Entropy from Entanglement

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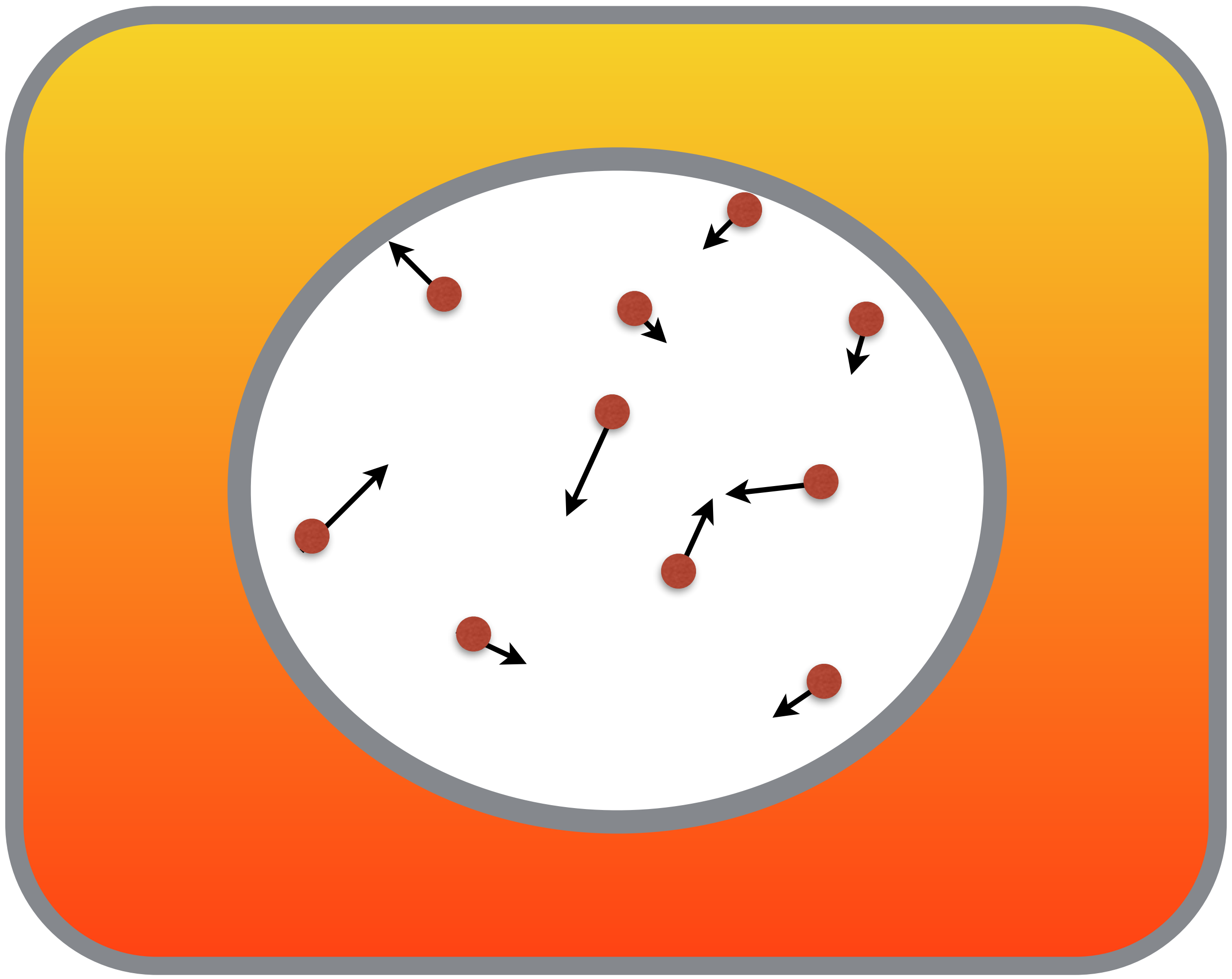
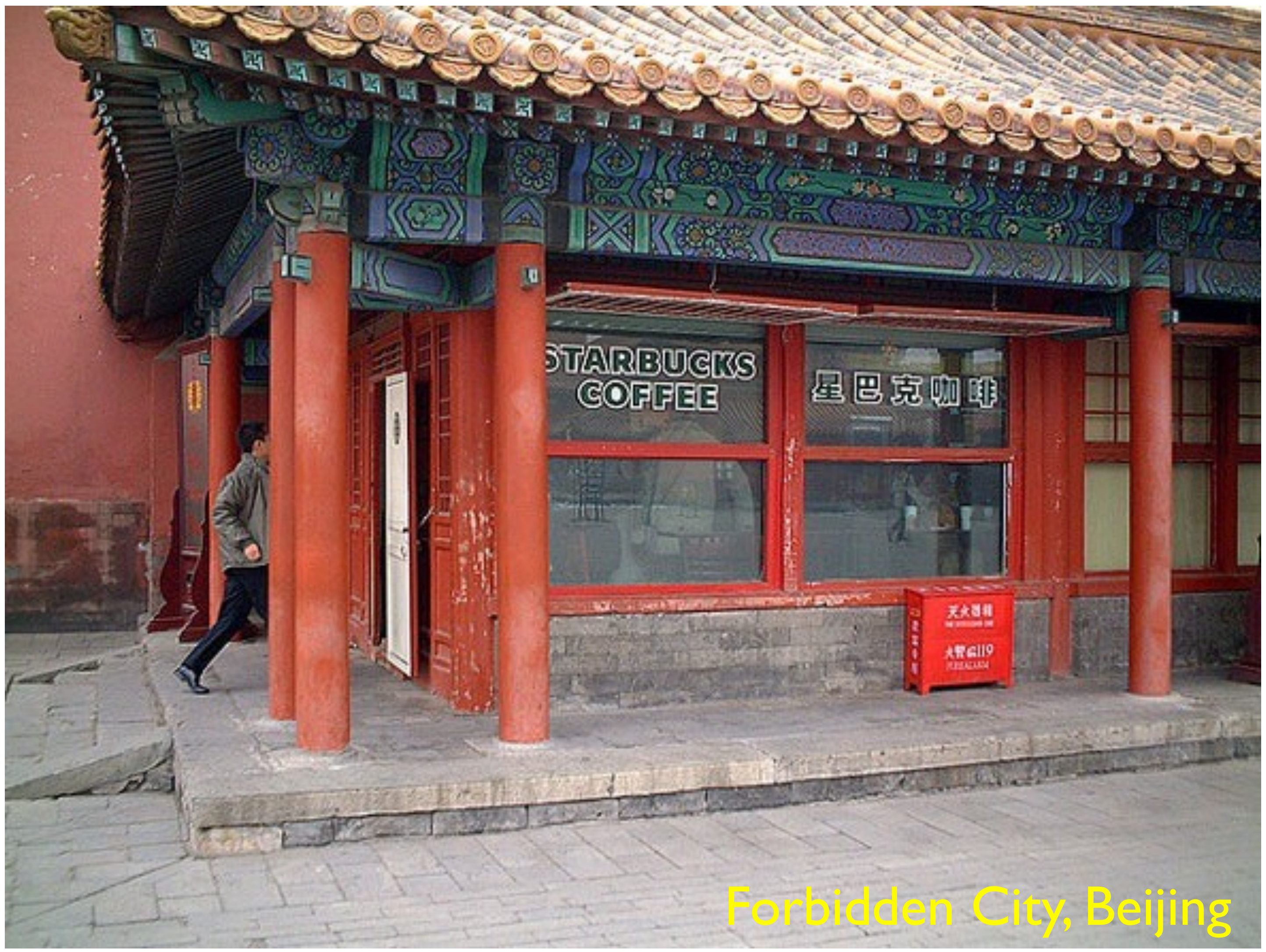
Saturday Mornings of Theoretical Physics
Oxford, November 17, 2018

Usually: system exchanges heat with environment



Intuitively: coupling to environment can limit the ‘menu’ of possible phenomena…



Forbidden City, Beijing

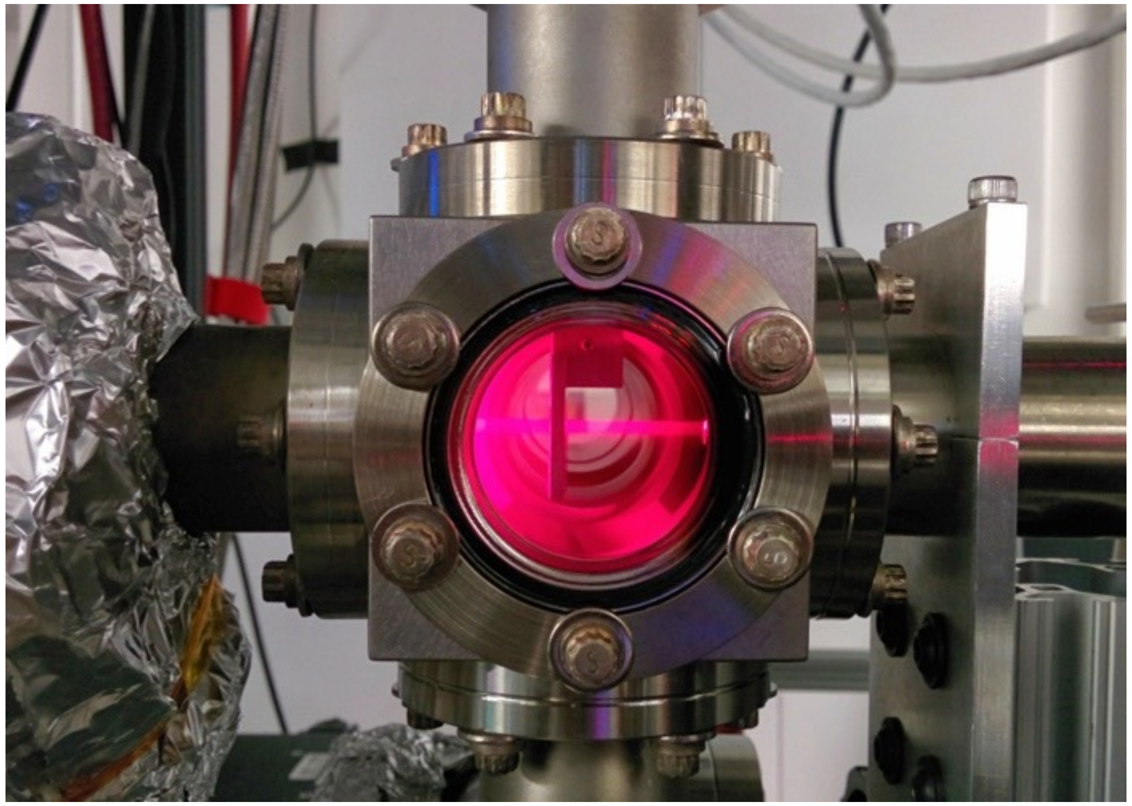
Images: businessinsider.com, captainsvoyage-forum.com

Isolated systems may offer more exotic possibilities.

