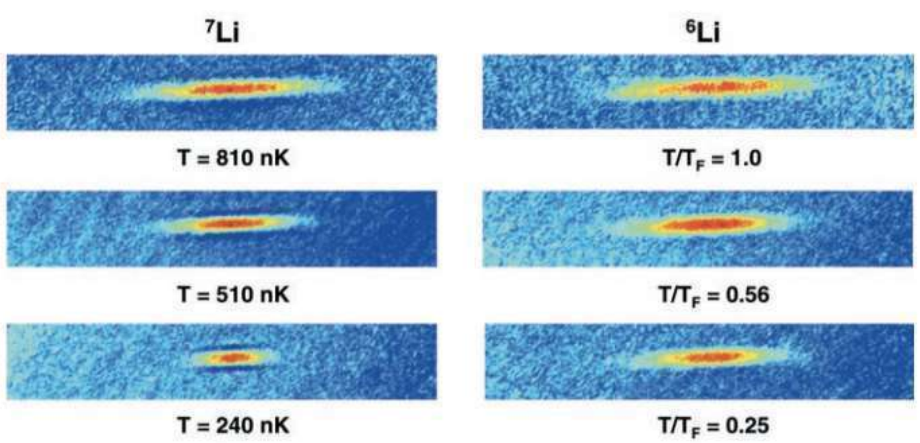

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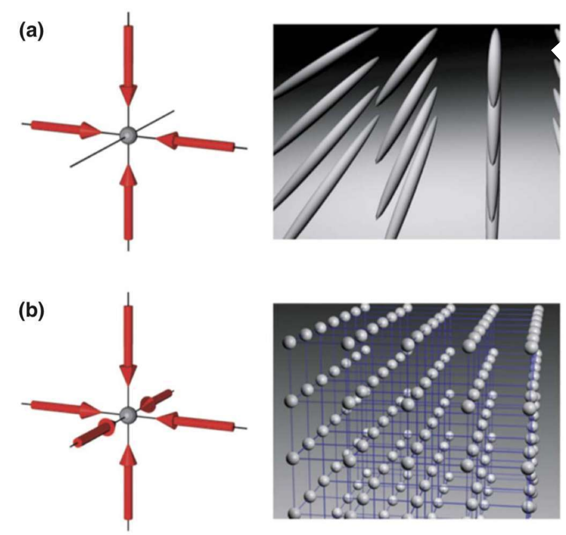

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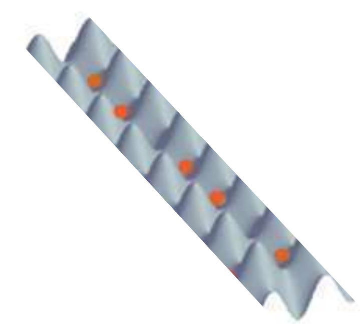
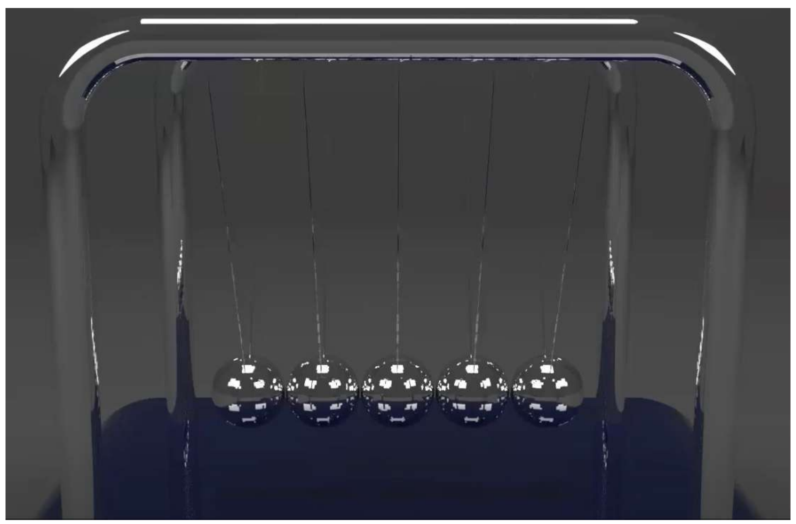


