Fluid-gravity duality and hydrodynamics of black holes

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Outline

Relativistic hydrodynamics

Black holes and thermodynamics: a reminder

Beyond black hole thermodynamics

Black hole membrane paradigm

Modern view: the holographic duality

Einstein = Navier-Stokes?

Gravity-inspired new advances in relativistic hydrodynamics

Relativistic Hydrodynamics

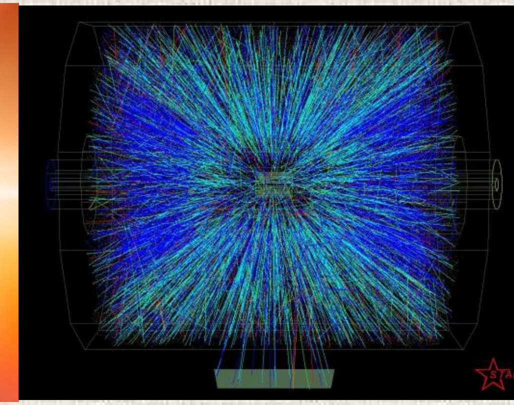
Relativistic hydrodynamics is necessary when fluids/gases move with the speeds comparable to the speed of light.

Such situations are not uncommon...

Relativistic Hydrodynamics

Astrophysics

High-energy cosmic rays



Quark-gluon plasma

In relativistic systems: energy, momentum and mass are no longer separate quantities

$$E = \sqrt{p^2c^2 + m^2c^4} \approx mc^2 + \frac{p^2}{2m} + \cdots$$

Instead, use energy density $\mathcal{E}$ and momentum density $p^a = mu^a$ combined in the object known as the energy-momentum tensor T^{ab}, where a and $b = 0, 1, 2, 3$ $\quad \partial_a T^{ab} = 0$

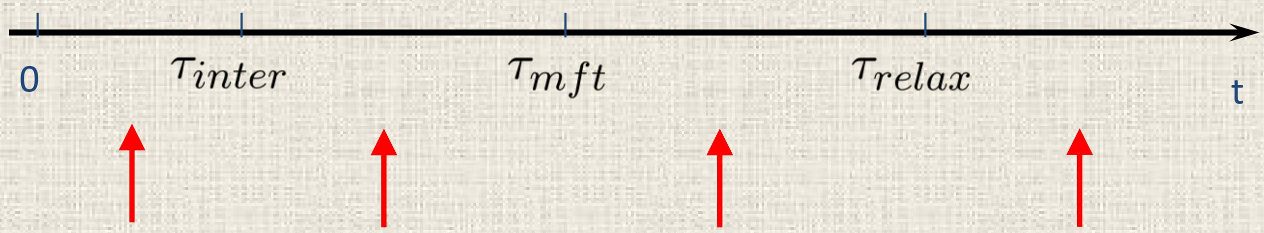
In relativistic systems: the number of particles is no longer conserved but there may be other conserved quantities such as baryonic charge or lepton number

Use (volume) density of charge J^0 Conservation law: $\partial_t J^0 + \partial_x J^1 + \partial_y J^2 + \partial_z J^3 = \partial_a J^a = 0$

The hydrodynamic regime

Fluid dynamics is an effective theory valid in the long-wavelength, long-time limit

If you wait long enough, the system equilibrates. Just BEFORE that, on large scales, it is characterized by time and space-dependent densities of conserved charges



Mechanical description

Coordinates, momenta of individual particles

Kinetic theory

Coordinate- and time-dependent distribution functions

Hydrodynamic approximation

Local densities of conserved charges

$\epsilon(x,t)$

Equilibrium thermodynamics

Globally conserved charges

E

Hydro regime: $\tau_{\text{micro}} \ll \tau \ll t_{\text{global}}$ $\quad l_{\text{micro}} \ll l \ll L_{\text{global}}$

Relativistic Hydrodynamics

Fluid dynamics is an effective theory valid in the long-wavelength, long-time limit

Fundamental degrees of freedom = densities of conserved charges

Equations of motion = conservation laws + constitutive relations

Example I

$$\left.\begin{aligned} &\partial_a J^a = 0 \\ &J^i = -D\,\nabla^i\,J^0 + \cdots \end{aligned}\right\} \qquad \partial_t J^0 = D\,\nabla^2 J^0 + \cdots$$

Gradient expansion

Example II

$$\left.\begin{aligned} &\partial_a T^{ab} = 0 \\ &T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots \end{aligned}\right\}$$

Navier-Stokes eqs
Burnett eqs
.....

** E.o.m. universal, transport coefficients depend on underlying microscopic theory

Consider relativistic neutral conformal fluid in a d-dimensional (curved) space-time

$$T^{ab} = \varepsilon u^a u^b + P(\varepsilon)\left(g^{ab} + u^a u^b\right) + \Pi^{ab} + \cdots$$

Including only terms with first and second derivatives of fluid velocity:

$$\begin{aligned}\Pi^{ab} = & -\eta\sigma^{ab} \\ & + \eta\tau_{\Pi}\left[{}^{\langle}D\sigma^{ab\rangle} + \frac{1}{d-1}\sigma^{ab}\left(\nabla\cdot u\right)\right] \\ & + \kappa\left[R^{\langle ab\rangle} - (d-2)u_c R^{c\langle ab\rangle d}u_d\right] \\ & + \lambda_1\sigma^{\langle a}{}_c\sigma^{b\rangle c} + \lambda_2\sigma^{\langle a}{}_c\Omega^{b\rangle c} + \lambda_3\Omega^{\langle a}{}_c\Omega^{b\rangle c}\end{aligned}$$

Transport coefficients (in conformal case): $\eta\,,\tau_{\Pi}\,,\kappa\,,\lambda_1\,,\lambda_2\,,\lambda_3$

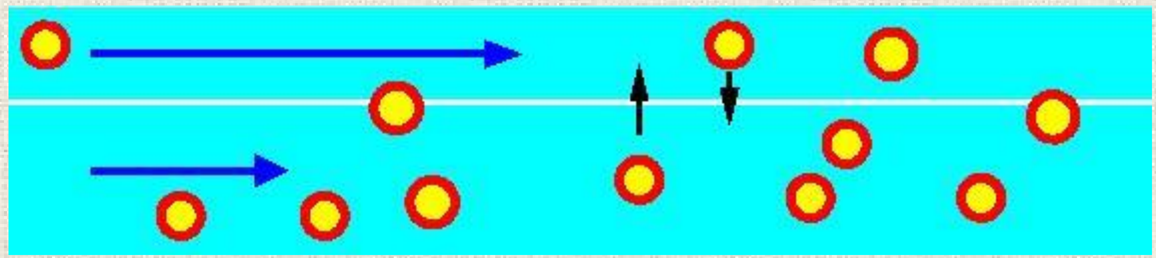
Non-conformal case: 2 first order coefficients, 15 (10) second order coefficients