Images: abovetopsecret.com/, tahitibycarl.com/

Isolated Quantum Systems

$T \sim 10^{-8}$ K

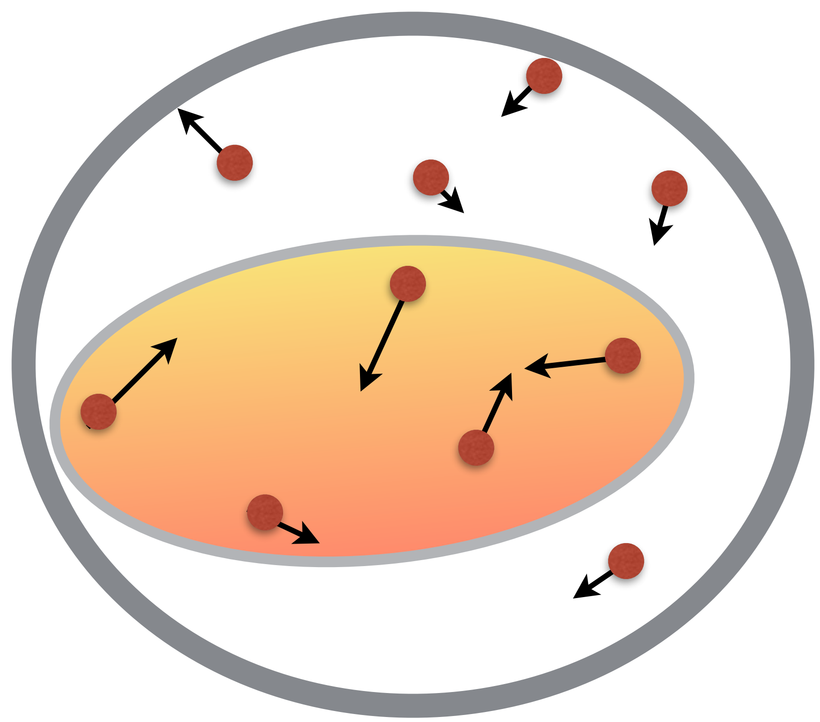


Atoms trapped in ultrahigh vacuum: almost perfect isolation

<u>Macroscopic</u> number of weakly interacting atoms

Image: David Weld, UCSB

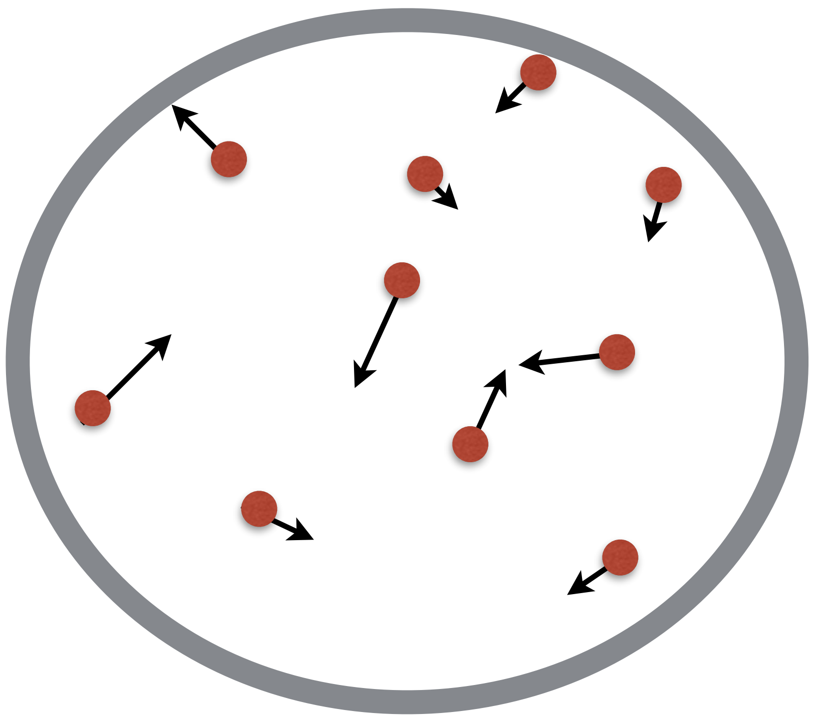
Can isolated systems self-generate 'environment'?



"Can a system be its *own* heat bath"
(need to go to the roots of statistical physics)

Statistical Physics of Classical Systems

Try to directly simulate a monoatomic ideal gas

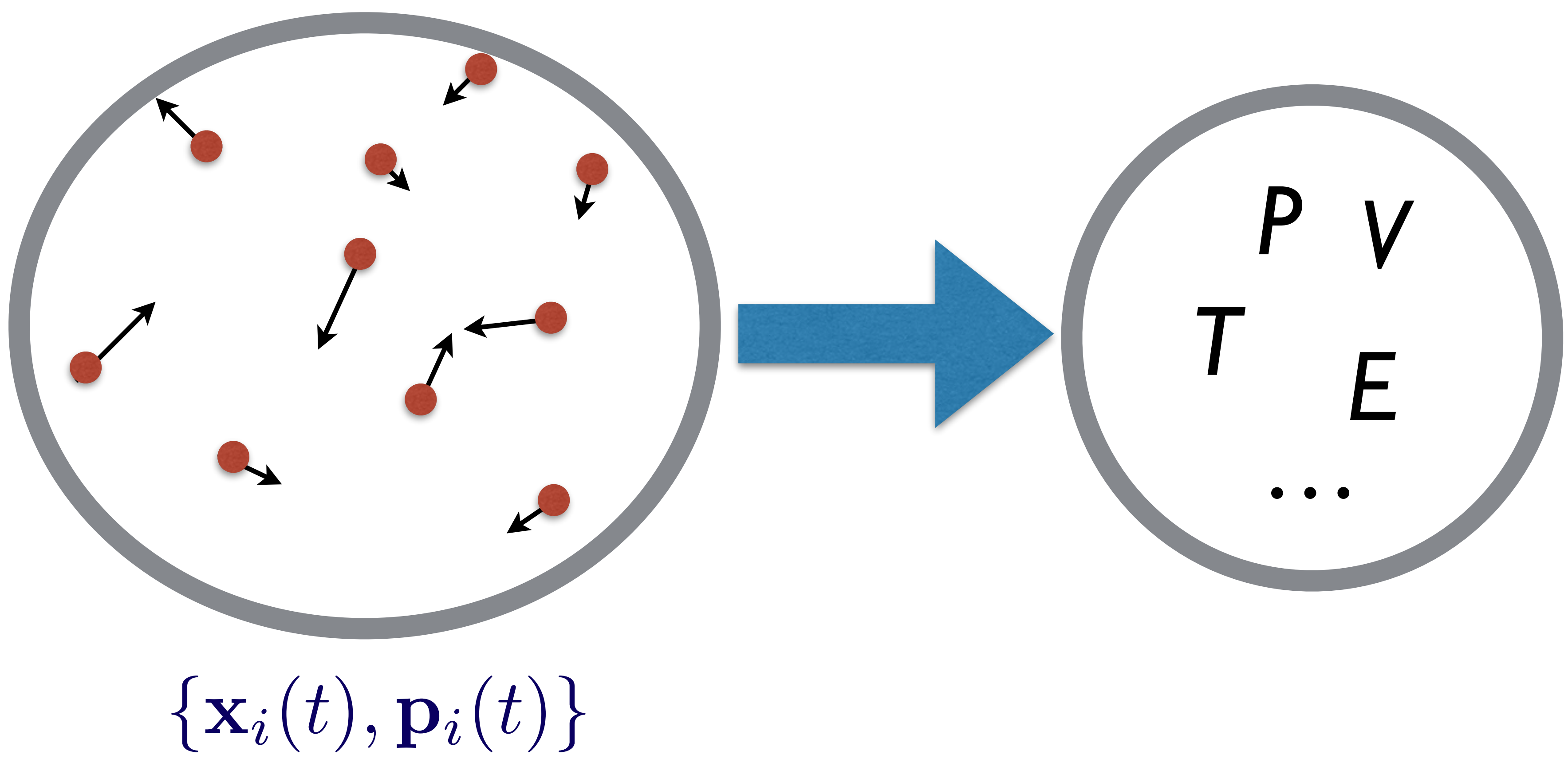


Statistical Mechanics
is how we solve this!

~10^{24} atoms
6 coordinates/atom
(x, y, z, p_x, p_y, p_z)

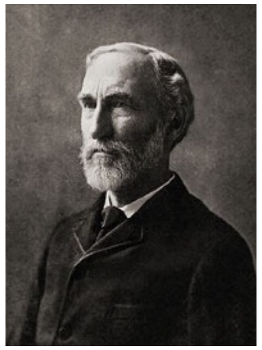
6 bits/molecule
$\Rightarrow 10^{12}$ Tb!