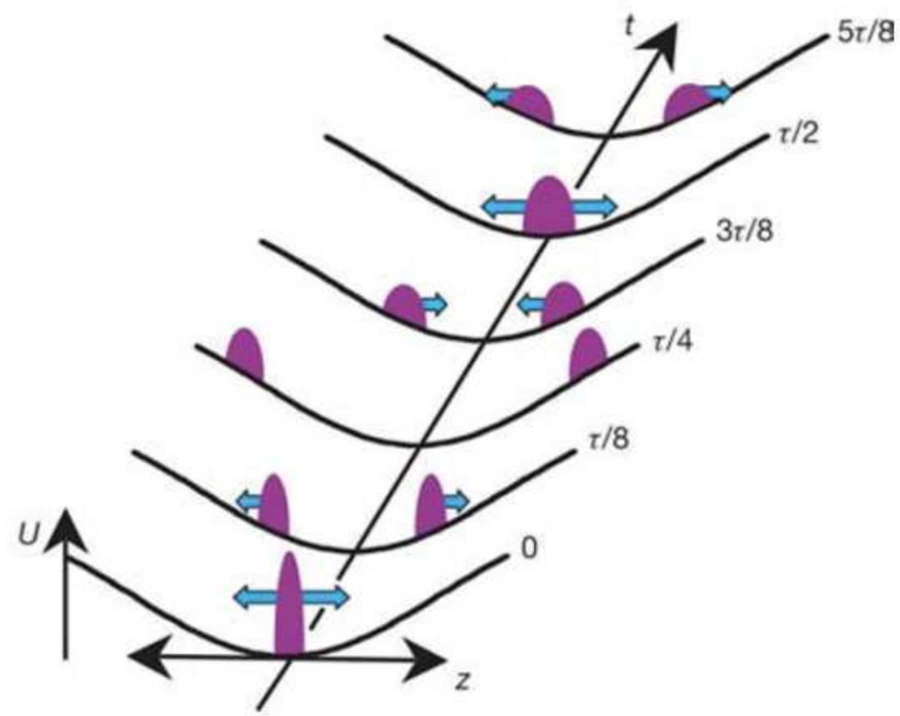
$d = 1$

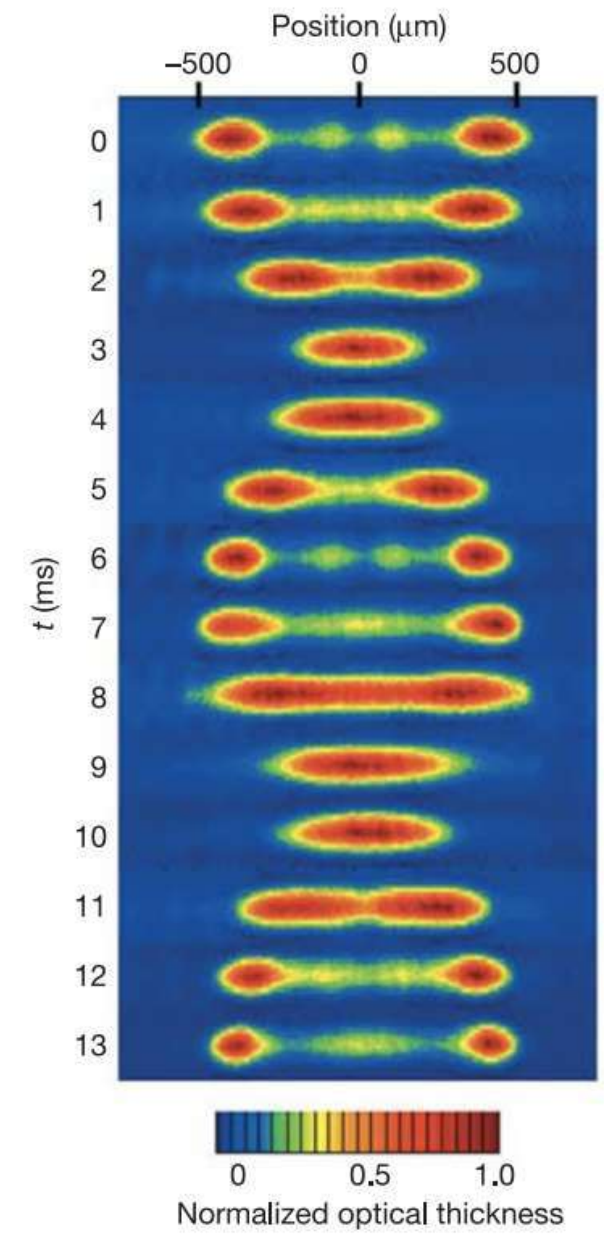
Quantum systems with extensive memory

A quantum Newton's cradle

NATURE|Vol 440|13 April 2006







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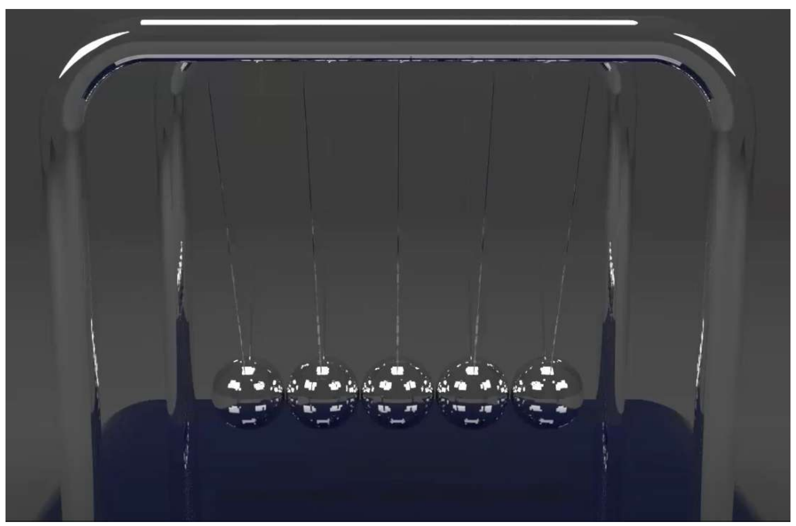
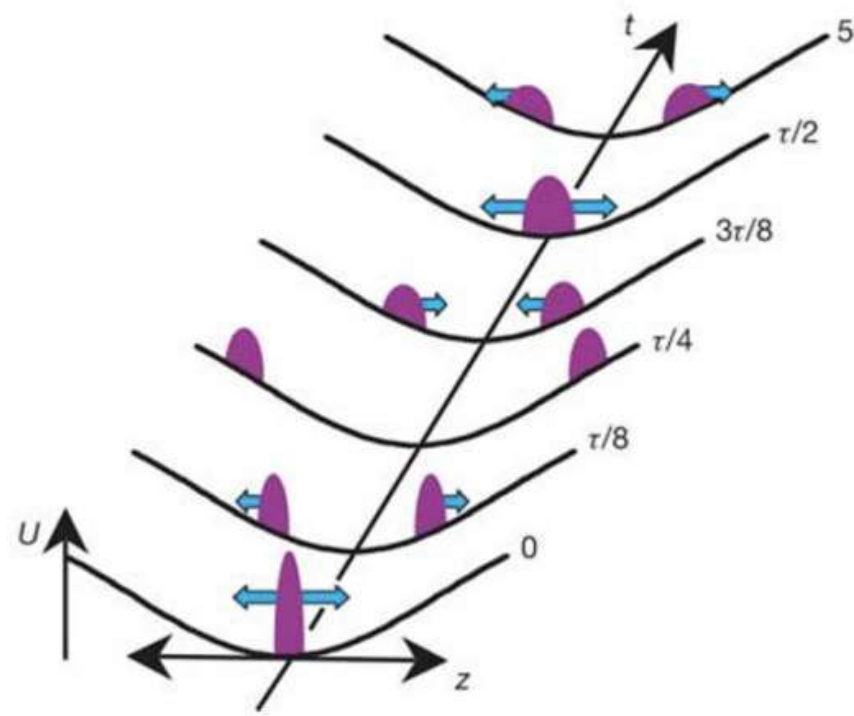
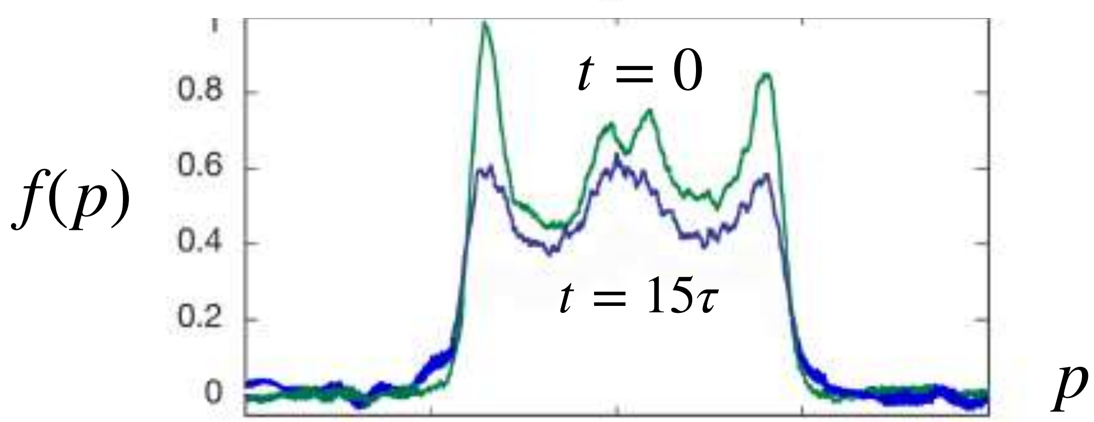
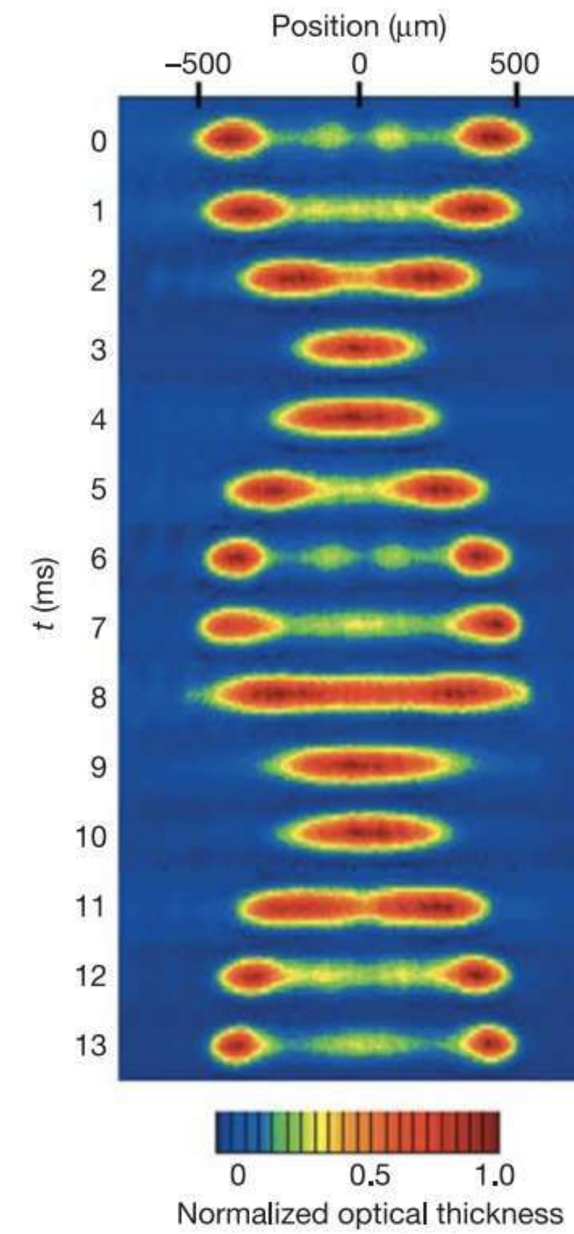
NATURE|Vol 440|13 April 2006









- "Remembers" the initial momentum distribution like 1d classical spheres
- Instead, in the 3d case it rapidly randomise

Can we describe these systems using hydrodynamics?

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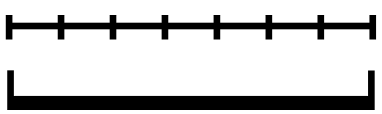
Important **practical** question

To describe a quantum system of N particles one needs a wavefunction of $3N+1$ variables

This is becomes extremely expensive for N large

Example: back to the 1d lattice of length L



- $\psi(r_1, \ldots, r_N, t) \longrightarrow L^N$ numbers

L

- $L \sim 10$; $N \sim 10^{23}$ for solids ($N \sim 10^5$ for cold atoms)

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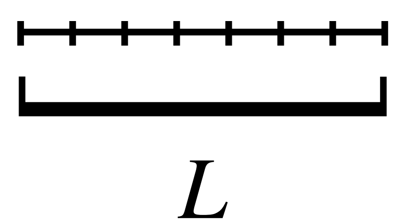

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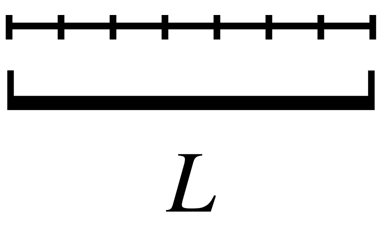

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Hydro description only requires a few functions of $1+1$ variables

Monumental Simplification!

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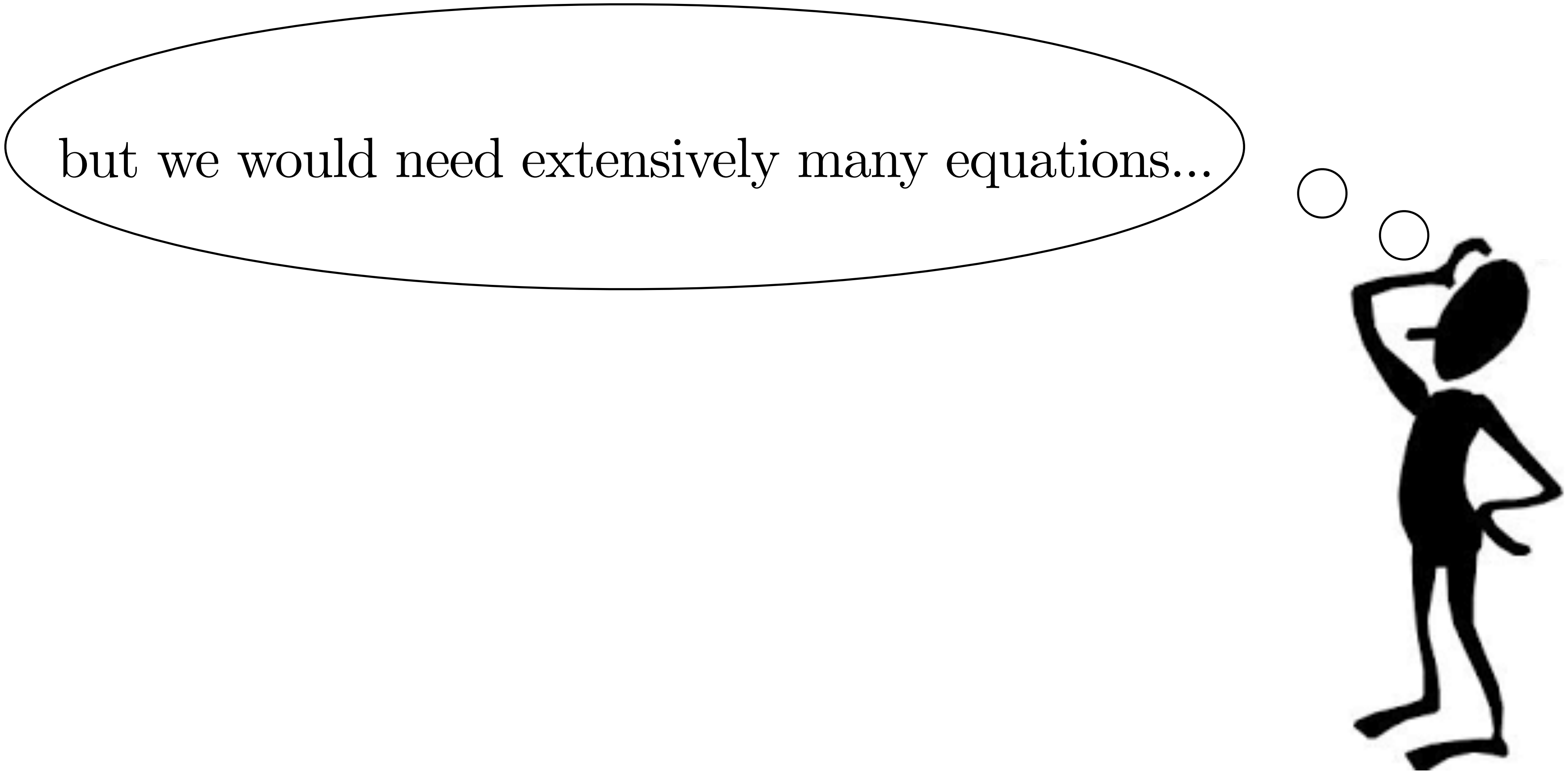