Shear viscosity

One of the most important characteristics of a fluid is shear viscosity

Shear viscosity can be understood as the measure of internal friction in liquid or gas

$v_2 > v_1$



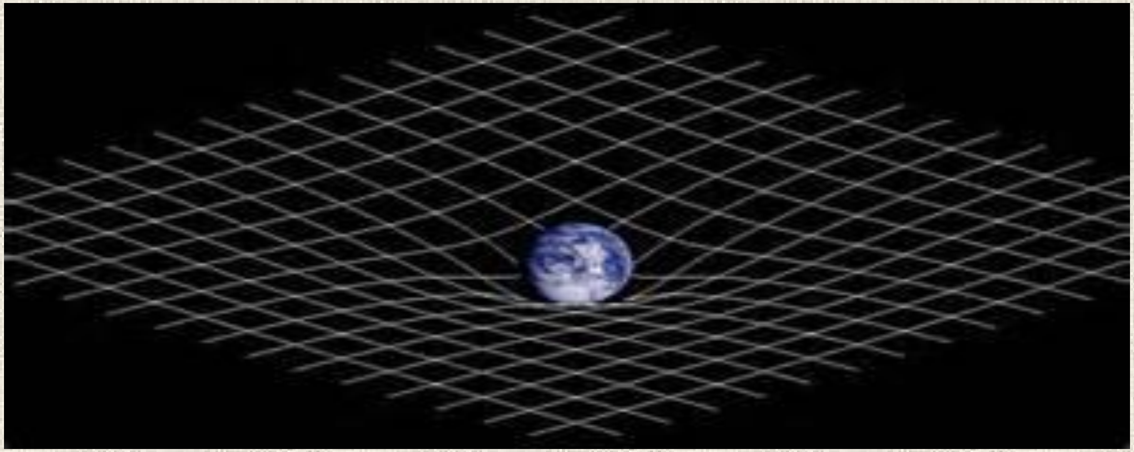
v_1

Momentum transfer between two layers of liquid or gas moving with different velocities leads to gradual equilibration of the velocities

GRAVITY
and
BLACK
HOLES

General Relativity is a theory of (classical) gravity

Einstein's equations determine the metric of space-time



Solve $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$ for $g_{\mu\nu}$

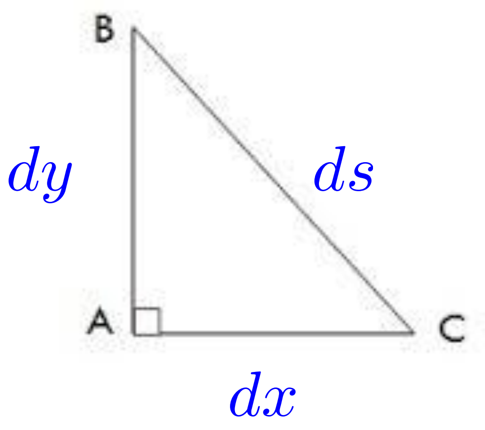
Geometry (metric) = Energy (and/or mass)

This is similar to Maxwell equations:

Solve $$\partial^\nu \partial_\mu A^\mu - \partial_\mu \partial^\mu A^\nu = \mu_0 J^\nu$$ for A^μ

Electromagnetic field = Currents (and/or charges)

The Metric Tensor



Infinitesimal distance in 2 dimensions:

$$ds^2 = dx^2 + dy^2$$

Can be written more generally as

$$ds^2 = g_{ij}dx^i dx^j = g_{11}dx^2 + g_{12}dxdy + g_{21}dydx + g_{22}dy^2$$

$$= dx^2 + dy^2$$

$$g_{ij} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Minkowski space-time metric: $ds^2 = -c^2dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$

The Schwarzschild metric

$$ds^2 = -c^2 \left(1 - \frac{2GM}{c^2 r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Describes the metric of space-time
OUTSIDE of a spherically-symmetric body of mass M

Corrections to “1” are very small for stars & planets

Note the (coordinate) singularity at

$$r = R_S = \frac{2GM}{c^2}$$

(the Schwarzschild radius)

Karl Schwarzschild (1873-1916)

Black holes (continued)

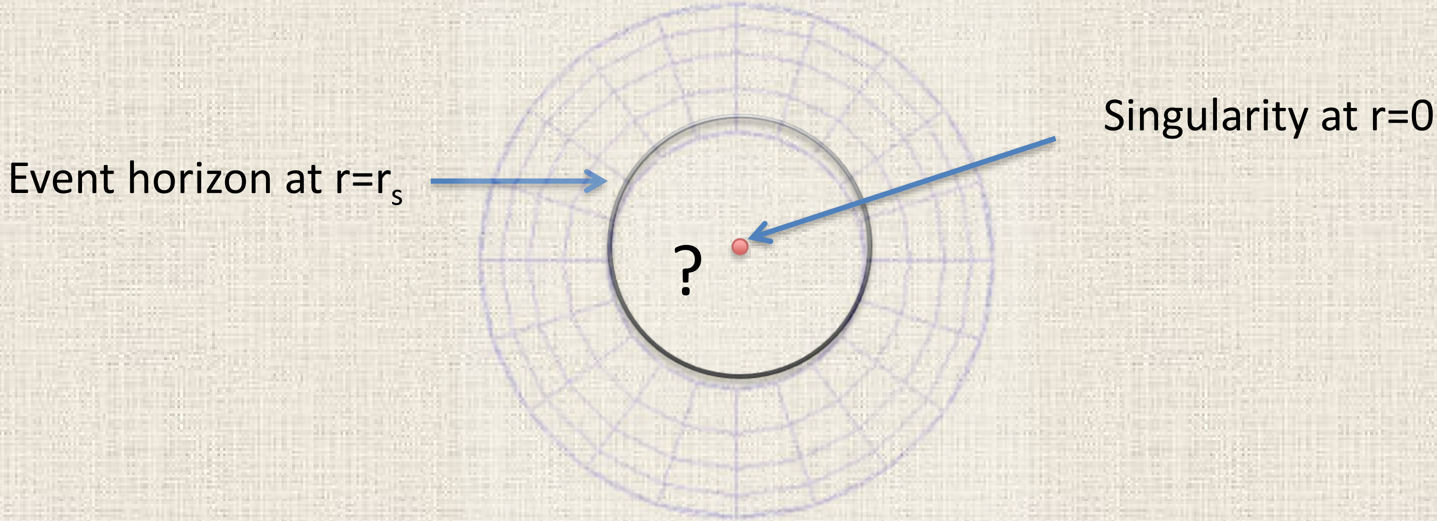
$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Describes the metric of space-time OUTSIDE of a spherically symmetric body of mass M

$$r = r_S = \frac{2GM}{c^2}$$

Earth: 1 cm; Sun: 3 km

However, if the matter is squeezed inside its Schwarzschild radius (e.g. in the process of a gravitational collapse of a star), we get a black hole