Microstates vs. Macrostates



Fundamental Postulate of Statistical Mechanics

An isolated system in equilibrium is equally likely to be in any of its accessible microstates (given a macrostate)



Josiah Willard Gibbs
(1839-1903)

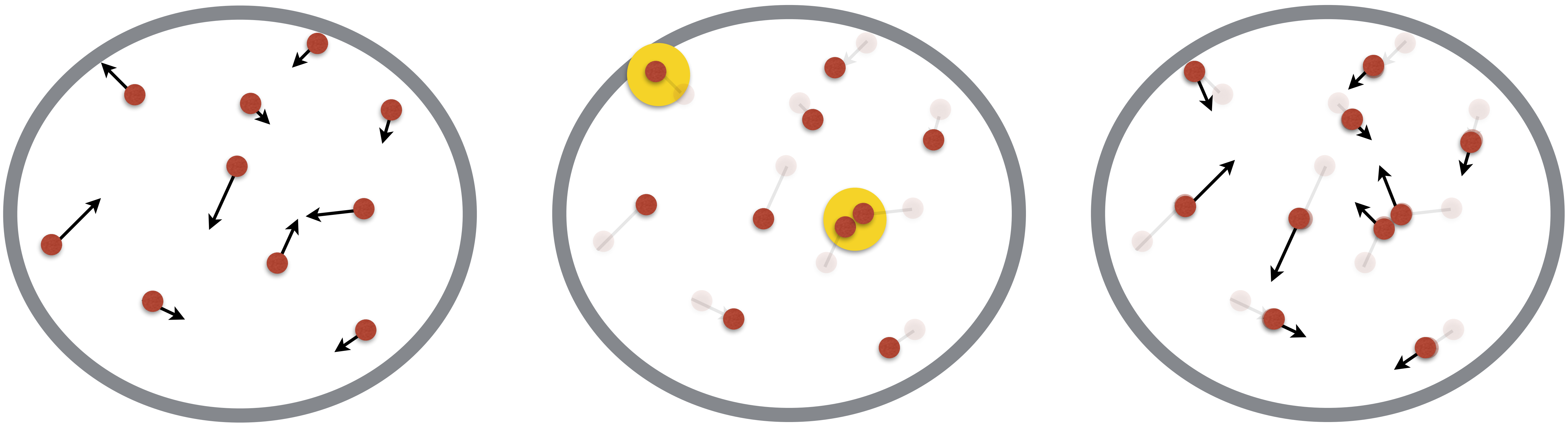
Ludwig Boltzmann
(1844-1906)

James Clerk Maxwell
(1831-1879)

Images: Wikipedia

Ergodicity

How do we go from microstates to macrostates?



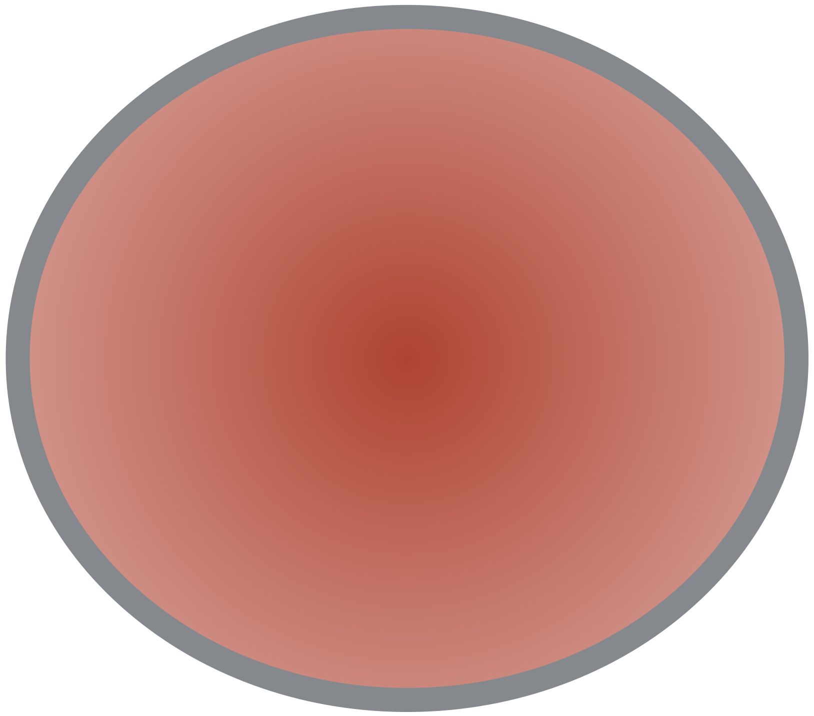
$t = 0$ $\quad$ $t = \Delta t - \varepsilon$ $\quad$ $t = \Delta t + \varepsilon$

$\{\mathbf{x}_i(0), \mathbf{p}_i(0)\}$

System "forgets" which microstate it started in

Ergodicity and Entropy

Given enough time, systems explore all accessible microstates consistent w/ macrostate

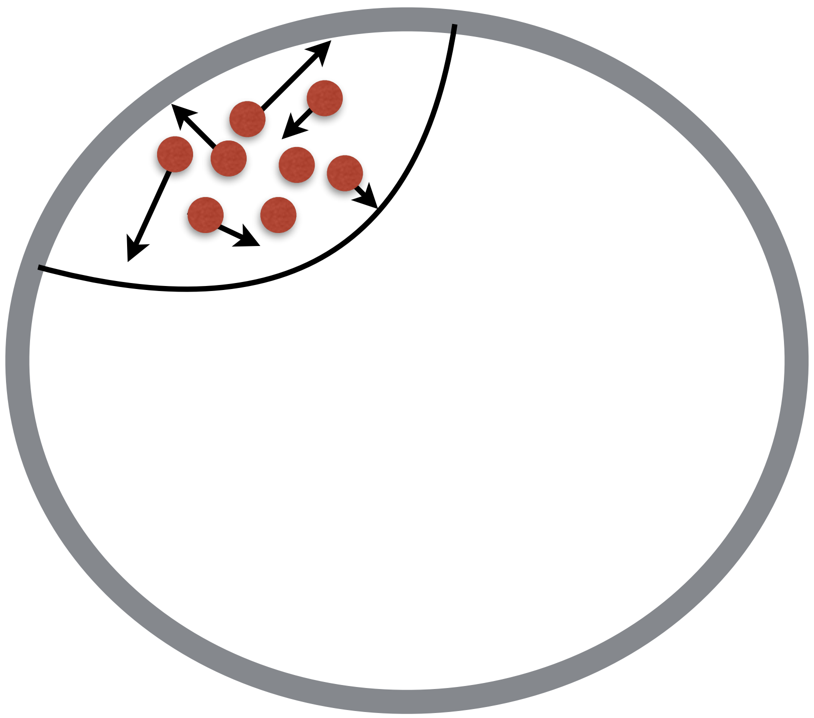


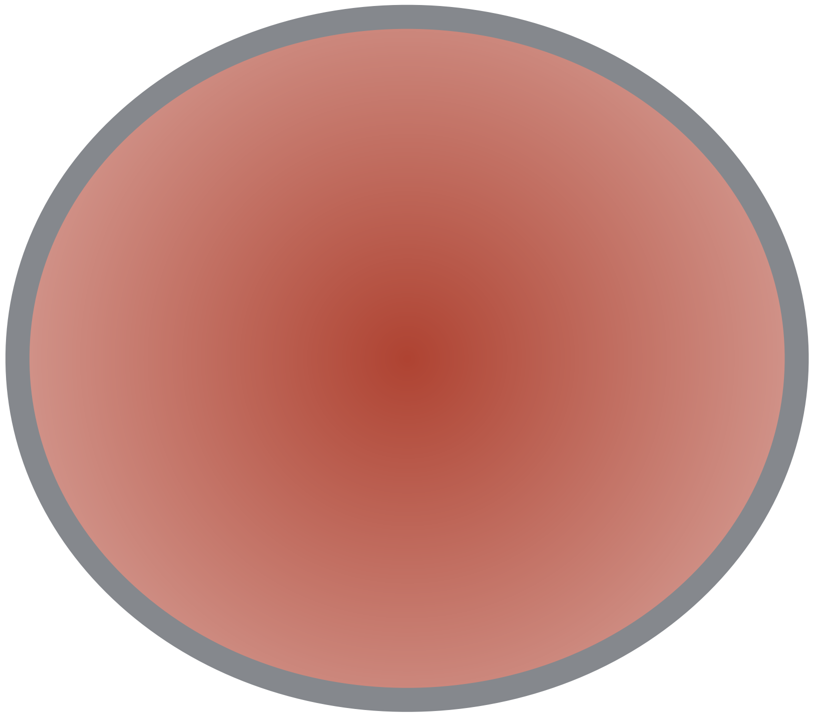
<u>Entropy</u>: how many microstates correspond to given macrostate?

$$S = k_B \log W$$

Thermalization

Given enough time, systems explore all accessible microstates consistent w/ macrostate





Time evolution is towards higher entropy.