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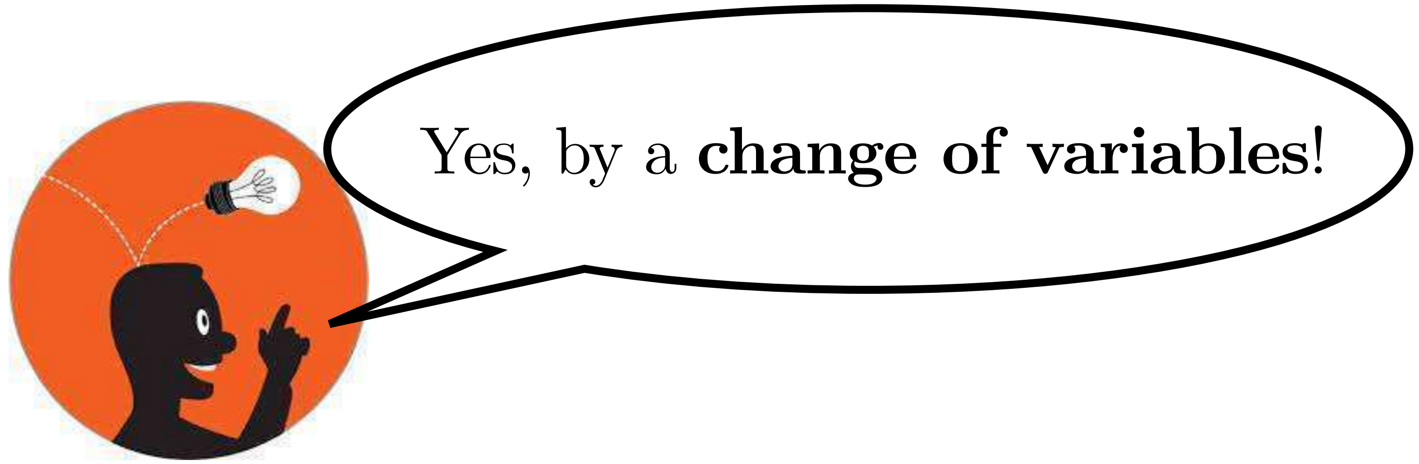

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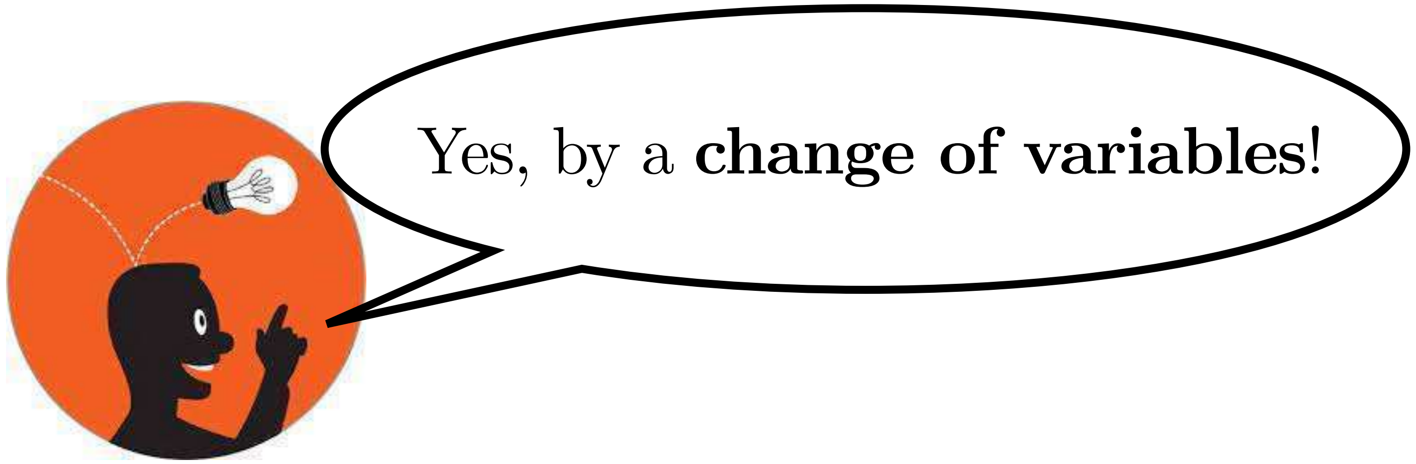
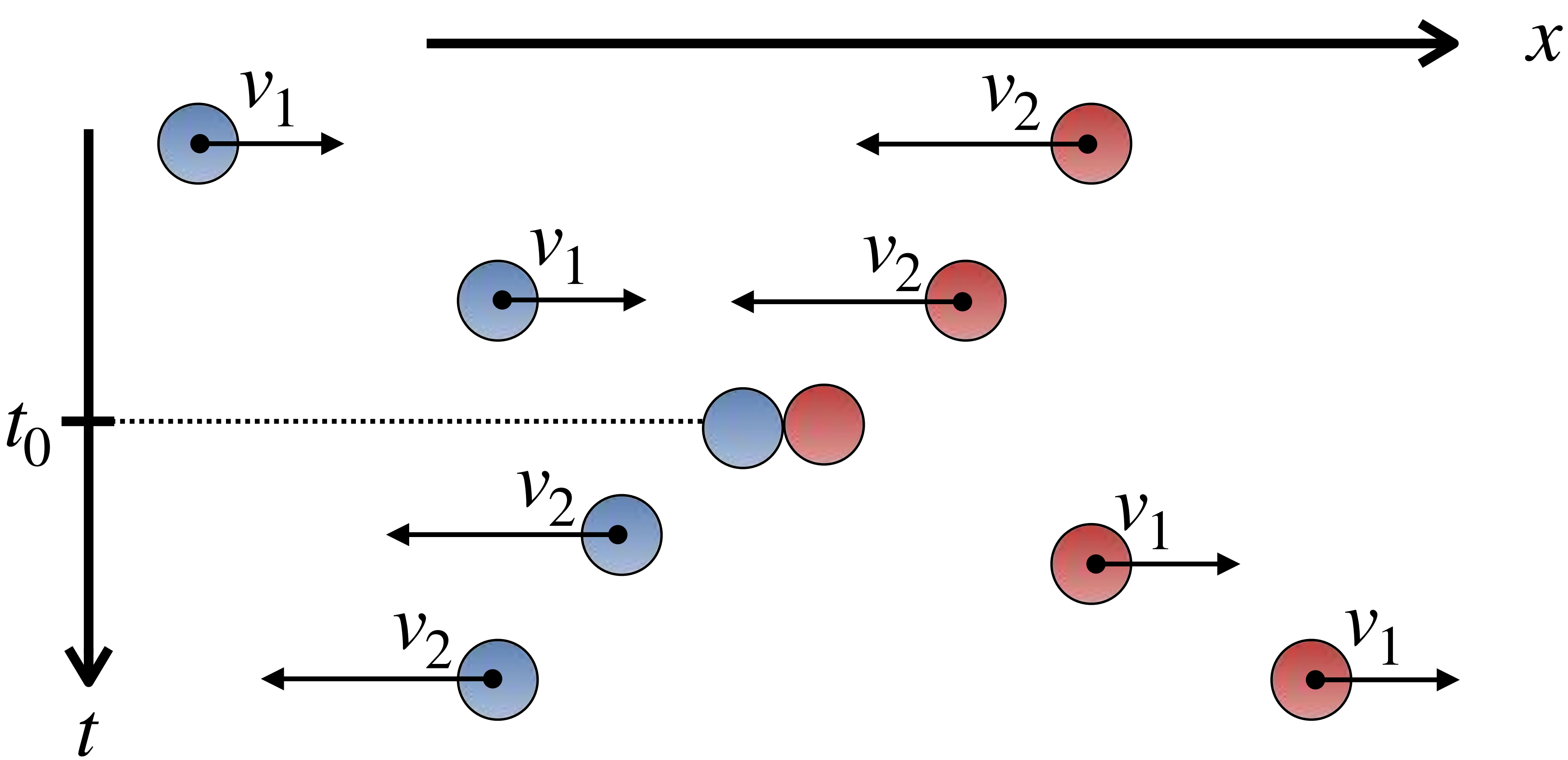


Back to spheres:



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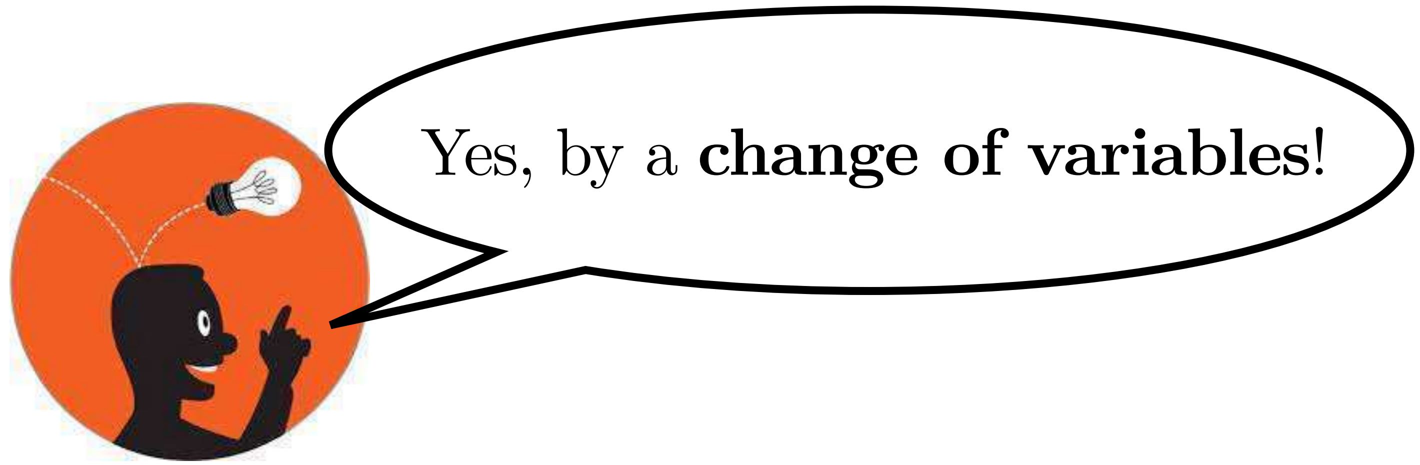

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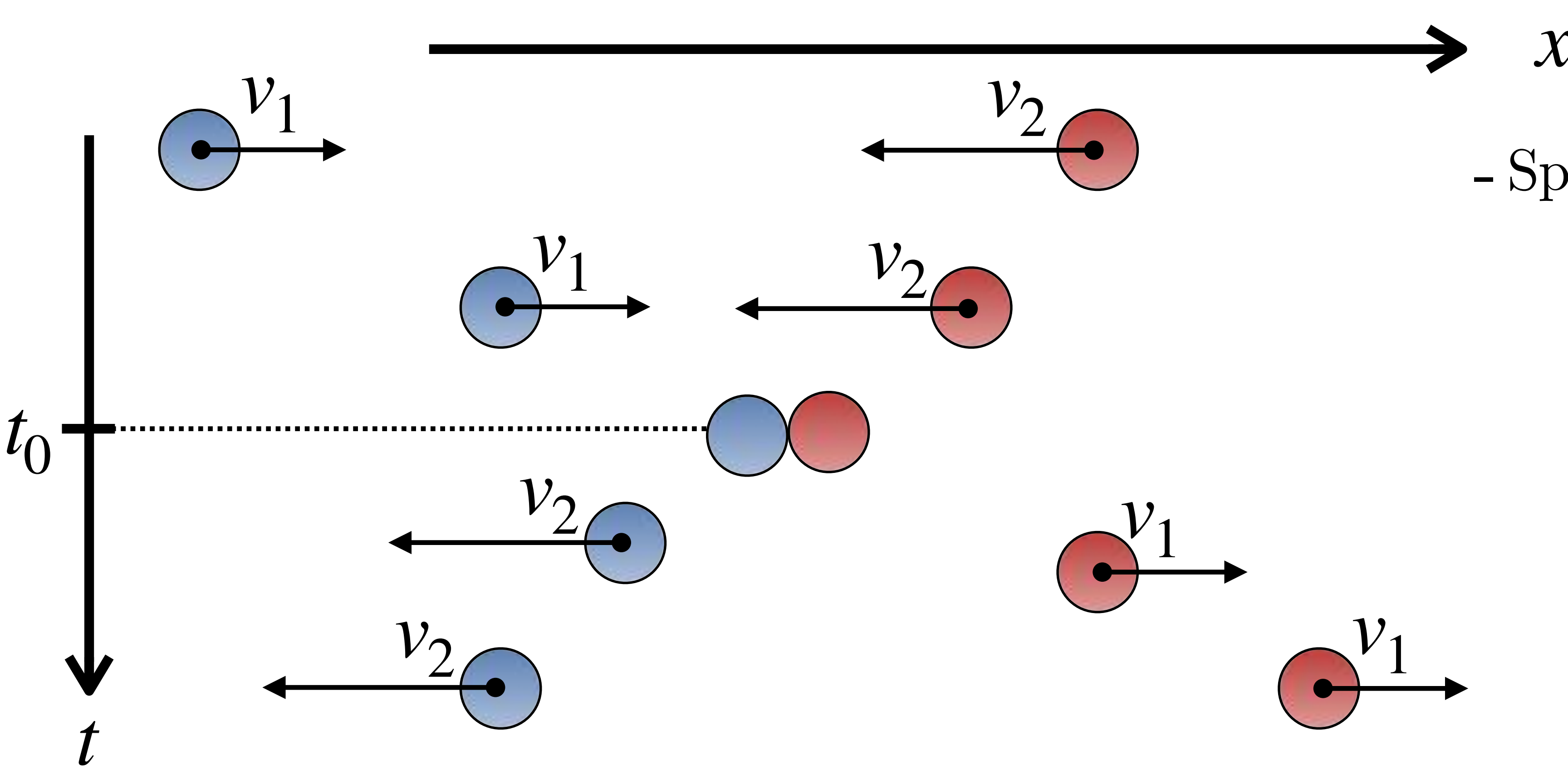

- Spheres follow complicated trajectory

$$x_1(t) = v_1 t_0 - v_2(t - t_0)$$

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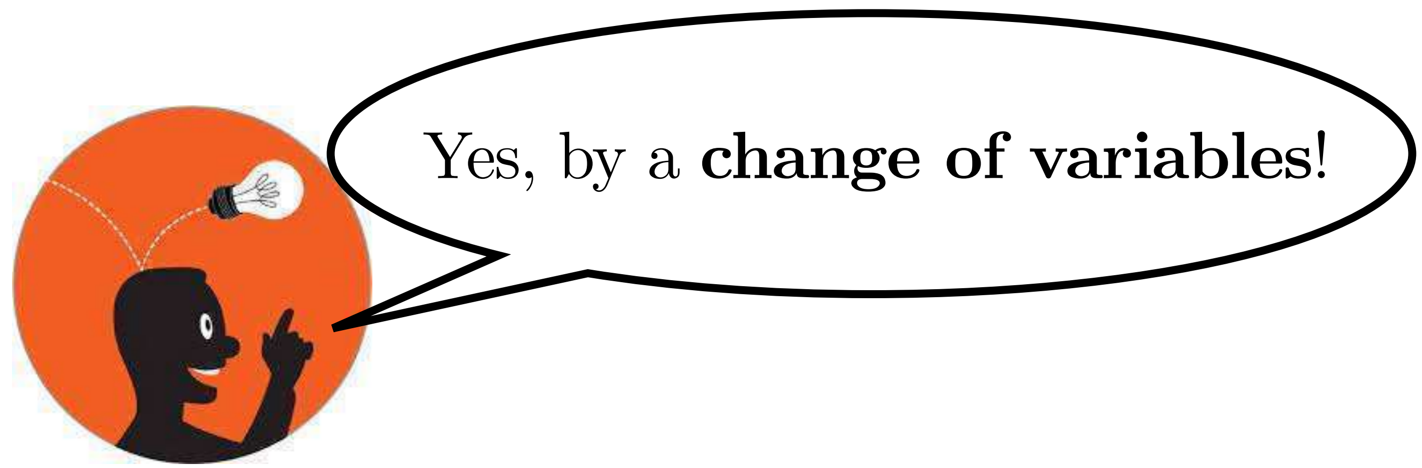

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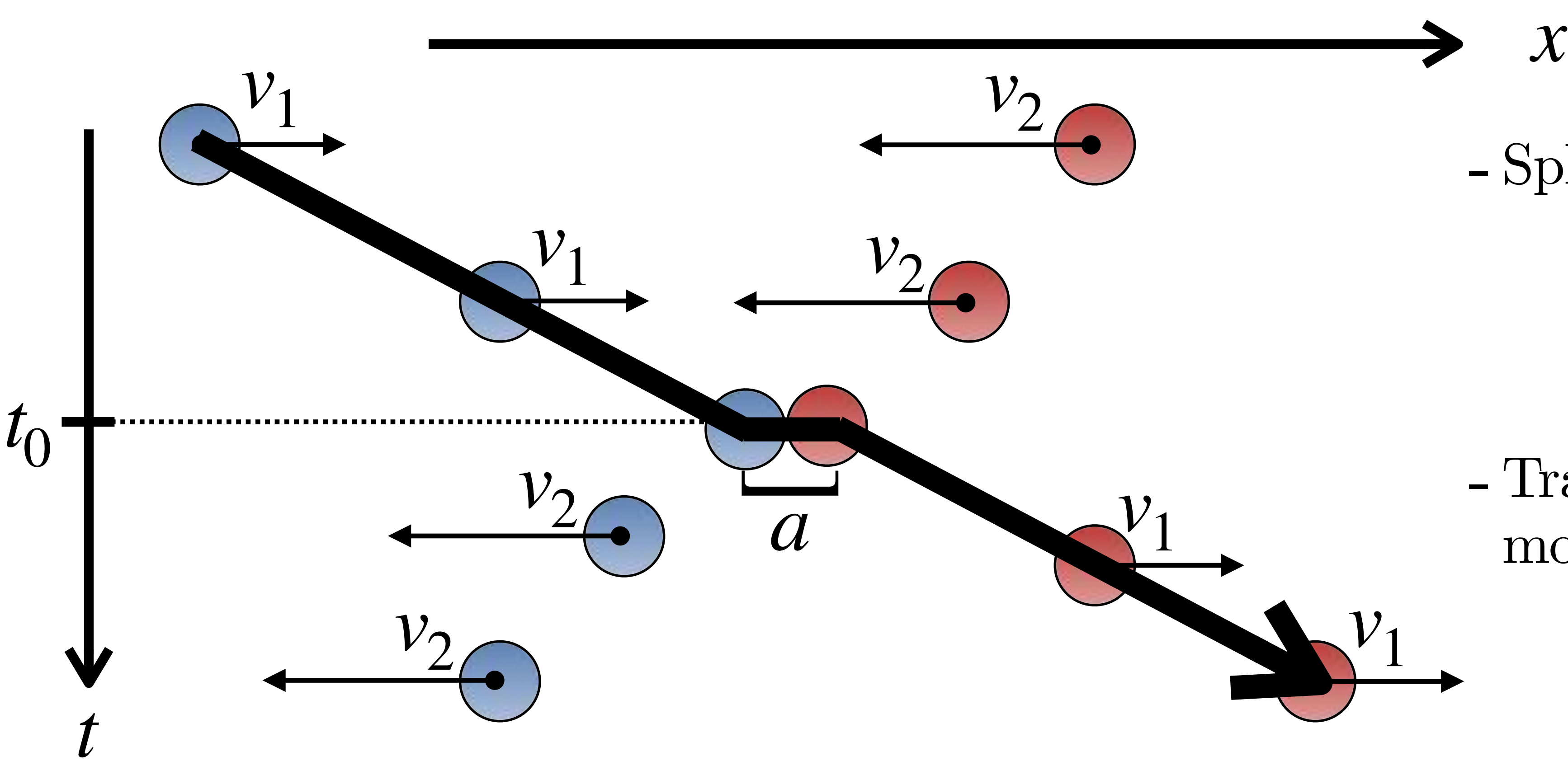

