Entropy: two short stories

John Chalker

Entropy

In thermodynamics

$$\Delta S = \frac{\Delta Q_{\mathrm{rev}}}{T}$$

In statistical mechanics

$$S = k_{\mathrm{B}} \ln W$$

Reflections on the Motive Power of Fire

Sadi Carnot (1824)

Everyone knows that heat can produce motion. That it possesses vast motive-power no one can doubt, in these days when the steam-engine is everywhere so well known.

Reflections on the Motive Power of Fire

Sadi Carnot (1824)

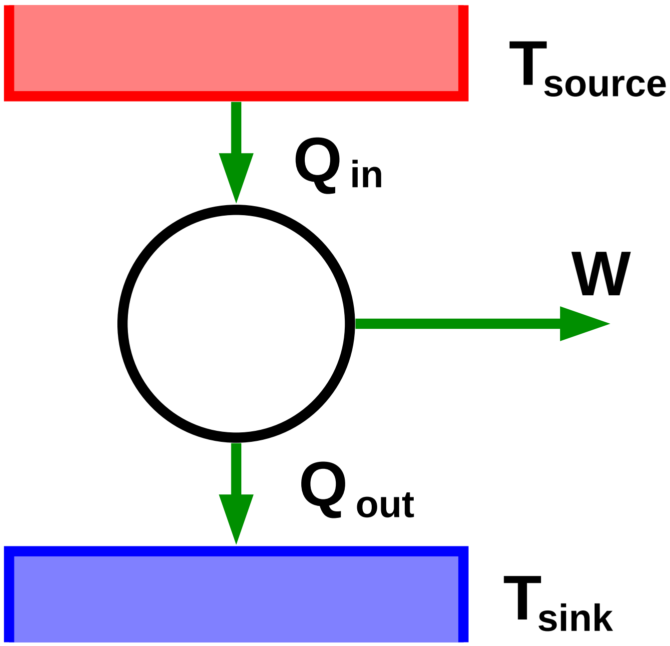
The question has often been raised whether the motive power of heat is unbounded, whether the possible improvements in steam-engines have an assignable limit, a limit which the nature of things will not allow to be passed by any means whatever; or whether, on the contrary, these improvements may be carried on indefinitely.

Reflections on the Motive Power of Fire

Sadi Carnot (1824)

In order to consider in the most general way the principle of the production of motion by heat, it must be considered independently of any mechanism or any particular agent. It is necessary to establish principles applicable not only to steam-engines but to all imaginable heat-engines.