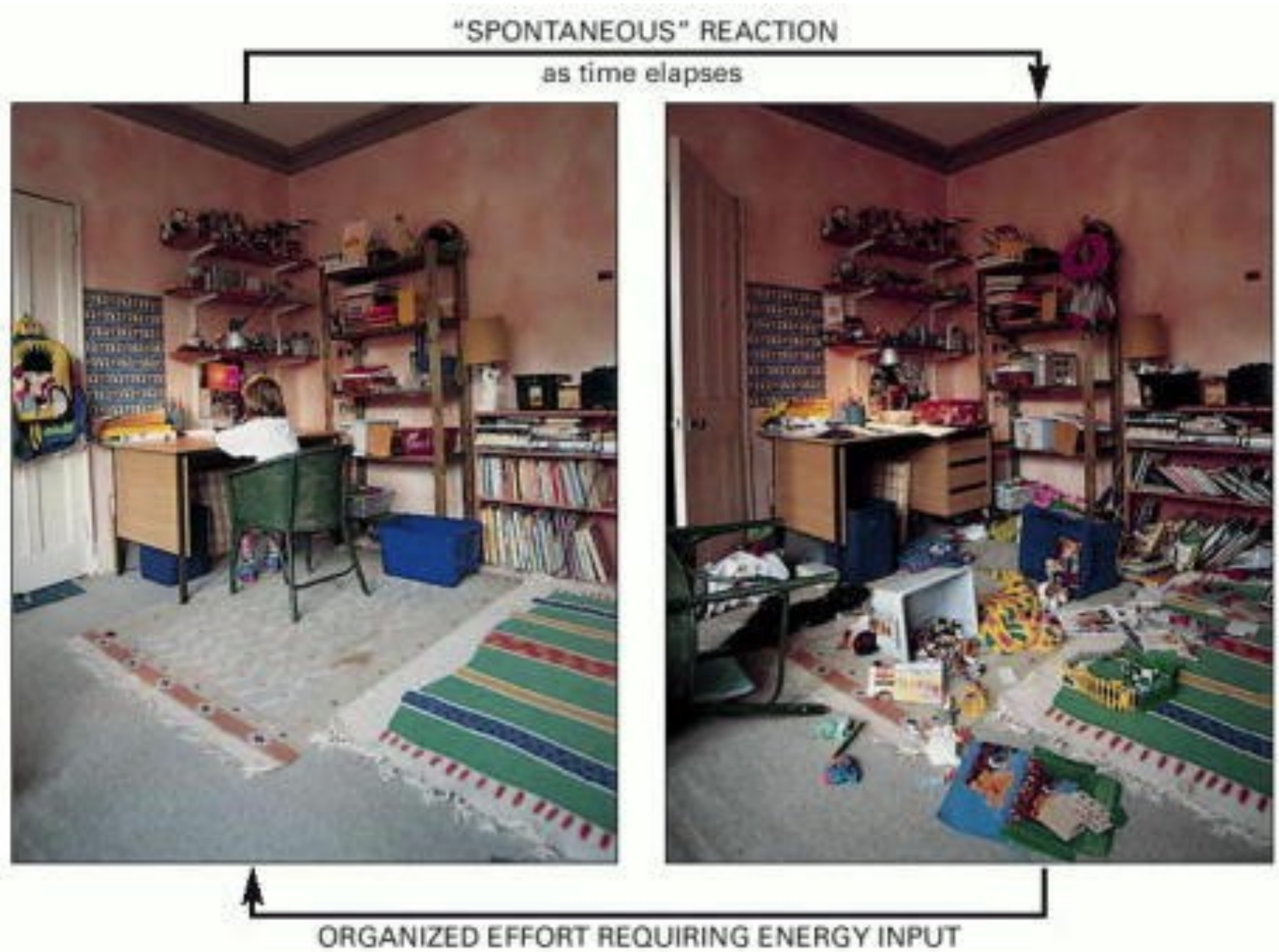


Image: *Molecular Biology of the Cell,* Alberts B, Johnson A, Lewis J, et al. New York: Garland Science; 2002.

What about isolated quantum systems?

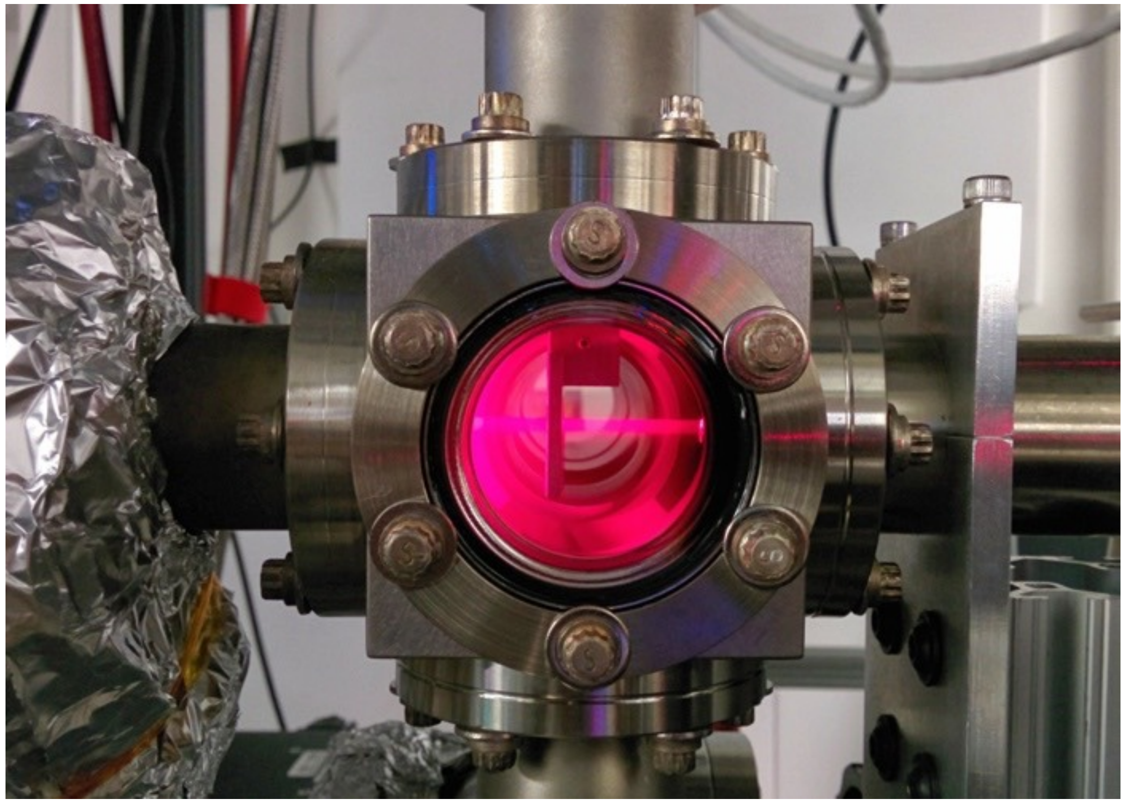
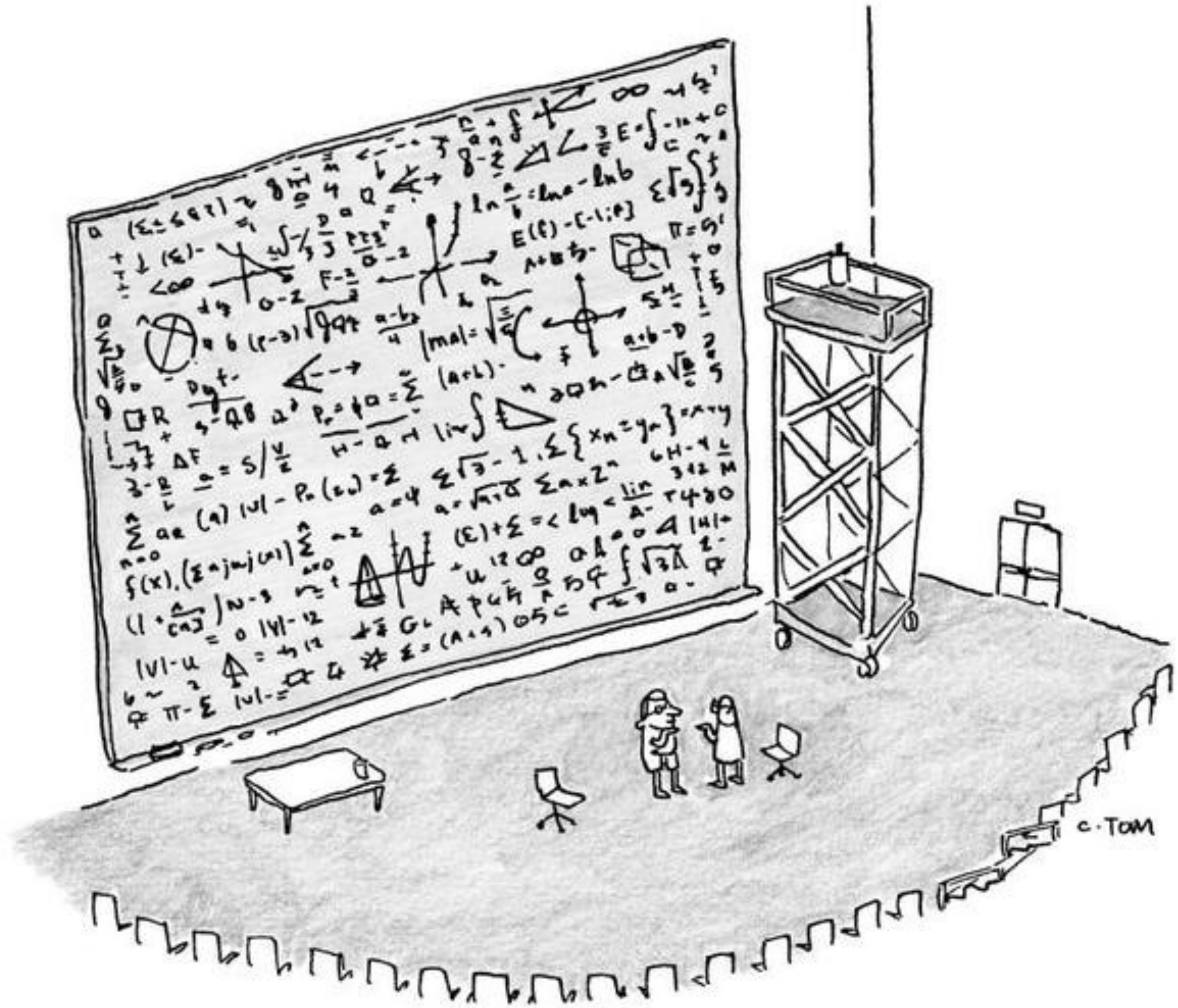


Image: David Weld, UCSB



*‘The math is right. It’s just in poor taste.’**

THE NEW YORKER

*Translated from the American by S. Parameswaran

Quantum Mechanics Reminder

State of system described by "state vector" $|\Psi\rangle$ "Dirac notation"

Properties of system captured by "Hamiltonian" $\hat{H}$

Time evolution: Schrödinger equation: $\hat{H}|\Psi\rangle = i\hbar\frac{\partial}{\partial t}|\Psi\rangle$

Solution simple in terms of special "eigenstates":

$$\hat{H}|\Psi_\alpha\rangle = E_\alpha|\Psi_\alpha\rangle \qquad |\Psi_\alpha\rangle \mapsto e^{-iE_\alpha t/\hbar}|\Psi_\alpha\rangle$$

$$|\Psi(t=0)\rangle = \sum_\alpha c_\alpha|\Psi_\alpha\rangle$$

$$\mapsto |\Psi(t)\rangle = \sum_\alpha c_\alpha e^{-iE_\alpha t/\hbar}|\Psi_\alpha\rangle$$