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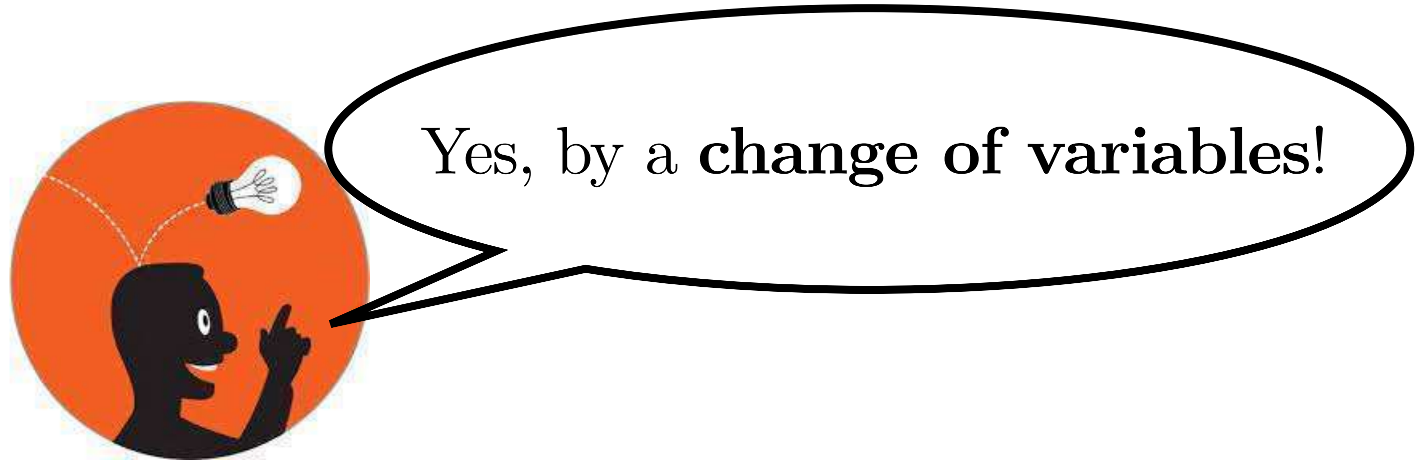

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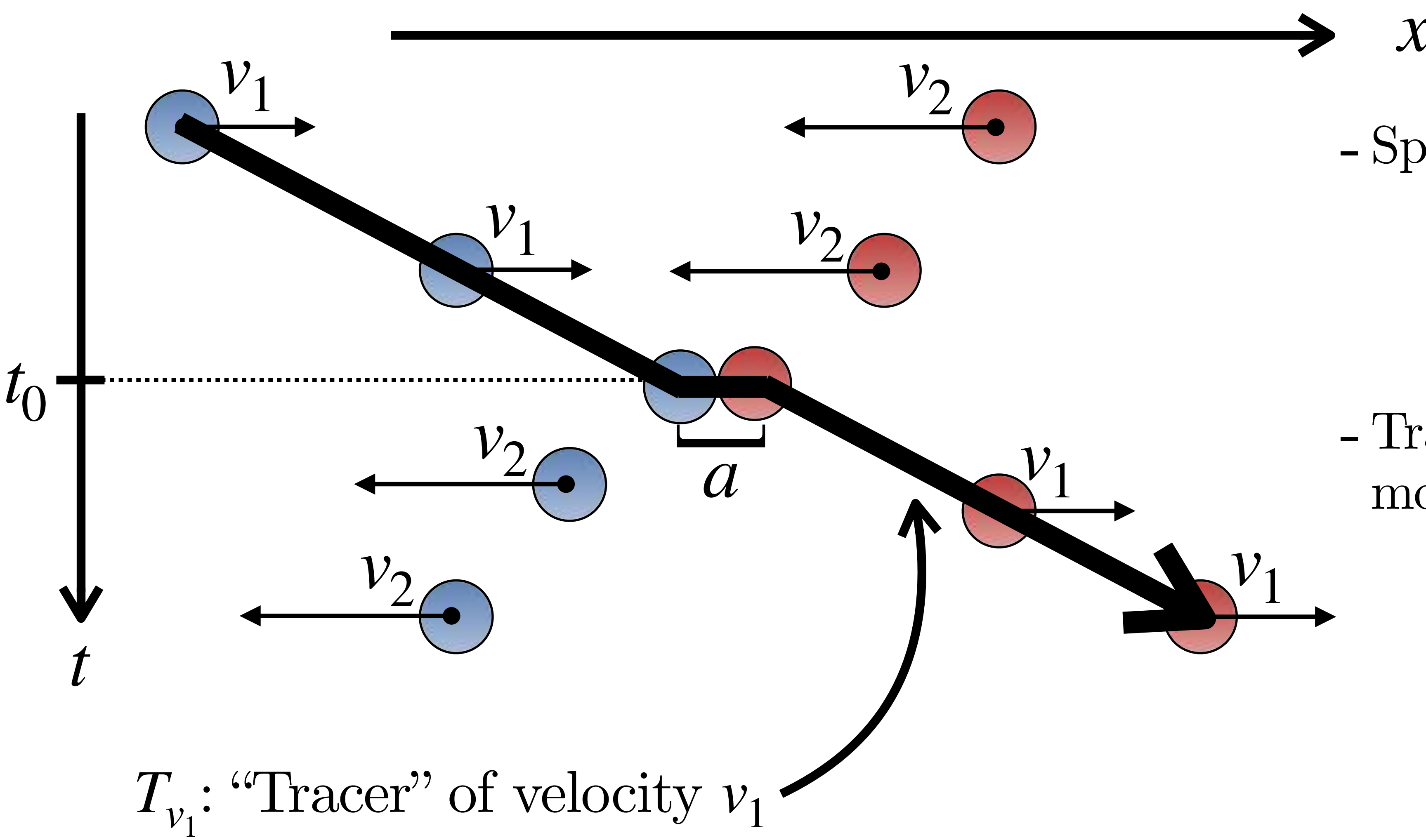

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$$x_{T_{v_1}}(t) = v_1 t + a$$

don't need to know t_0!

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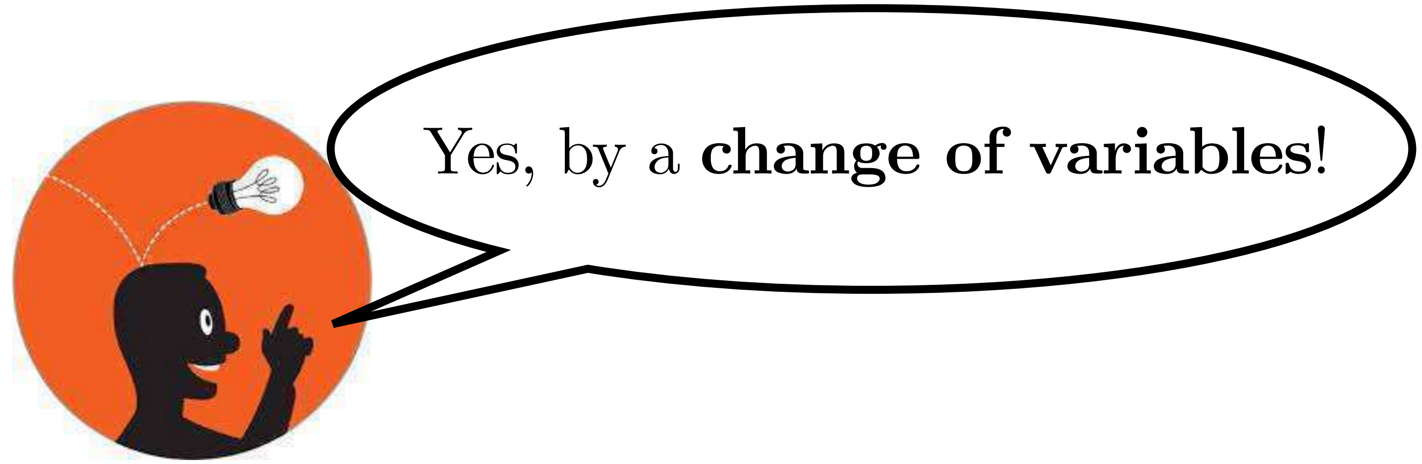
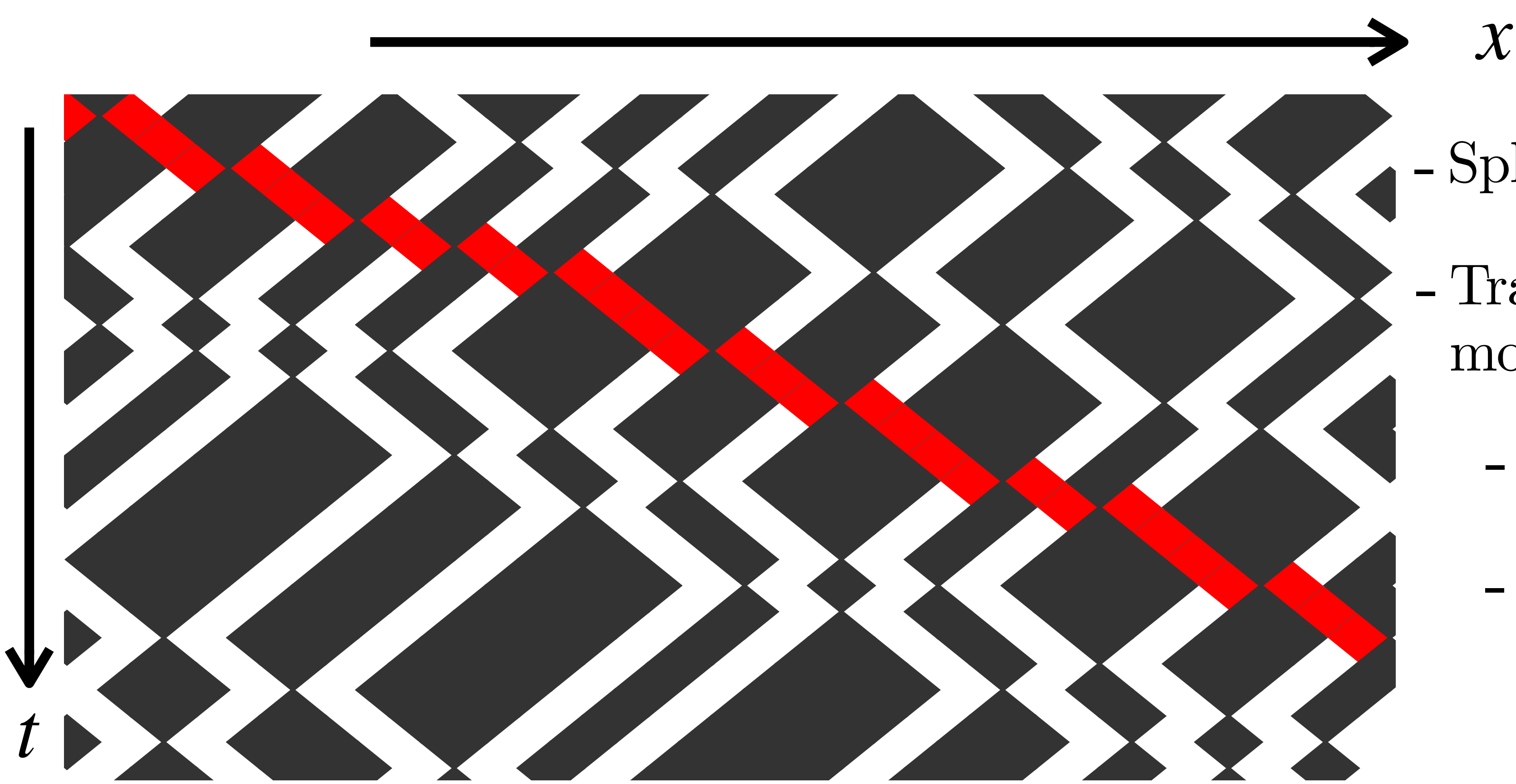