Black holes

Schwarzschild metric

Describes space-time OUTSIDE of a spherically symmetric body of mass M

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Reissner–Nordström metric

Describes space-time OUTSIDE of a body of mass M and charge Q

$$ds^2 = -c^2\left(1 - \frac{2GM}{c^2 r} + \frac{Q^2 G}{4\pi\varepsilon_0 r^2}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r} + \frac{Q^2 G}{4\pi\varepsilon_0 r^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Kerr-Newman metric

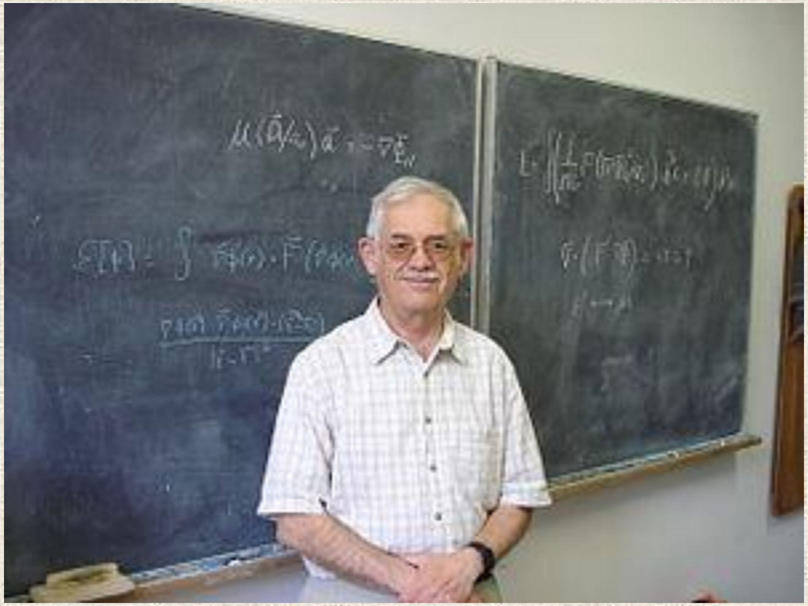
Describes space-time OUTSIDE of a body of mass M, charge Q and angular momentum J

Black holes have interesting properties...

Entropy and temperature associated with black holes

In 1972, Jacob Bekenstein suggested that a black hole can be assigned an entropy proportional to the horizon area

(See John Chalker's talk at Saturday Morning "Entropy" event on November 17, 2018)



Jacob Bekenstein

Stephen Hawking

In 1974 Hawking demonstrated that black holes emit radiation at a quantum level and so one can associate a temperature with them

Hawking temperature and Bekenstein-Hawking entropy

Hawking showed that black holes emit radiation with a black-body spectrum at a temperature

$$T = \frac{\hbar c^3}{8\pi k_B G M} \approx \frac{1.2 \times 10^{23}\,\mathrm{kg}}{M} K$$

(a black hole of one solar mass has a Hawking temperature of about 50 nano-Kelvin)

This fixes the coefficient of proportionality in Bekenstein's conjecture:

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar} = \frac{k_B A}{4 l_P^2}$$

Immediate consequences and problems:

- Black holes "evaporate" with time
- Information loss paradox
- *What are the microscopic degrees of freedom underlying the BH thermodynamics?*

Laws of black hole mechanics vs laws of thermodynamics

(J.Bardeen, B.Carter, S.Hawking, 1973)

Zeroth Law (BH): The horizon of a stationary black hole has constant surface gravity κ

Zeroth Law (TD): The system in thermal equilibrium has a constant temperature T

First Law (BH): In perturbations of stationary black holes, the change of mass M is related to the change of charge Q, angular momentum J and horizon area A by

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ$$

First Law (TD): In thermodynamic processes the change of energy E is related to the change of entropy S (plus relevant work terms)

$$dE = T dS - p\, dV - \mu\, dN$$

Second Law (BH): The horizon area A is a non-decreasing function of time

Second Law (TD): The entropy S is a non-decreasing function of time

Third Law: It is impossible to achieve zero surface gravity by a physical process

Third Law: It is impossible to achieve zero temperature by a physical process

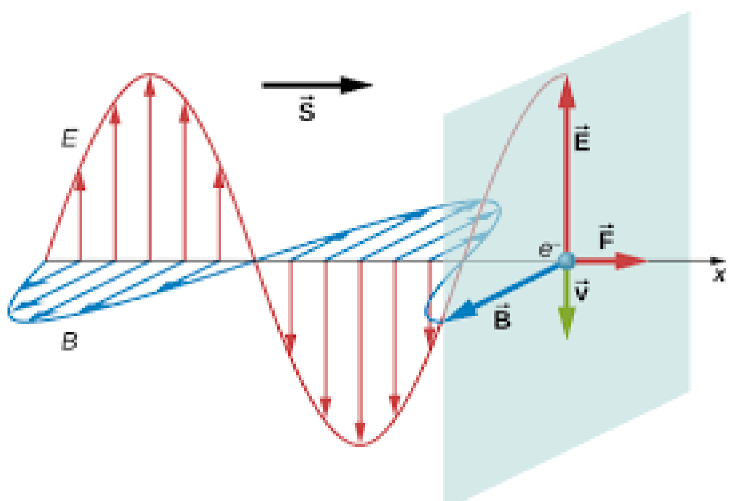
Beyond black hole thermodynamics

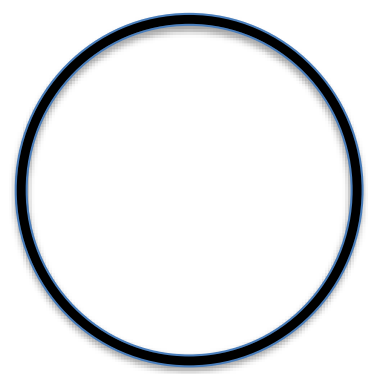
Entropy and temperature are characteristics of a system in thermal equilibrium

In near-equilibrium regime, “normal” systems are described by hydrodynamics

What happens in case of near-equilibrium black holes?