$\hbar = 1$ rest of this talk

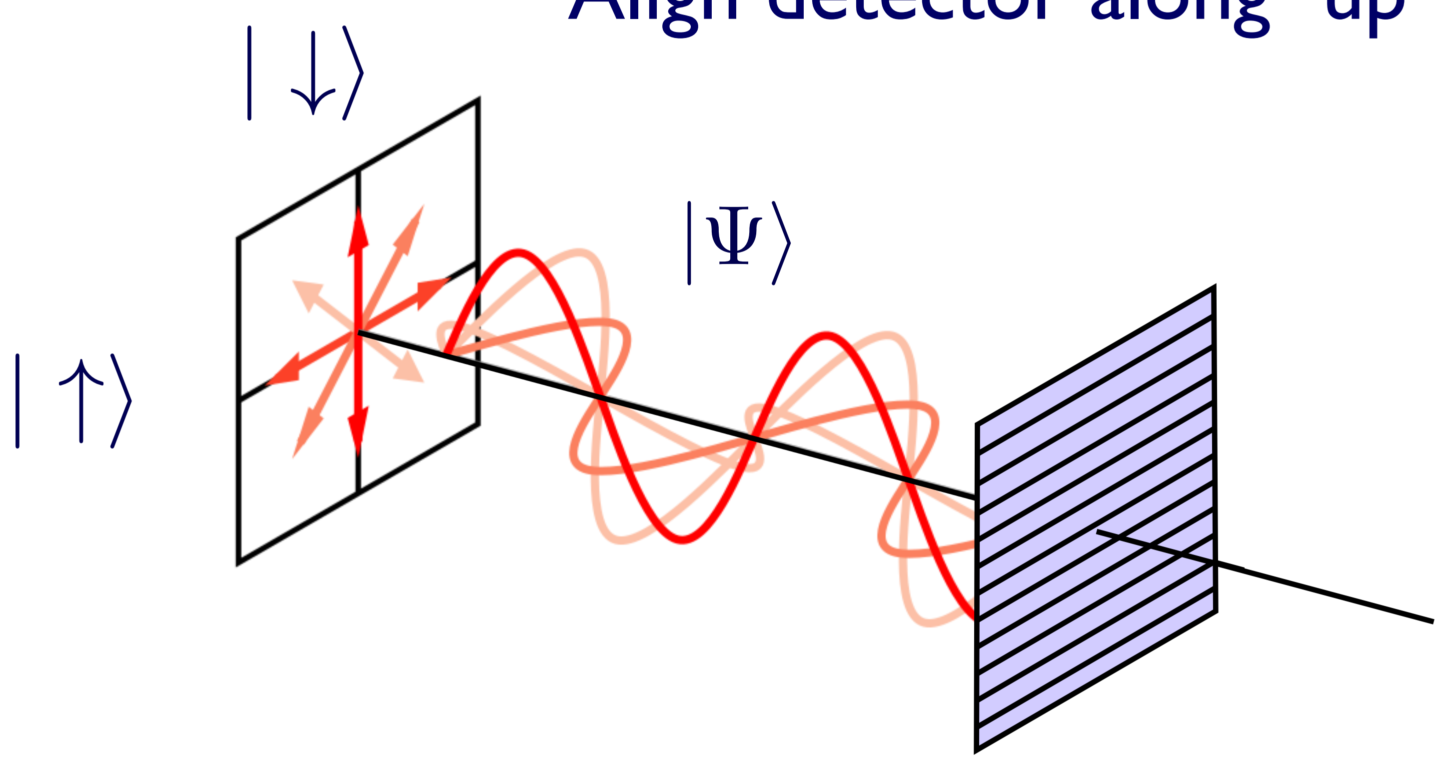
Quantum Superposition

Simplest quantum system: single spin, “up” or “down” $|\uparrow\rangle \quad |\downarrow\rangle$

analogous to horizontal/vertical polarization:
“superposition” means some polarization in between

$$|\Psi\rangle = \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$$

Align detector along “up”/horizontal



See signal w/ probability $\cos^2\theta$
no signal w/ probability $\sin^2\theta$

Image: Wikipedia/Bob Mellish

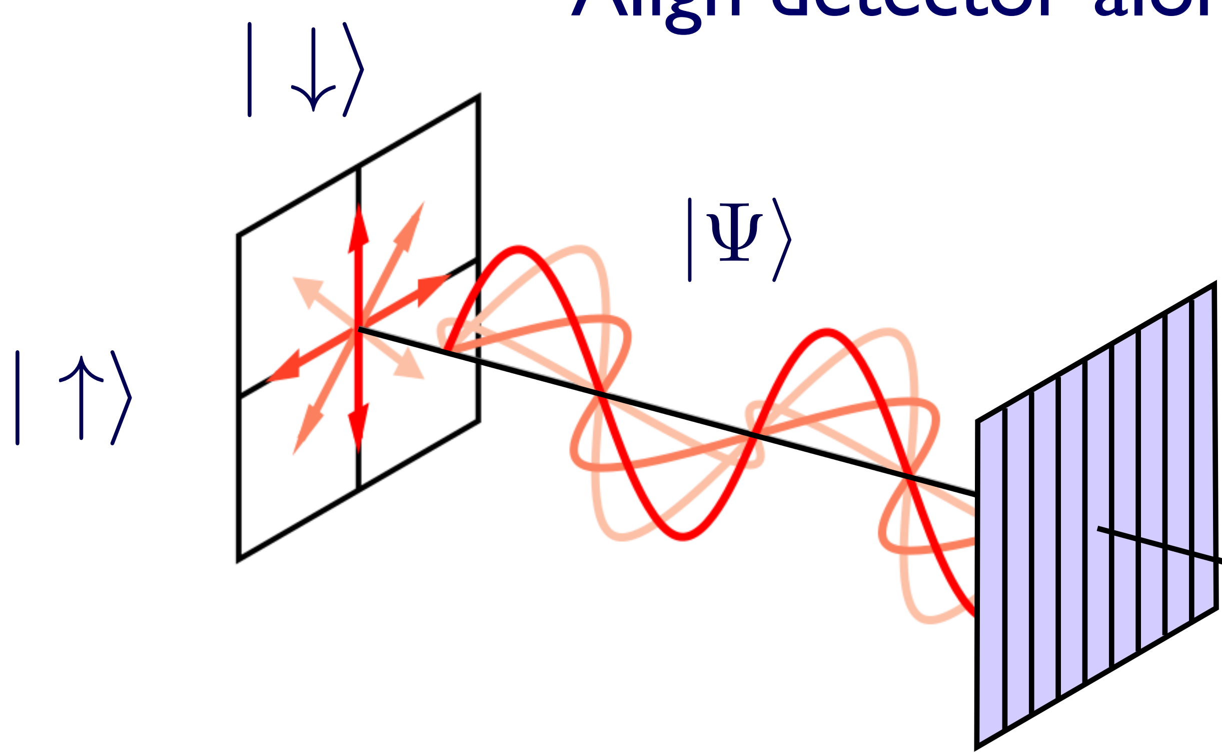
Quantum Superposition

Simplest quantum system: single spin, "up" or "down" $|\uparrow\rangle \quad |\downarrow\rangle$

analogous to horizontal/vertical polarization:
"superposition" means some polarization in between

$$|\Psi\rangle = \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$$

Align detector along "down"/vertical



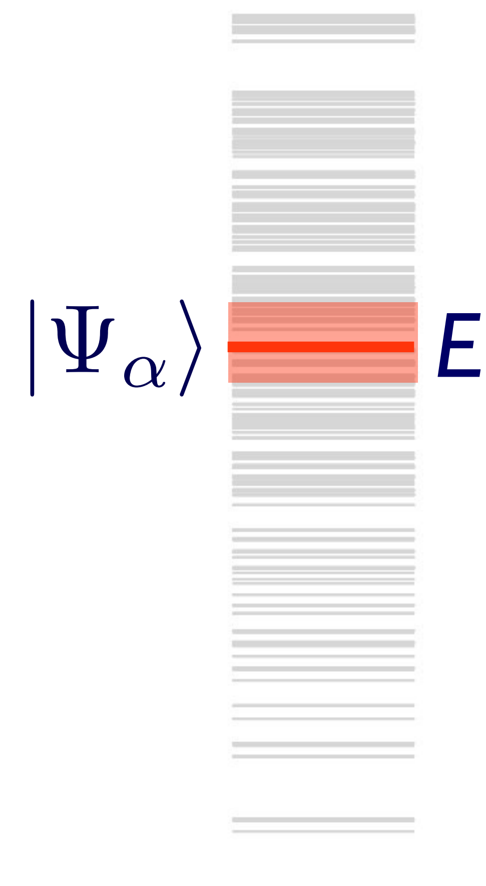
See signal w/ probability $\sin^2\theta$
no signal w/ probability $\cos^2\theta$

Image: Wikipedia/Bob Mellish

Isolated quantum systems

Microstates: single eigenstate of the Hamiltonian

$$\hat{H}|\Psi_\alpha\rangle = E_\alpha|\Psi_\alpha\rangle$$



"Macrostate": set approximate energy ~ *E*

Must also fix an initial state: $$|\Psi(0)\rangle = \sum_\alpha c_\alpha |\Psi_\alpha\rangle$$

$$\sum_\alpha |c_\alpha|^2 = 1$$ (probabilities add to 1)

Isolated quantum systems

Time evolution:

$$|\Psi(t)\rangle = \sum_\alpha c_\alpha e^{-iE_\alpha t}|\Psi_\alpha\rangle$$

Probability to be in microstate (eigenstate) α:

$$P(\alpha) = |c_\alpha|^2$$

doesn't change with time!

As long as it had different probabilities of being in different microstates *initially*, system *never* forgets!

We need to think differently about statistical physics in isolated quantum systems

Observables

Recall that quantum mechanics is a theory of measurement

Consider measuring an observable

$$\langle \hat{\mathcal{O}}(t) \rangle \equiv \langle \Psi(t) | \hat{\mathcal{O}} | \Psi(t) \rangle = \sum_{\alpha,\beta} c_\alpha^* c_\beta e^{i(E_\alpha - E_\beta)t} \langle \Psi_\alpha | \mathcal{O} | \Psi_\beta \rangle$$

Off-diagonal terms oscillate, diagonal terms constant:

$$\langle \hat{\mathcal{O}}(t) \rangle \underset{t \to \infty}{\approx} \sum_\alpha |c_\alpha|^2 \langle \Psi_\alpha | \hat{\mathcal{O}} | \Psi_\alpha \rangle$$

Even observables seem to "remember" the microstate!

How can the system thermalize, i.e. "forget" its initial state?

Thermalization

Only possible if expectation values just depend on macrostate properties

$$\langle \Psi_\alpha | \hat{\mathcal{O}} | \Psi_\alpha \rangle \approx f(E_\alpha)$$

$$\langle \hat{\mathcal{O}}(t) \rangle \underset{t\to\infty}{\approx} \sum_\alpha |c_\alpha|^2 f(E_\alpha) \approx f(E) \sum_\alpha |c_\alpha|^2 \approx f(E)$$

(as long as the initial state is not too spread out in energy)

The logical corollary is that we could just work with a *single* eigenstate, dispensing with the c_α

How can we define entropy in this setting?