More spheres:



- Spheres follow complicated trajectory
- Tracers (or labels) of the velocities move in a simpler way
  - "almost" **uniform linear motion**
  - **interactions** just **change the** value of the **velocity**

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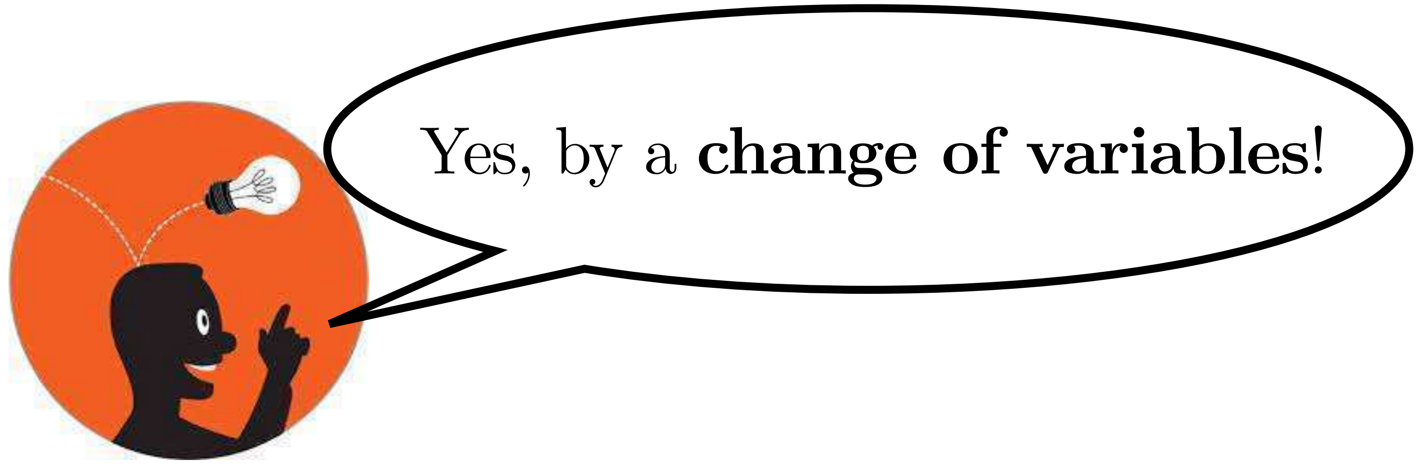

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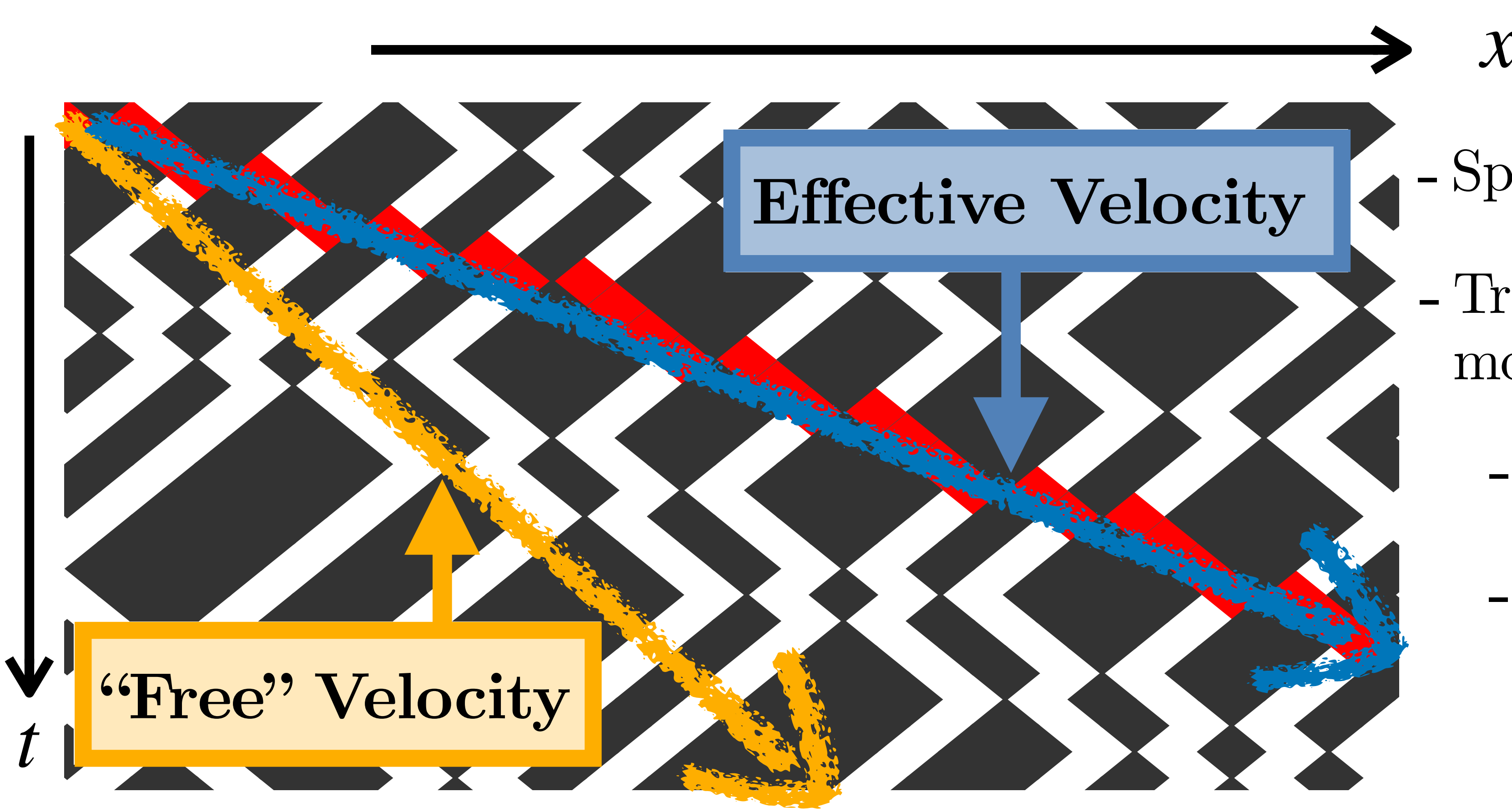