Example of a normal system: a conducting sphere in an external electromagnetic field





Sphere of conductivity σ

Expect surface currents and the Ohm’s law: $\vec{J} = \sigma \vec{E}$

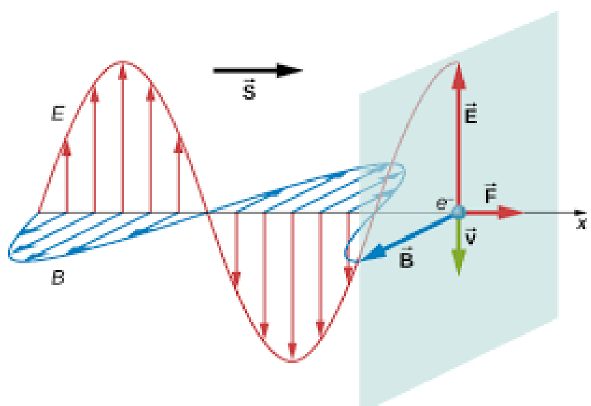
Note: we use only Maxwell’s equations and boundary conditions, no assumptions about microscopic structure of the sphere

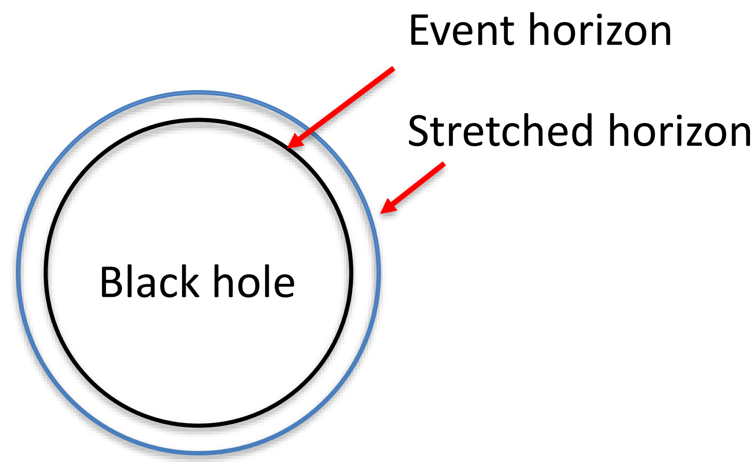
Black hole "membrane paradigm"

Now consider a black hole in an external electromagnetic field

Hajicek, 1974; Hanni, Ruffini, 1976; Znajek, 1976-78

Use Einstein-Maxwell equations to analyze fields on the "stretched horizon"
(a time-like surface just outside the true horizon)





It turns out the surface current on the stretched horizon obeys the Ohm's law $\vec{J} = \sigma \vec{E}$

Stretched horizon behaves as an ohmic conductor with a surface resistance

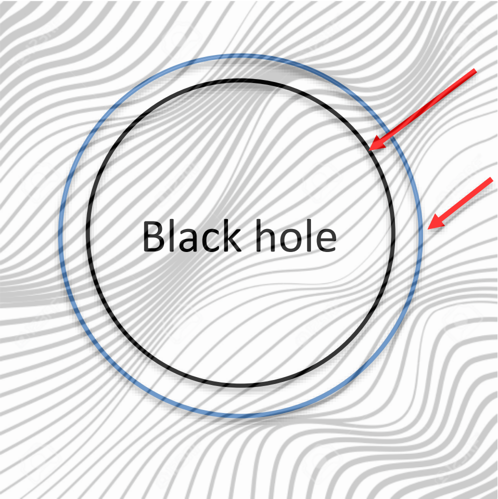
$$R_s = \frac{4\pi}{c} \approx 377\,\mathrm{Ohm}/\Box$$ (Roman Znajek, 1976-78)

Compare: Metal foils $R_s \sim 0.1\,\mathrm{Ohm}/\Box$ Polycrystalline silicone $R_s \sim 1 - 400\,\mathrm{Ohm}/\Box$

Black hole “membrane paradigm” (continued)

Similarly, we can consider a (Schwarzschild) black hole in an external gravitational field (for example, in a gravitational wave)