Entanglement

<u>Simplest example</u>: 2 quantum spins, A, B, each "up" or "down"

Two distinct states:

$$|\Psi_1\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\uparrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B)$$

$$|\Psi_2\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B)$$

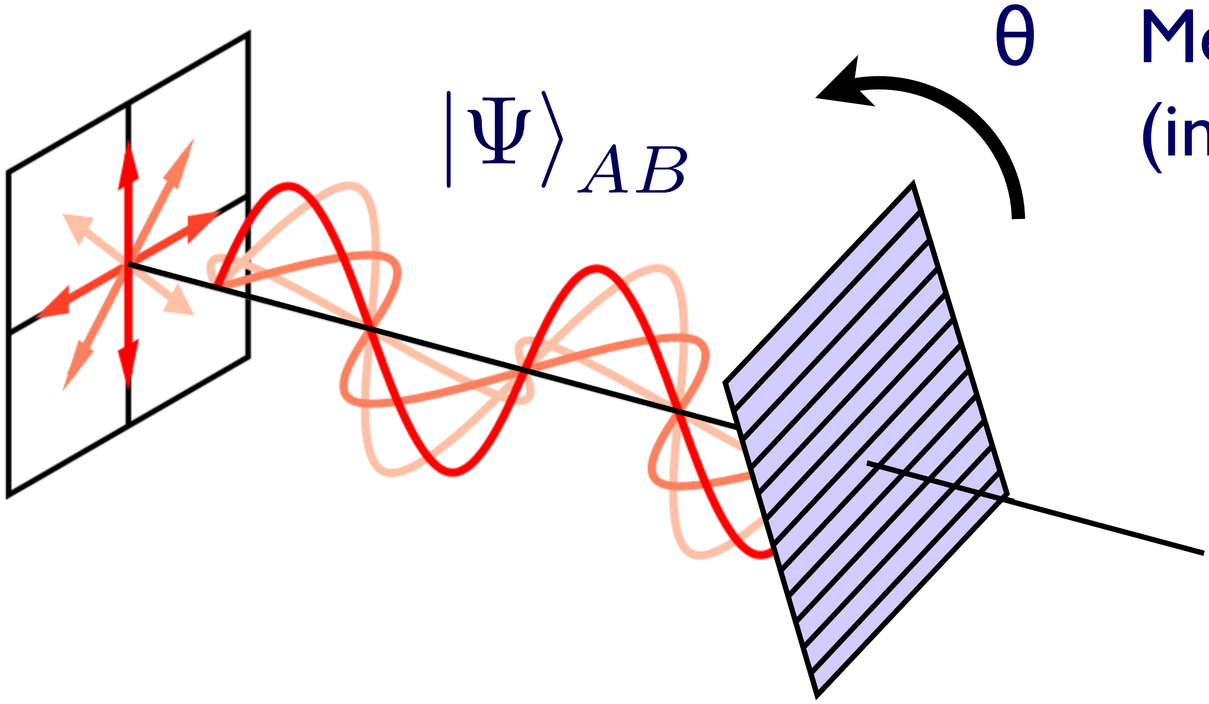
One is entangled, the other is not.

What's the difference? How can we tell?

Entanglement

"Constructive ignorance": give up some information.

Agree never to measure the spin B; I'll keep handing you copies of the state, and you can measure spin A all you like.



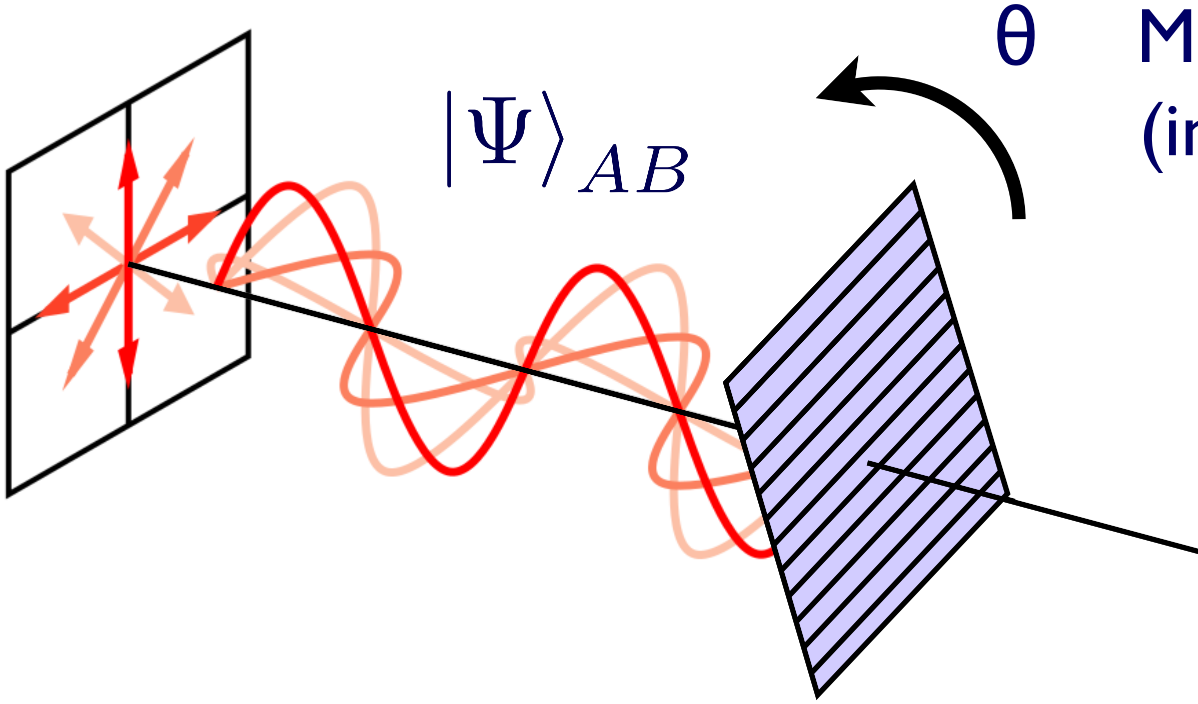
Measure only *A* (in direction θ)

How does signal depend on θ?

Image: Wikipedia/Bob Mellish

Entanglement

$$|\Psi_1\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\uparrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B)$$



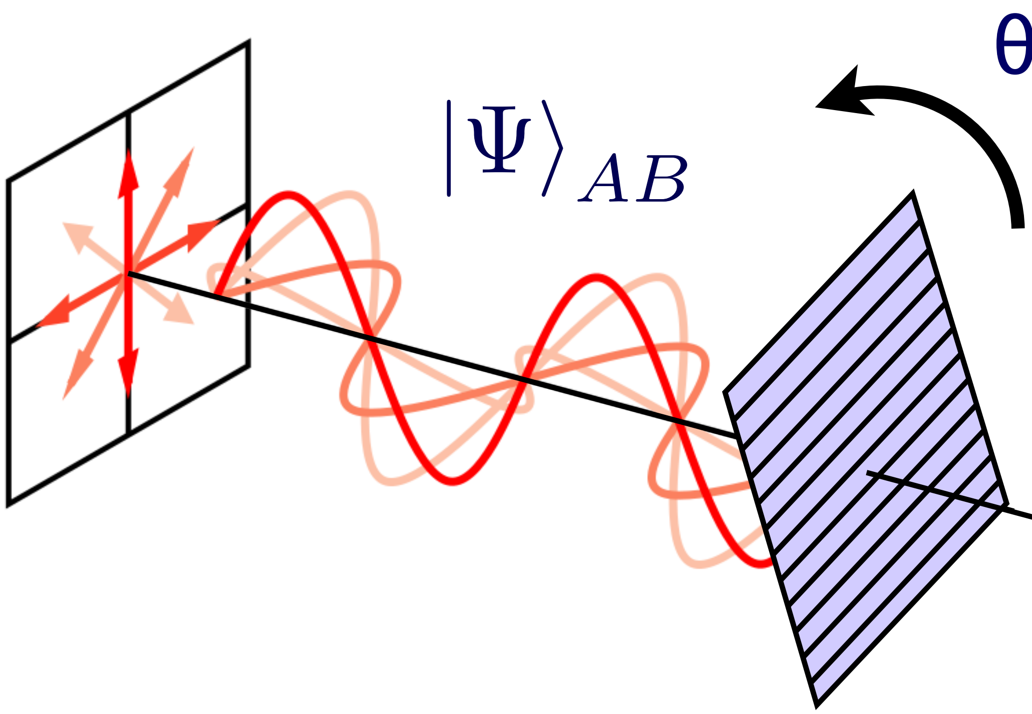
Measurement result ~ polarized light: <u>quantum</u> superposition

$$|\Psi_1\rangle_{AB} = |u\rangle_A|v\rangle_B$$ unentangled "product state"

Image: Wikipedia/Bob Mellish

Entanglement

$$|\Psi_2\rangle_{AB} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B)$$



Measure only *A*
(in direction θ)

signal ~ 1/2
(independent of θ)

Measurement result ~ unpolarised light: <u>classical</u> randomness

$$|\Psi_2\rangle_{AB} \neq |u\rangle_A|v\rangle_B$$

entangled,
can't write as "product state"

Image: Wikipedia/Bob Mellish

Entanglement

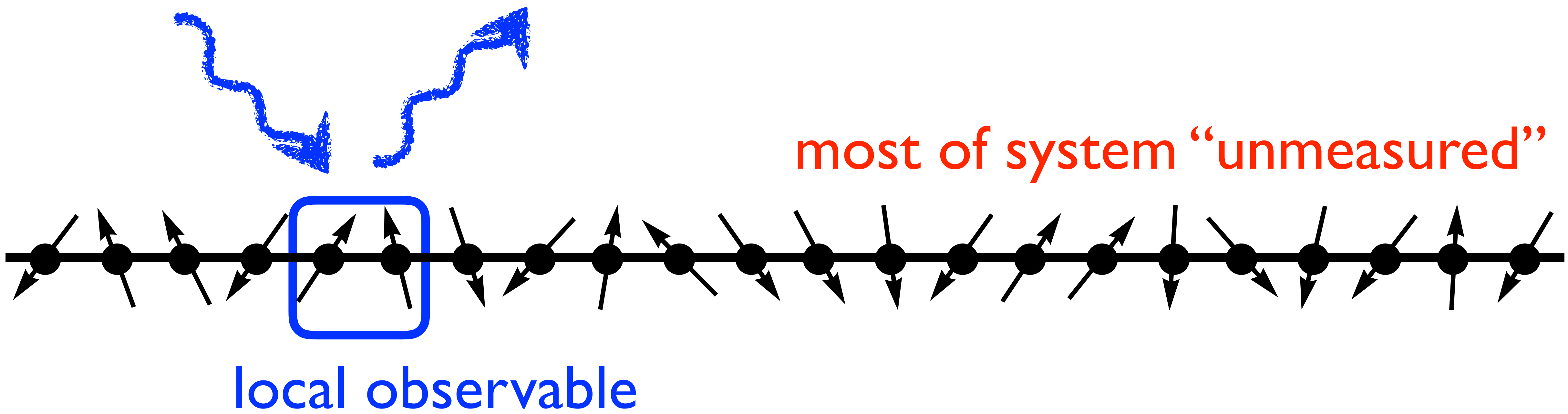
For an unentangled state, giving up information on spin B doesn't change the fact that spin A is in a quantum superposition state

For an entangled state, giving up information on spin B makes spin A have classical uncertainty.