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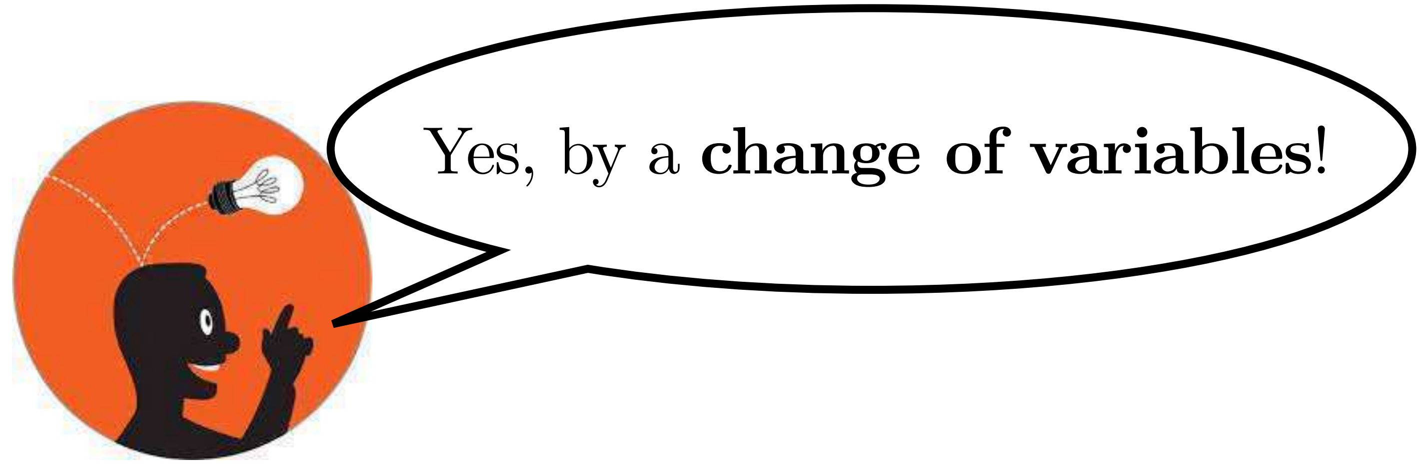

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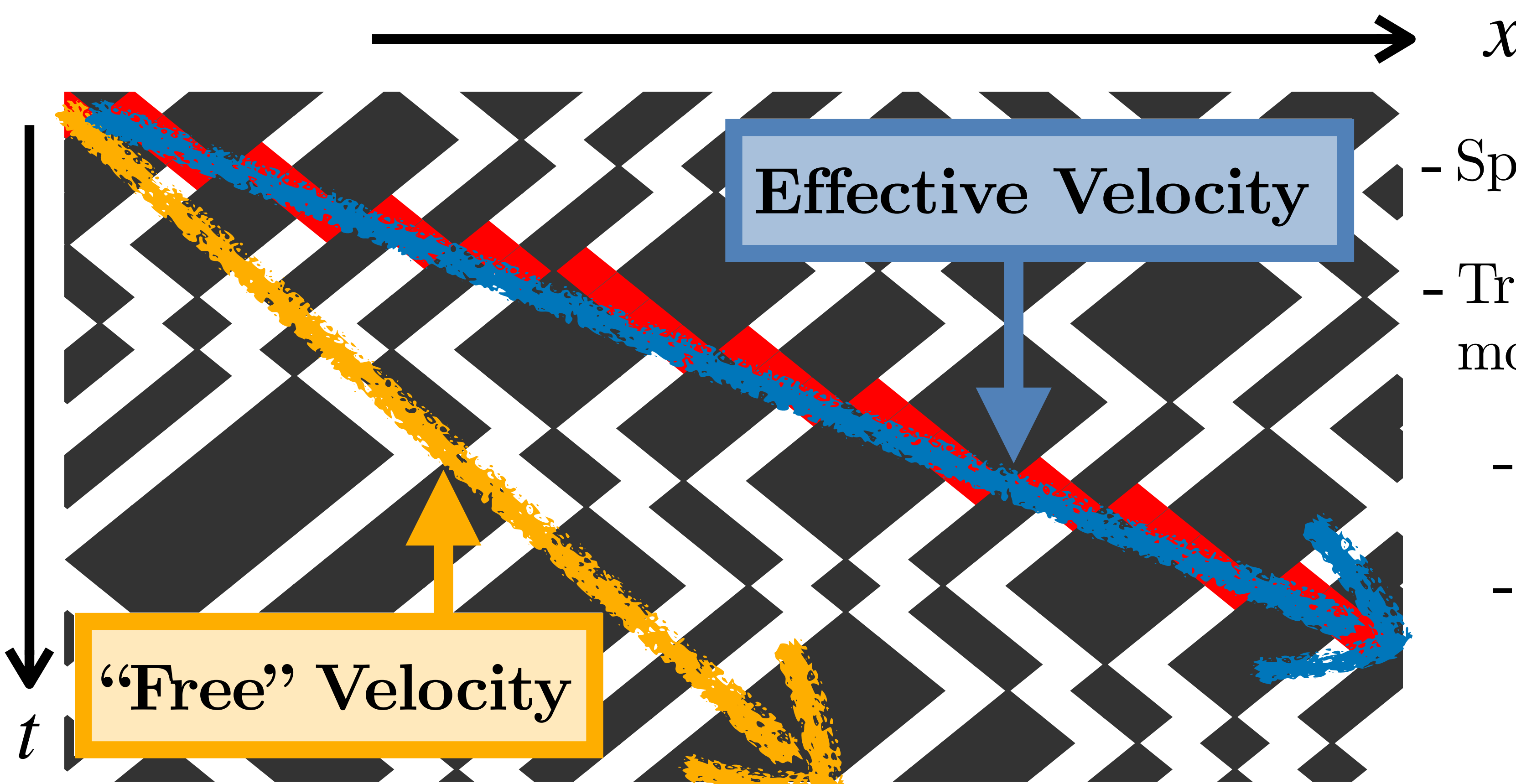

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Describe the system **using tracers instead of spheres!**

Can we describe these systems using hydrodynamics?

Yes, by a **change of variables**!

Describe the system **using tracers instead of spheres**!

Key Fact of Nature:
In many cases, complex interacting systems of many particles can be described by *"quasiparticles"*, i.e. *emergent* degrees of freedom that behave as weakly interacting particles in vacuum

Tracers = Quasiparticles

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Tracers = Quasiparticles

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Density of quasiparticles

Some equations:

$$\{q_n\} \leftrightarrow \rho(v, x, t)$$

Densities of conservation laws

- Specify the state of the system using $\rho(v, x, t)$

Can we describe these systems using hydrodynamics?

Density of quasiparticles

Some equations:

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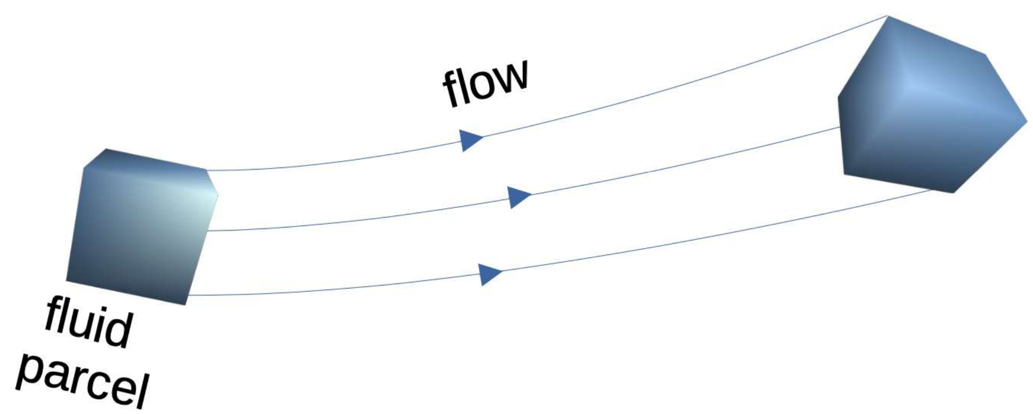
Densities of conserved charges

- Specify the state of the system using $\rho(v, x, t)$

- How does it evolve?



Change in the number of quasiparticles in the parcel = Flux of quasiparticles through the surface

$$\partial_t \rho(v, x, t) + \partial_x (v_{\text{eff}}(v, x, t)\rho(v, x, t)) = 0$$

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$v_{\text{eff}}(v) t = vt + a$ number of jumps of the quasiparticle

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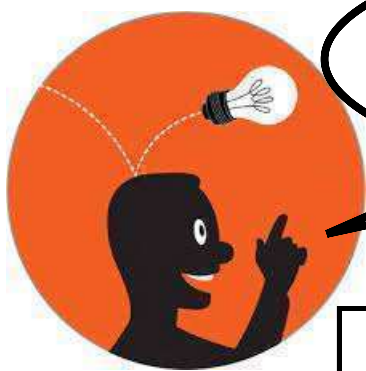

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$$v_{\text{eff}}(v) = v + a \int \mathrm{d}w \; \rho(w)(v_{\text{eff}}(v) - v_{\text{eff}}(w))$$

- The **velocity depends** on the **state of the system**

Can we describe these systems using hydrodynamics?

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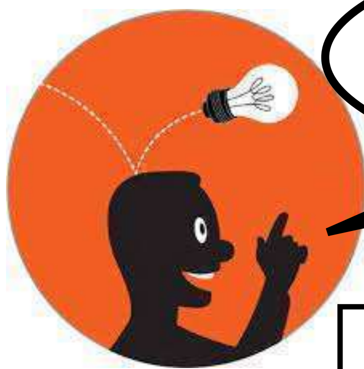

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"Generalised hydrodynamics"

- The **velocity depends** on the **state of the system**

Can we describe these systems using hydrodynamics?

The same description applies to **all quantum integrable models**!

Can we describe these systems using hydrodynamics?

The same description applies to **all quantum integrable models**!