Hawking, Hartle, 1972-74; Damour, 1978-82; Znajek, 1978-84

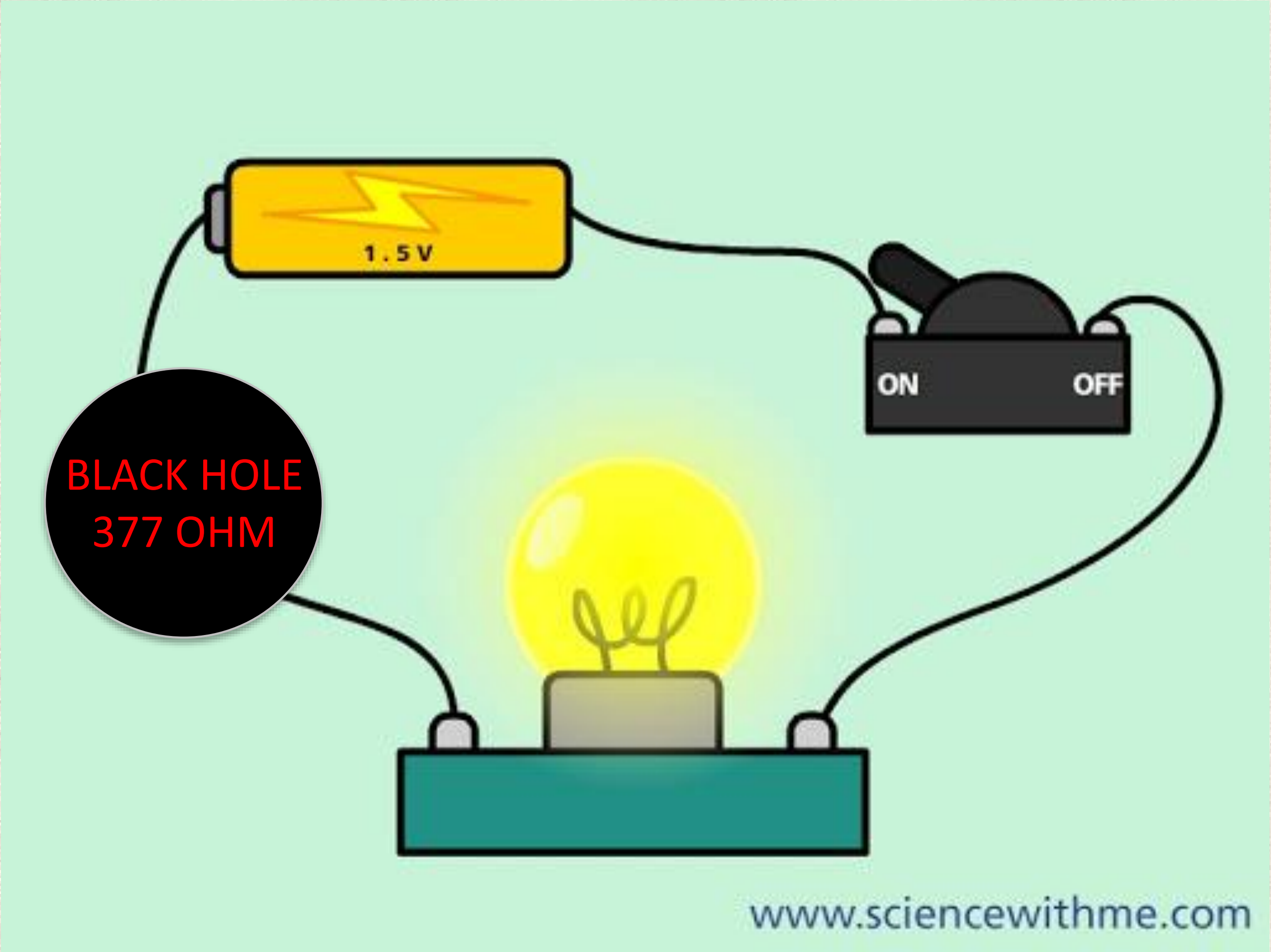


It turns out that the stretched horizon behaves as an elastic membrane or fluid with shear viscosity η and bulk viscosity ζ

$$\eta = \frac{c^3}{16\pi G} = \frac{\hbar}{16\pi l_P^2} \qquad \zeta = -\frac{c^3}{16\pi G} = -\frac{\hbar}{16\pi l_P^2}$$

Note: $s = S_{BH}/A = \frac{k_B c^3}{4G\hbar} = \frac{k_B}{4l_P^2}$ implies $\eta/s = \hbar/4\pi k_B$

Black hole horizons have properties of a physical medium such as conductivity and viscosity



Curious: The word "viscosity" is derived from the Latin "viscum" (mistletoe). Viscum refers to a viscous glue derived from mistletoe berries.

Holographic principle

In thermodynamic systems without gravity, the entropy is extensive (proportional to volume)

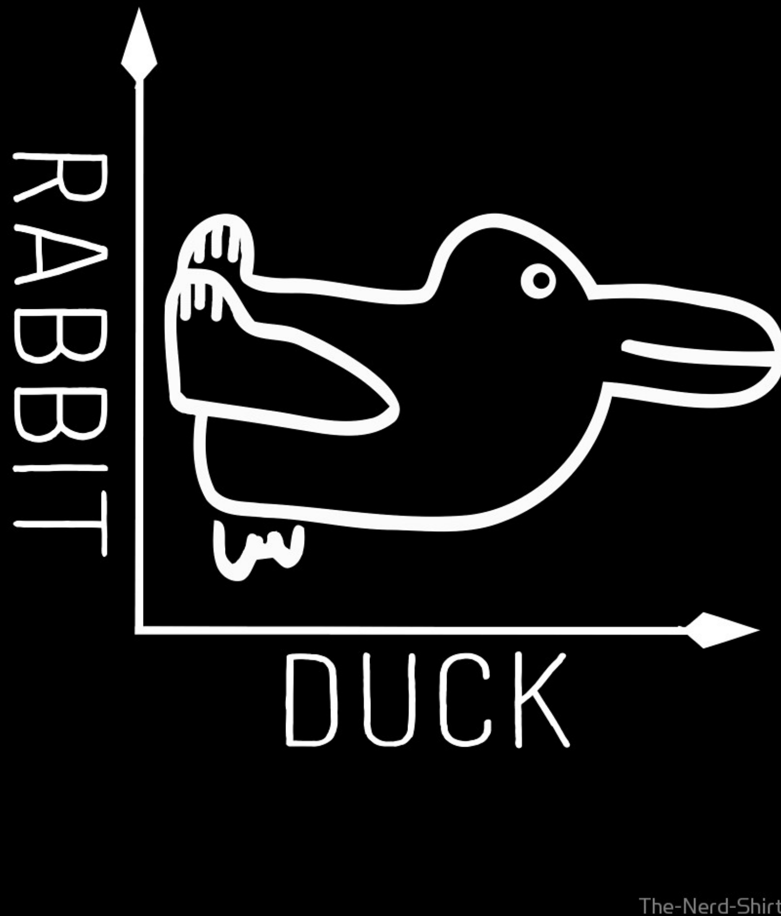
In gravitational systems, it is proportional to the AREA

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar} = \frac{k_B A}{4 l_P^2}$$

It seems that gravitational degrees of freedom in D dimensions are effectively described by a theory in D-1 dimensions (‘tHooft, Susskind, 1992)

A brief introduction
to
string theory and gauge-string duality

Ludwig Wittgenstein's view of duality (1892; 1953)



(The analogy stolen from Shamit Kachru's talk at Simons Foundation, New York, Feb 27, 2019)

Gauge-string duality (a.k.a. AdS-CFT correspondence)

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

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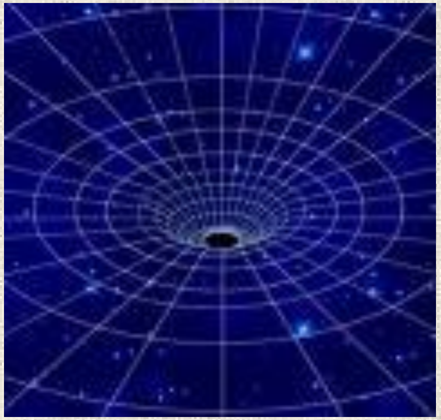


Open strings picture:
dynamics of strings & branes at low energy
is desribed by a
quantum field theory without gravity

Closed strings picture: dynamics of strings
& branes at low energy is described by
gravity and other fields
in higher dimensions

$$Z_{field\ theory}[\alpha] \Longleftrightarrow Z_{string\ theory}\left[\frac{1}{\alpha}\right]$$

conjectured
exact equivalence

Partition function of
field theory in 3+1 dim

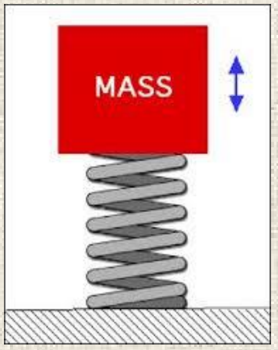
strong coupling

Partition function of
string theory in 10 dim

weak coupling

Black holes beyond equilibrium

Undisturbed black holes are characterized by global charges: M, Q, J…



A thermodynamic system in equilibrium is characterized by conserved charges: E, Q, J…