The classical uncertainty yields an entropy of ln 2:
This is the entropy of entanglement, denoted S_E

Entanglement

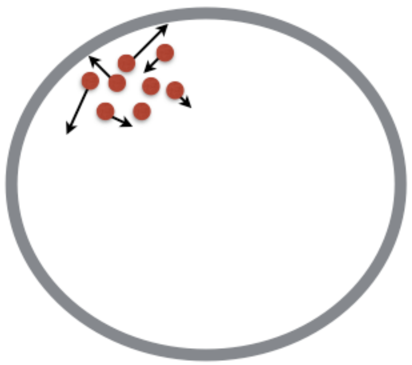
Local observables: ignorant about *most* of the system:
"see" a lot of classical uncertainty - entropy even in a single state!



In a typical quantum state of a many-spin system,
S_E is extensive ($\propto$ volume), just like the usual "thermal" entropy

Entanglement Growth

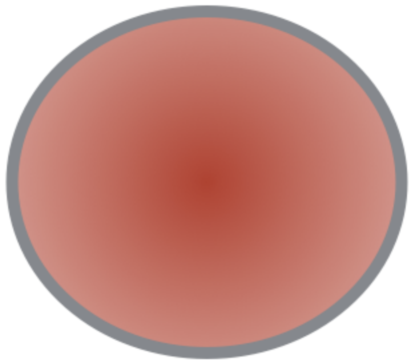
The analogue of the "special" low-entropy state



is a product state of spins: e.g. $|\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\ldots\rangle$

(zero entanglement between any of the spins)

What is the analogue of "entropy growth"?



Entanglement Growth

Consider two spins with eigenstates and energies as follows:

$$|\Psi_+\rangle = \frac{|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B}{\sqrt{2}} \qquad E_+ = +E$$

$$|\Psi_-\rangle = \frac{|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B}{\sqrt{2}} \qquad E_- = -E$$

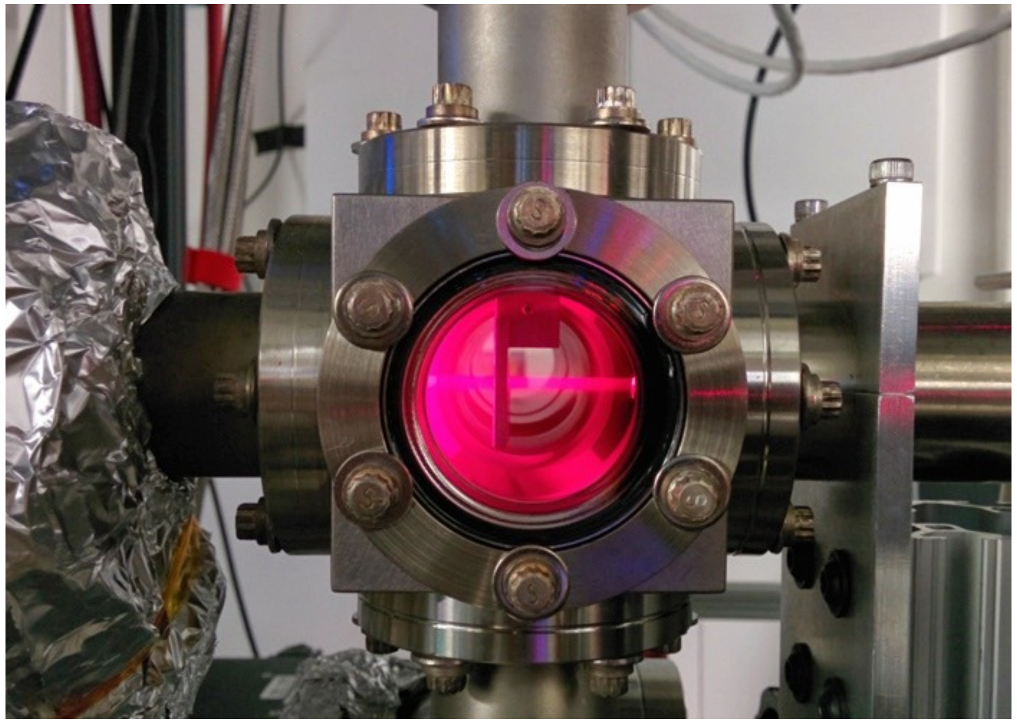
<u>Initial state:</u> $$|\Psi(t=0)\rangle = |\uparrow\rangle_A|\downarrow\rangle_B = \frac{|\Psi_+\rangle + |\Psi_-\rangle}{\sqrt{2}}$$

$$|\Psi(t)\rangle = \frac{e^{-iEt}|\Psi_+\rangle + e^{iEt}|\Psi_-\rangle}{\sqrt{2}}$$

$$= \cos(Et)|\uparrow\rangle_A|\downarrow\rangle_B - i\sin(Et)|\downarrow\rangle_A|\uparrow\rangle_B$$ entangled!