- State of the system described by **emergent quasiparticles**

- Move with **effective velocities depending on the state**

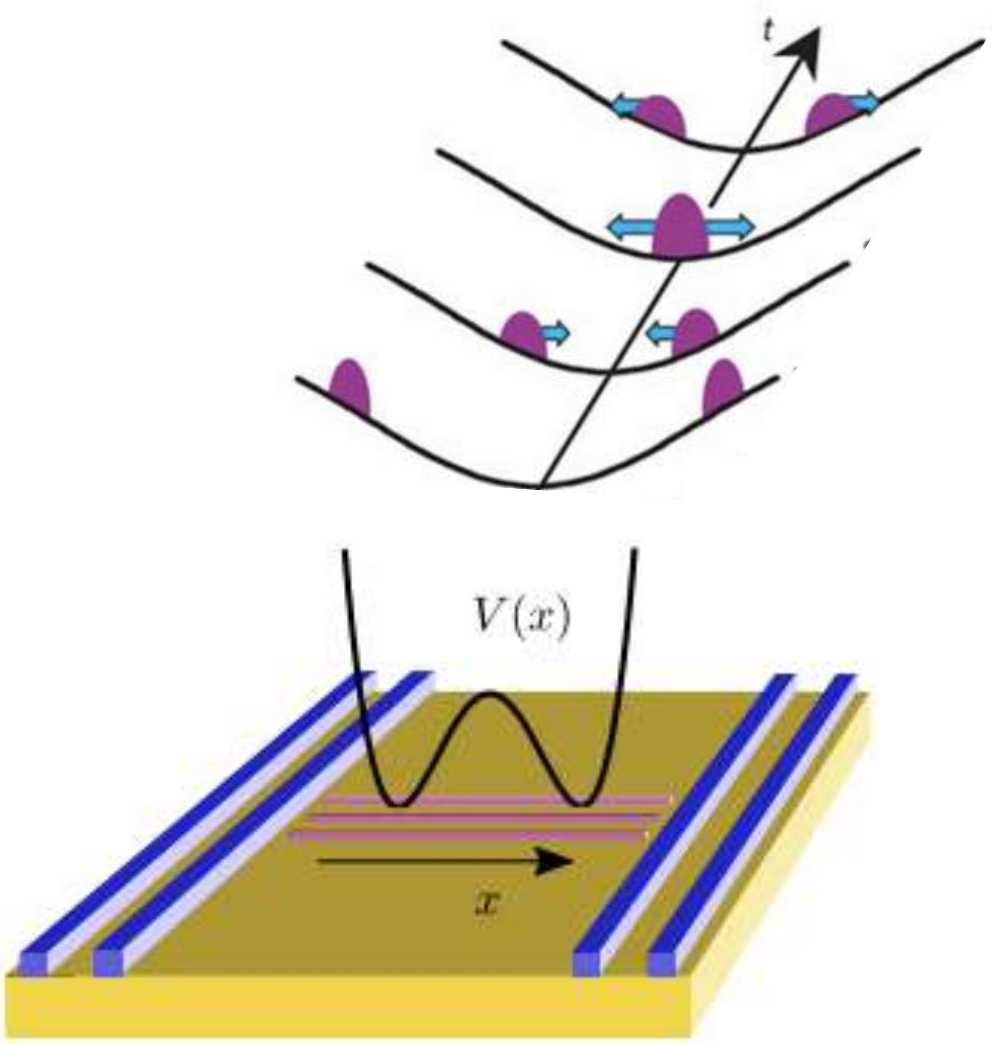
- **Same equations** (with velocity-dependent jumps)

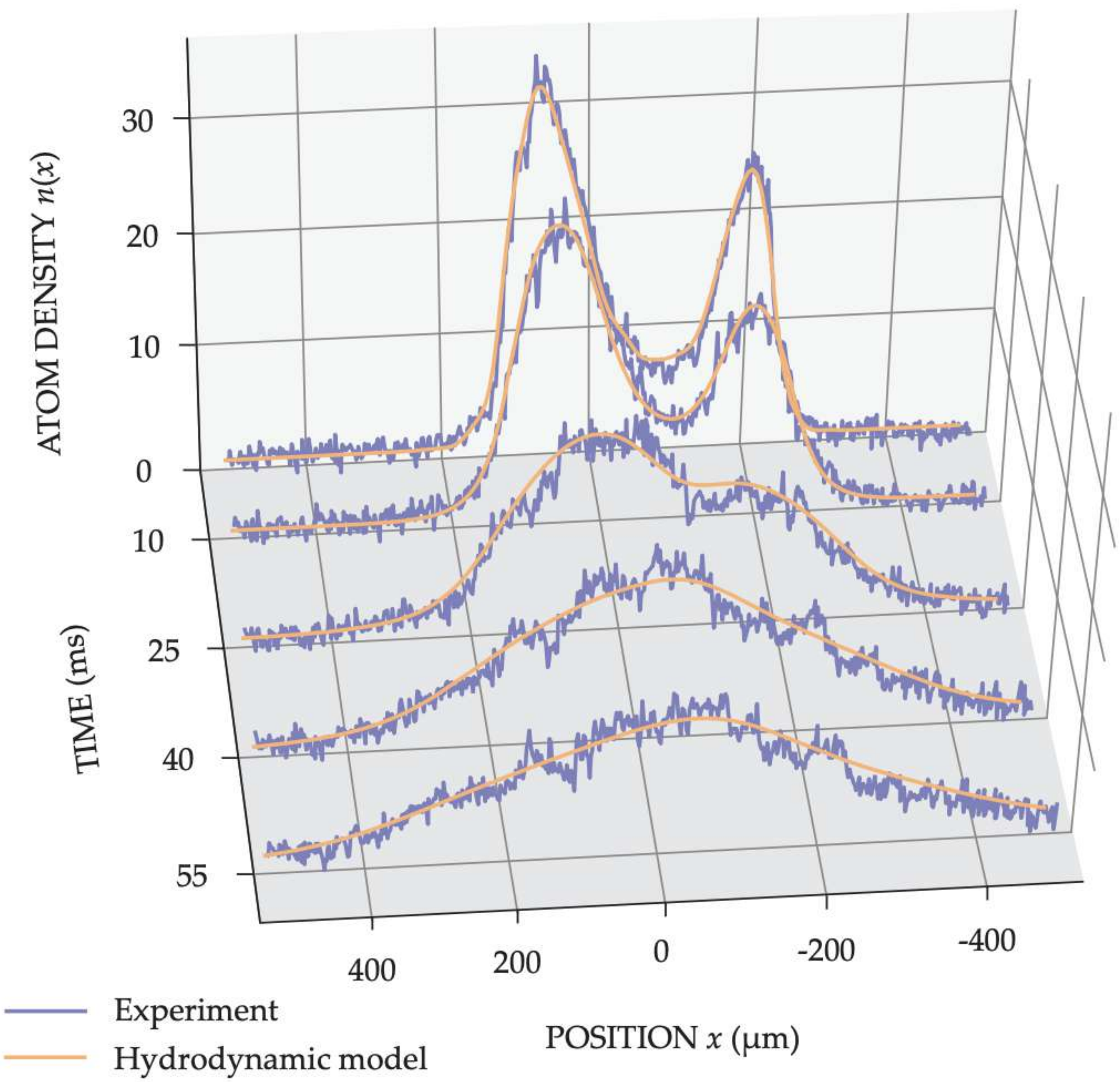
$$\partial_t \rho(v) + \partial_x (v_{\text{eff}}(v)\rho(v)) = 0$$

$$v_{\text{eff}}(v) = v + \int \mathrm{d}w \; \rho(w)(v_{\text{eff}}(v) - v_{\text{eff}}(w))a(v,w)$$

Does it work?

Quantum Newton's Cradle Revisited





PHYSICAL REVIEW LETTERS **122,** 090601 (2019)

Summary

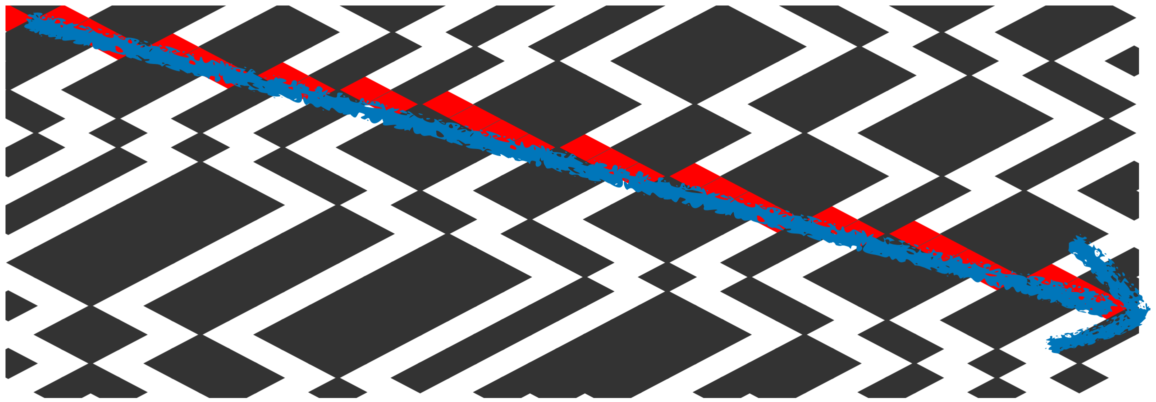
- Some interesting physical systems have an **extensive number of conservation laws**

- In these systems hydrodynamics can be defined by describing the state of the system in terms of **emergent quasiparticles**

- The **nature of quasiparticles depends on the state of the system**

Future Directions

Are there **higher order** terms in Generalised Hydrodynamics (e.g. Navier-Stokes)?
Up to what "scale" does it holds?



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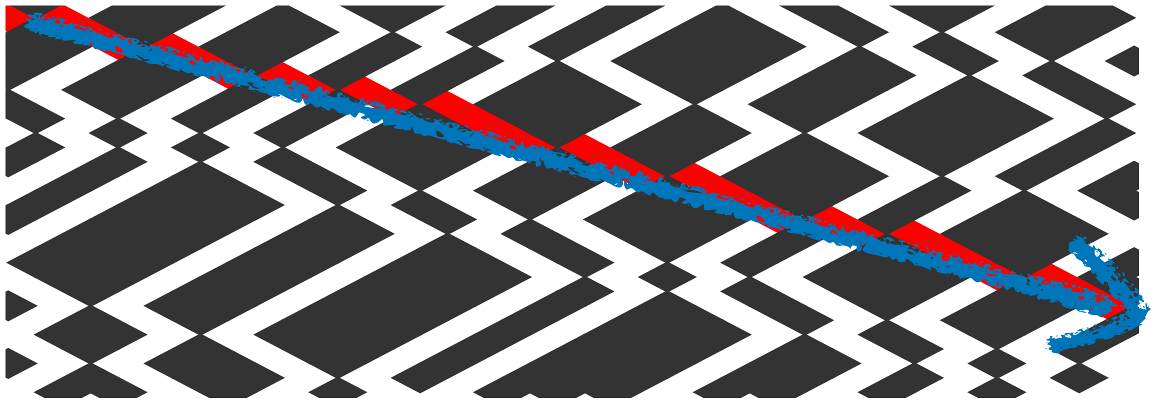

The theory is **classical: Where did $\hbar$ go? How and why** does (Generalised) Hydrodynamics emerge from the quantum dynamics?

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