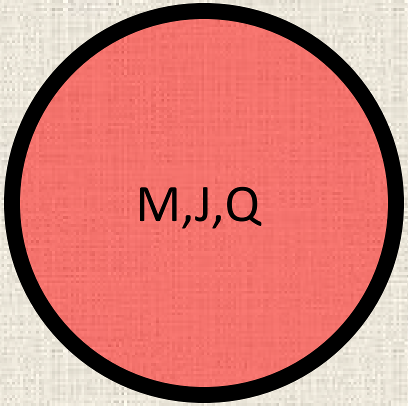
If one perturbs a non-gravitational system (e.g. a spring pendulum), it will oscillate with eigenfrequencies (normal modes) characterizing the system

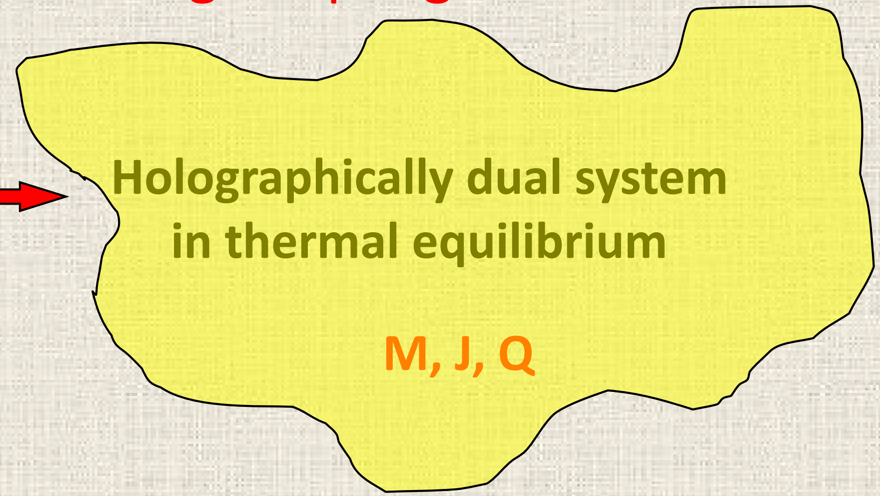
$$\omega = \sqrt{\frac{k}{M}}$$

What happens if one perturbs a black hole?

10-dim gravity

4-dim gauge theory – large N, strong coupling





T_{Hawking} $\quad$ $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T $\quad$ S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$$g^{(0)}_{\mu\nu} + h_{\mu\nu} \quad \Longleftrightarrow \quad ????$$

$$j_i = -D\partial_i j^0 + \cdots$$

$$"\Box" \; h_{\mu\nu} = 0 \text{ and B.C.} \quad \Longleftrightarrow \quad \partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

Black hole's quasinormal spectrum encodes properties of a dual microscopic system

Comparing eigenfrequencies of a black hole

$$\omega = \pm \frac{c}{\sqrt{3}} k - \frac{i}{6\pi T} k^2 + \frac{3 - 2\ln 2}{24\pi^2 \sqrt{3} T^2} k^3 + \cdots$$

with eigenfrequencies of a dual microscopic system (described by fluid mechanics)

$$\omega = \pm v_s k - i \frac{2\eta}{3sT} k^2 + \cdots$$

one can compute viscosity-entropy ratio and other quantities of a microscopic system

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} + \cdots$$

Moreover, one can directly relate Navier-Stokes equations and Einstein's equations…

Bhattacharyya, Minwalla, Rangamani, Hubeny, 2007

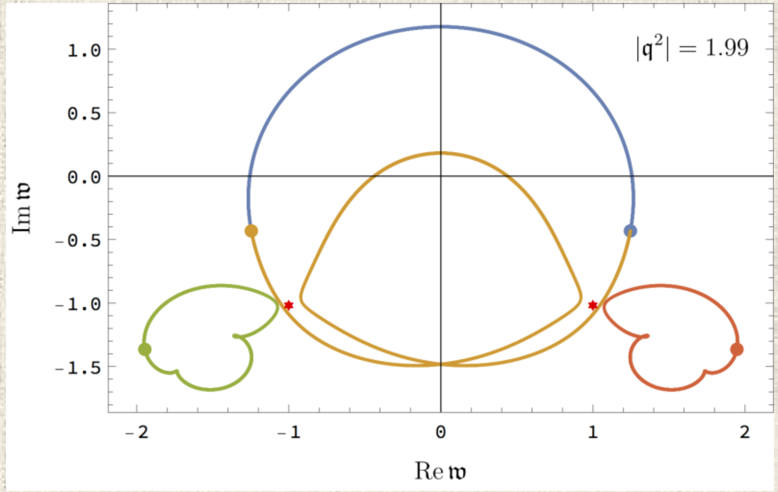
The “unreasonable effectiveness of hydrodynamics” and “hydrodynamisation” vs thermalisation

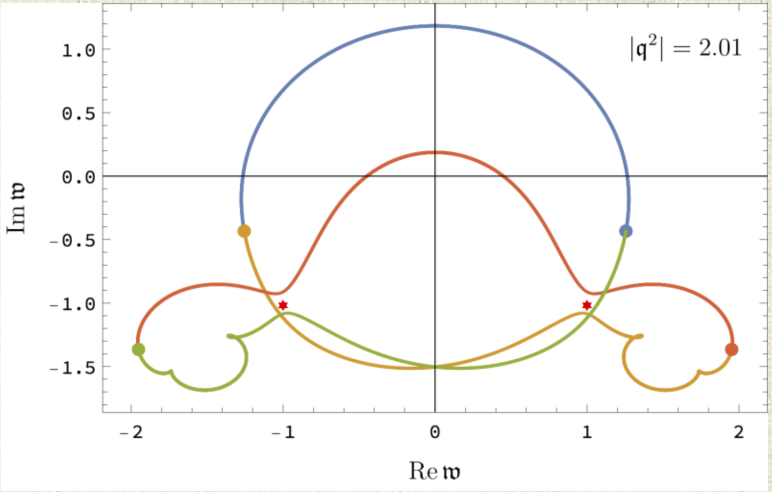
[A paraphrase (one of many) of Wigner’s "The Unreasonable Effectiveness of Mathematics in the Natural Sciences” (1960)]