<u>Many spins</u>: each spin highly entangled with several others

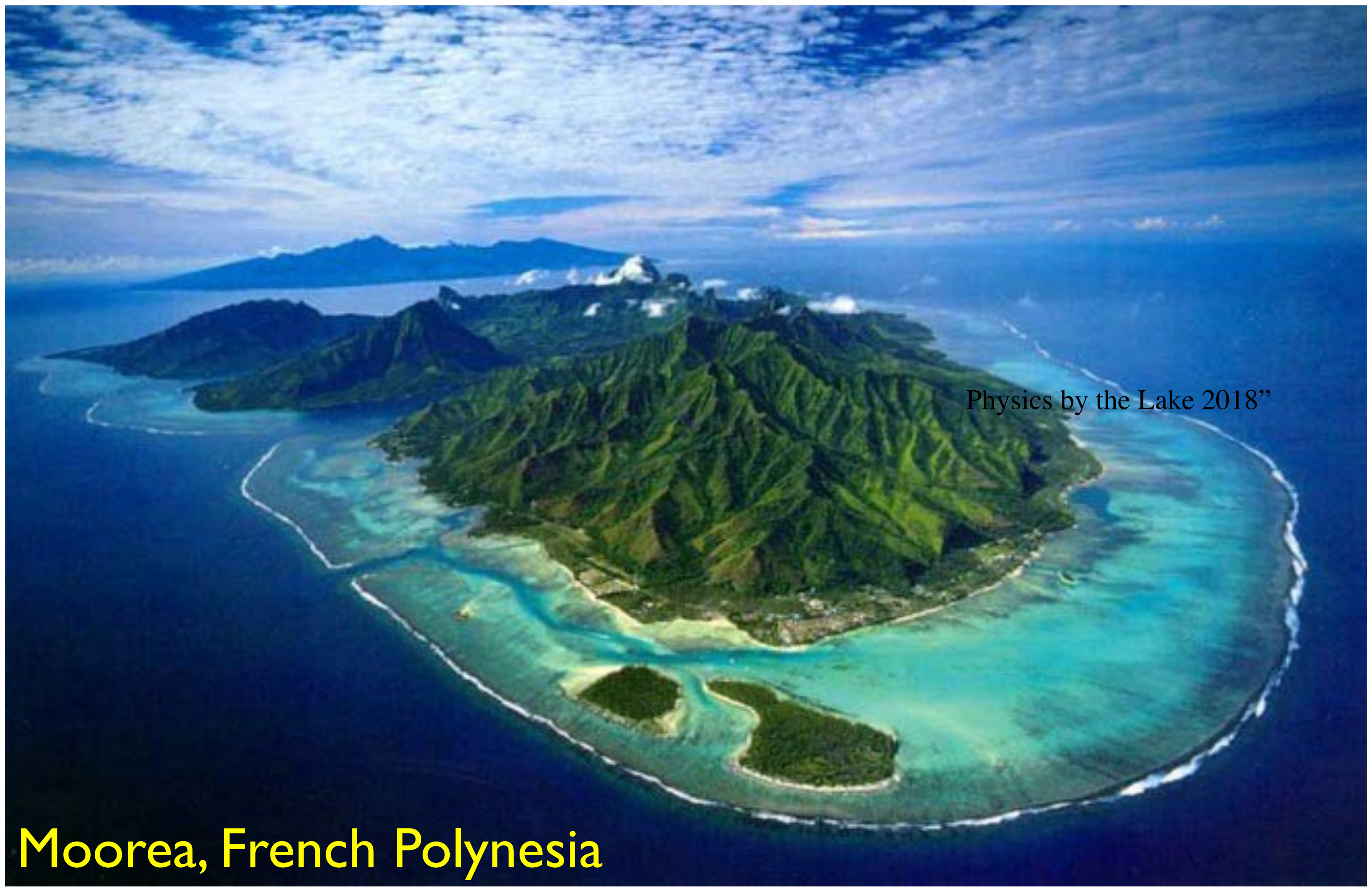
To sum up…



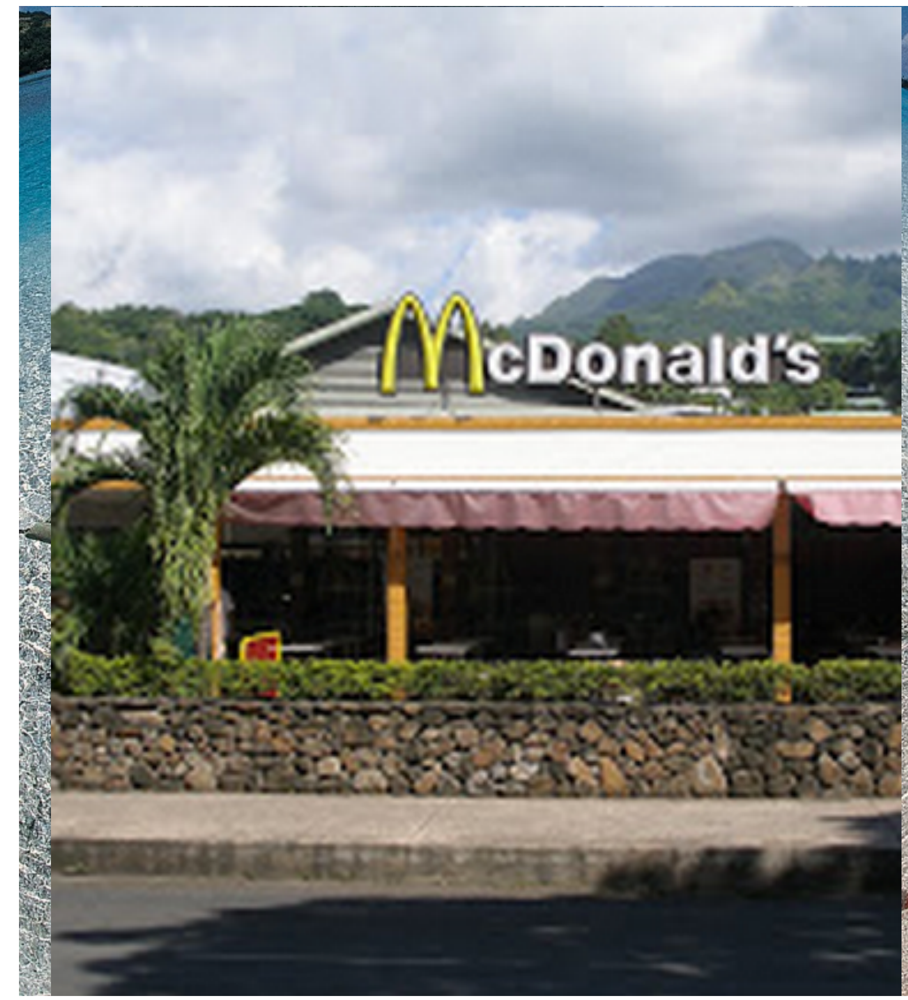
Isolated quantum systems can thermalize by "self-generating" classical uncertainty and thus entropy via entanglement.

Image: David Weld, UCSB

Started this talk by suggesting isolated systems may allow us to study “exotic” things



Moorea, French Polynesia



The news is not good…(?)

Images: abovetopsecret.com/, tahitibycarl.com/

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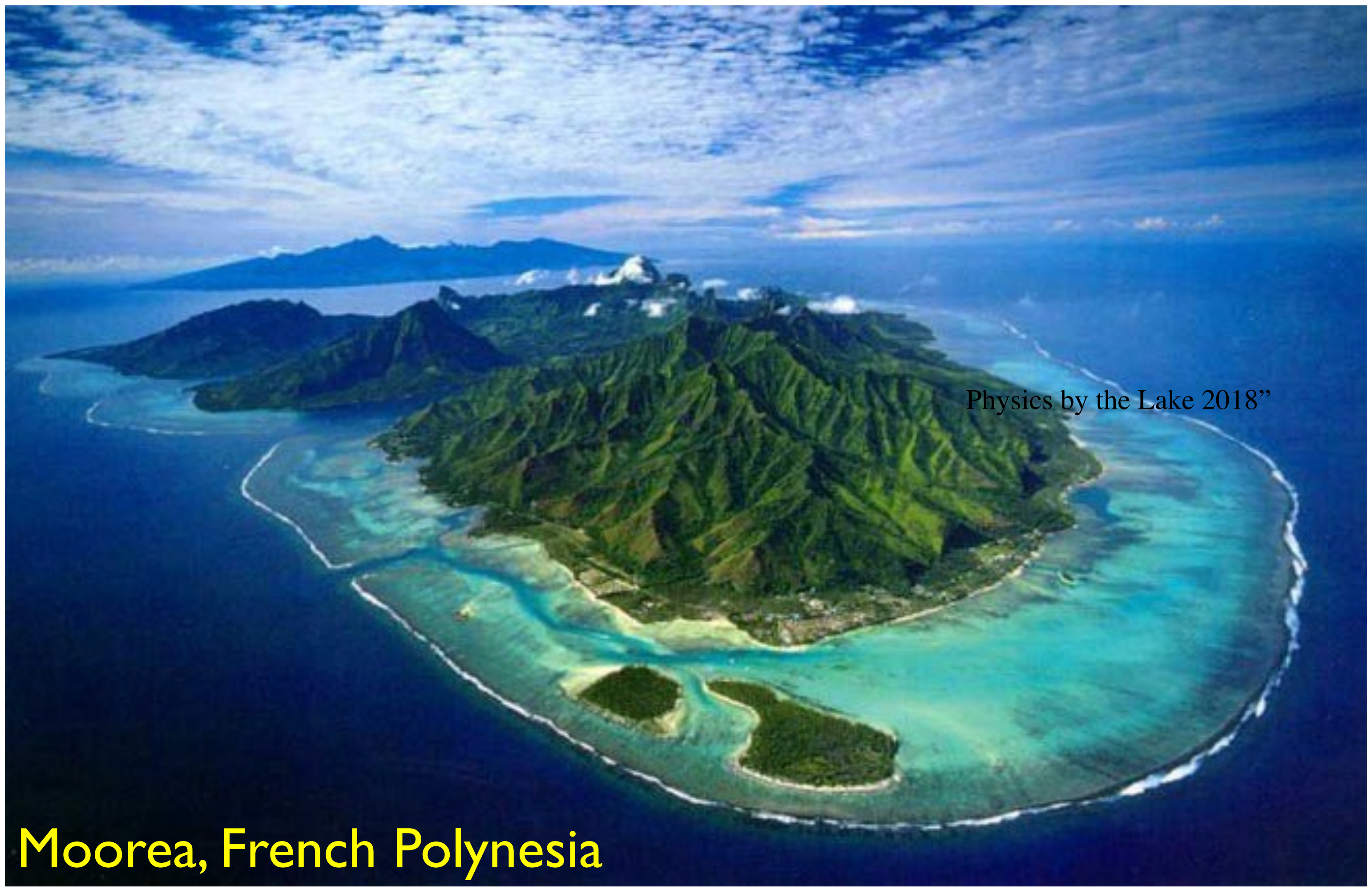

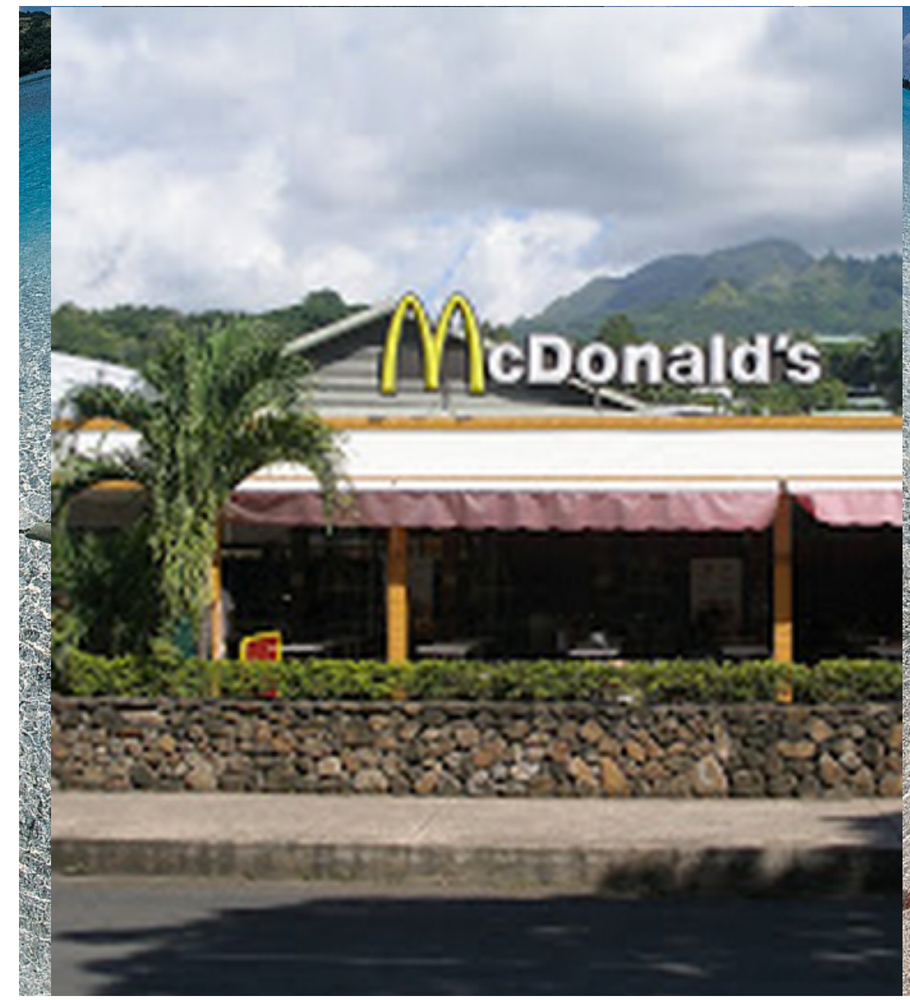

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Amazingly, some quantum systems can and do evade the tyranny of entropy.

Counter-intuitively, these are often <u>imperfect</u>, so that spins are no longer able to cooperatively generate entanglement.

This is one of the frontiers of research today.

“It is by avoiding the rapid decay into the inert state of “equilibrium” that an organism appears so enigmatic”.

- E. Schrödinger, *What is Life?*

Image: Wikipedia