Shear mode: $\mathfrak{w} = \mathfrak{w}(\mathfrak{q}) = -i\frac{\eta}{\epsilon + P}\mathfrak{q}^2 + \cdots$ $\quad |q_*| \approx 1.49131 \times (2\pi T)$

Sound mode: $\mathfrak{w} = \mathfrak{w}_\pm(\mathfrak{q}) = \pm v_s \mathfrak{q} - i\frac{\zeta + \frac{4}{3}\eta}{\epsilon + P}\mathfrak{q}^2 + \cdots$ $\quad |q_*| = \sqrt{2} \times (2\pi T)$

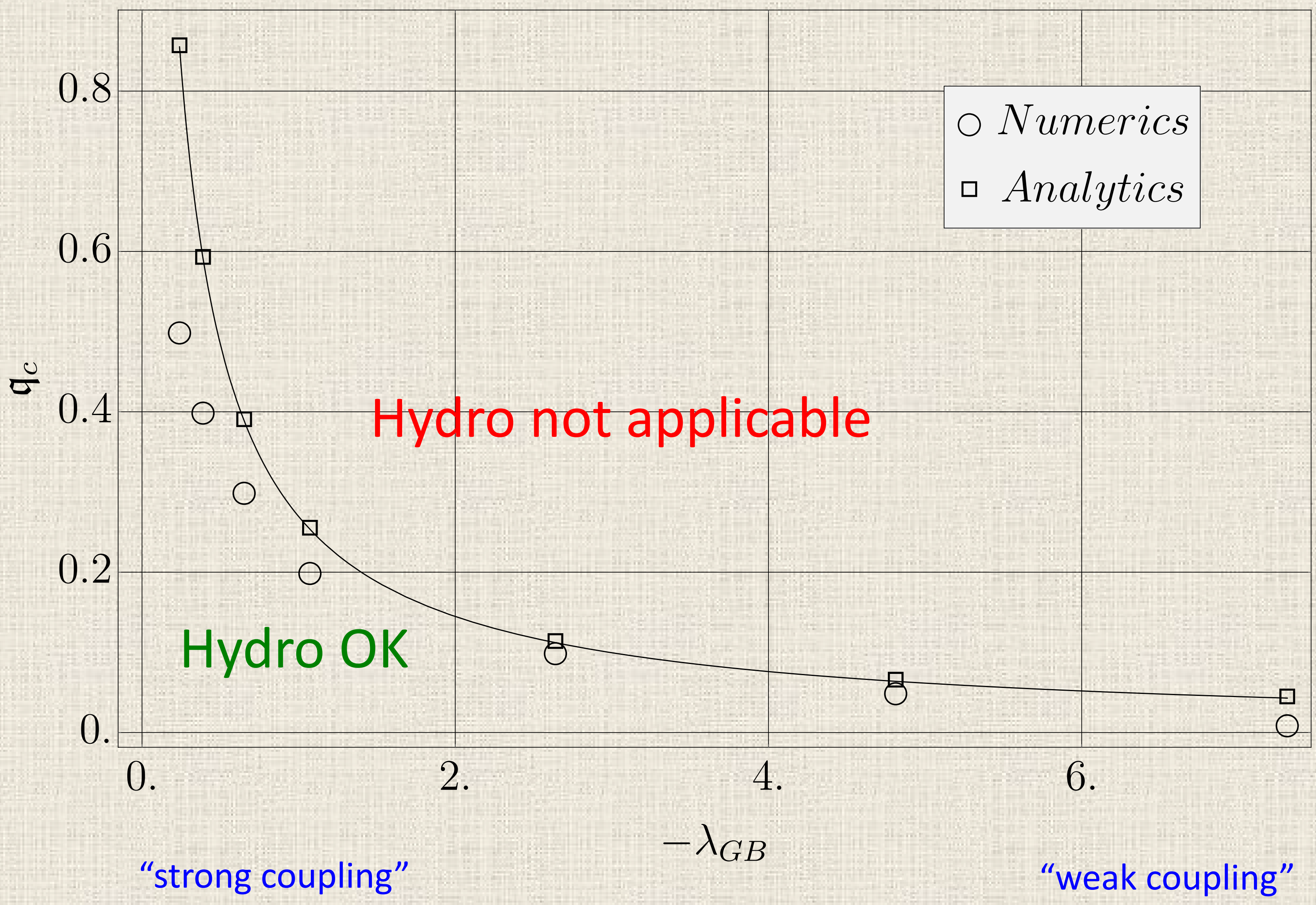
$\mathfrak{w} = \frac{\omega}{2\pi T}$, $\mathfrak{q} = \frac{q}{2\pi T}$; η, ζ – shear & bulk viscosities; v_s – speed of sound; ϵ, P – energy density & pressure





Grozdanov, Kovtun, Starinets, Tadic, 2019-2021

"Applicability of hydrodynamics" as a function of coupling



Conclusions

Black holes have entropy and temperature, and behave (in equilibrium) like thermodynamic systems, and we think we know why (holographic principle)

Moreover, out of equilibrium, horizons of black holes exhibit fluid-like properties (membrane paradigm)

String theorists' work on black hole physics has led to the (accidental) discovery of AdS/CFT (or gauge-string or holographic or "rabbit-duck") duality (conjecture)

Black hole thermodynamics and membrane paradigm are fully embedded into modern holographic picture

Eigenmodes of black holes are used to study thermalization and discover new phenomena such as "hydrodynamization"

Inspired by holography, relativistic hydrodynamics has been recently re-written to deal with the problem of causality – with possible applications in astrophysics (P.Kovtun, 2018-2020; J.Noronha, 2018, 2021)

THANK YOU!

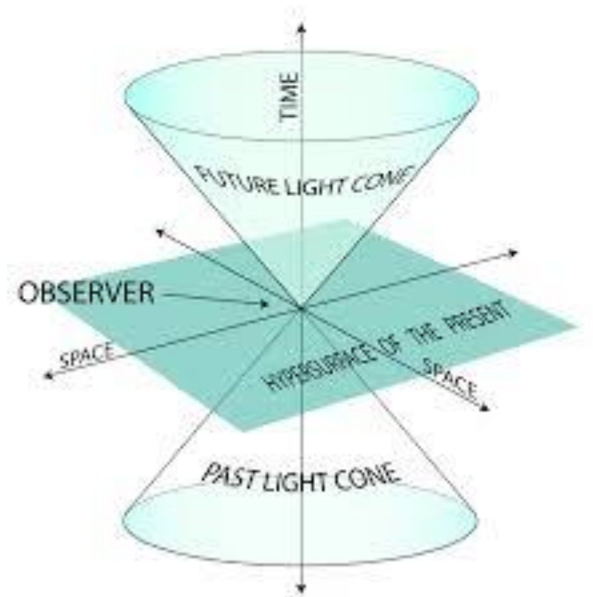
Observers: Freely Falling (FREFOS) vs Fiducial (FIDOS)

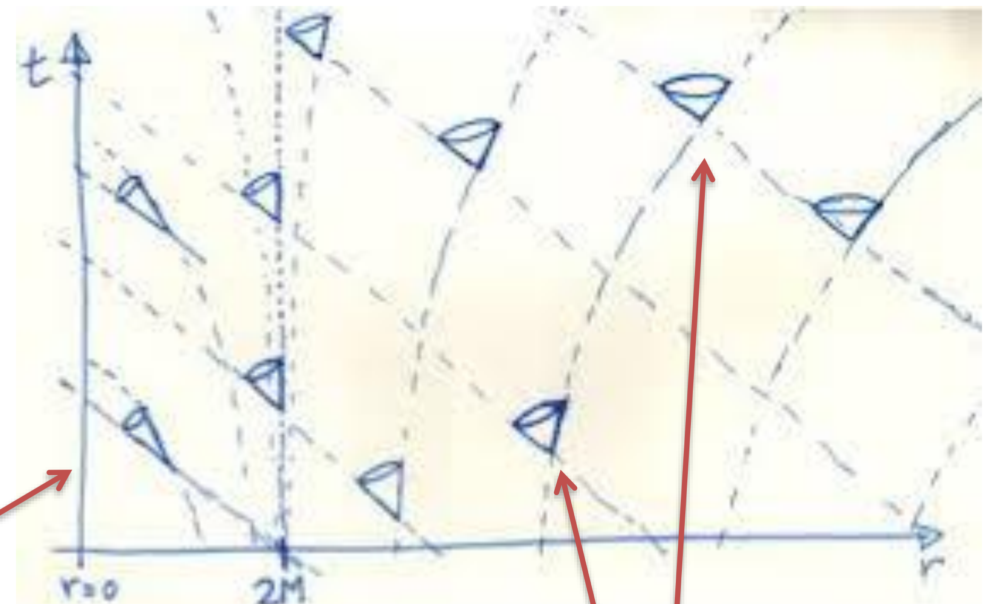
Not everyone is ready to become a freely falling observer...

Susskind & Lindesay, “Black holes, information and the string theory revolution”, World Scientific, 2005

Black hole horizon

The fate of light cones



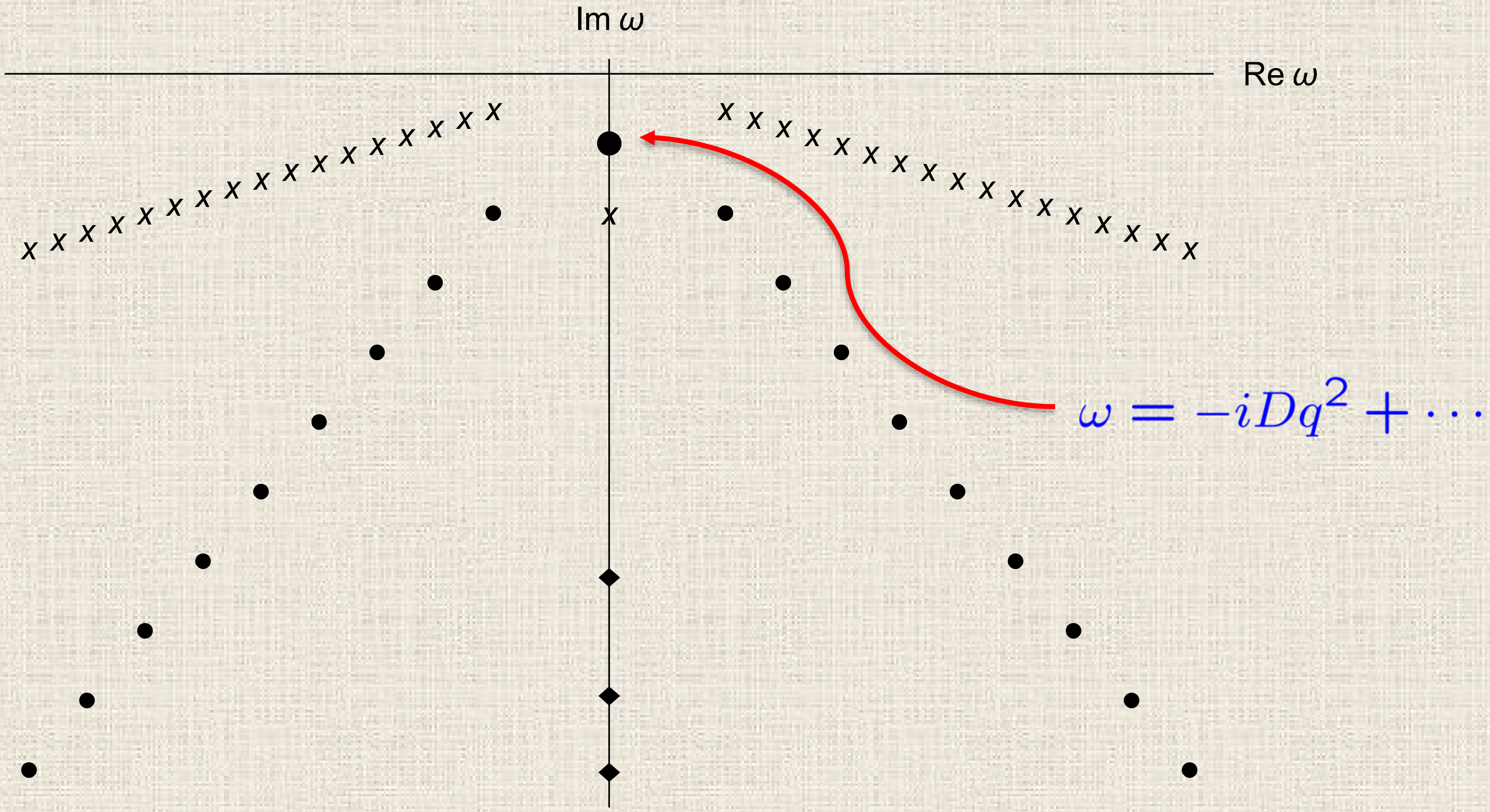


Singularity

Event horizon

Light cones outside the horizon

Quasinormal spectrum of dual black holes encodes hydro and non-hydro degrees of freedom



One can study thermalization (and "hydrodynamization") directly in a class of theories