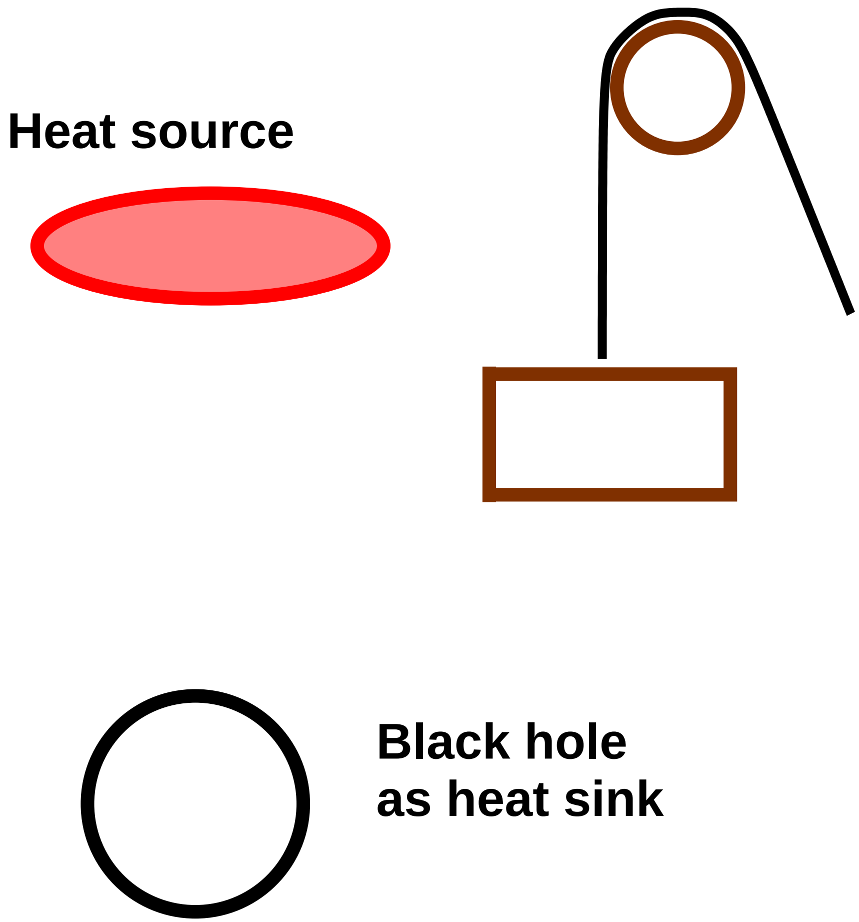
Heat engine



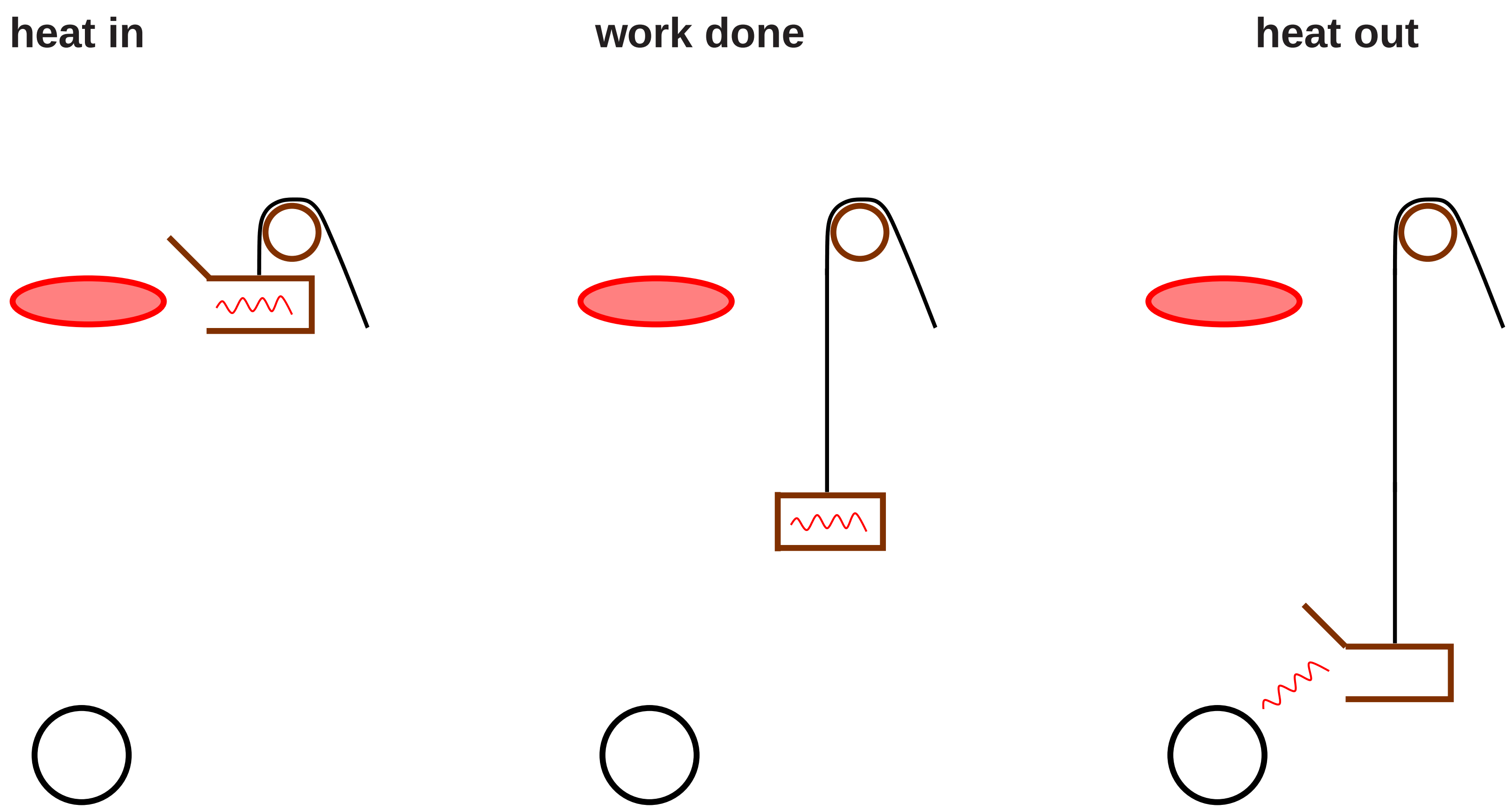
Efficiency η

$$\eta = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \leq 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

Black hole-based heat engine



Operation cycle



Black hole revision

Escape velocity from body of mass M

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

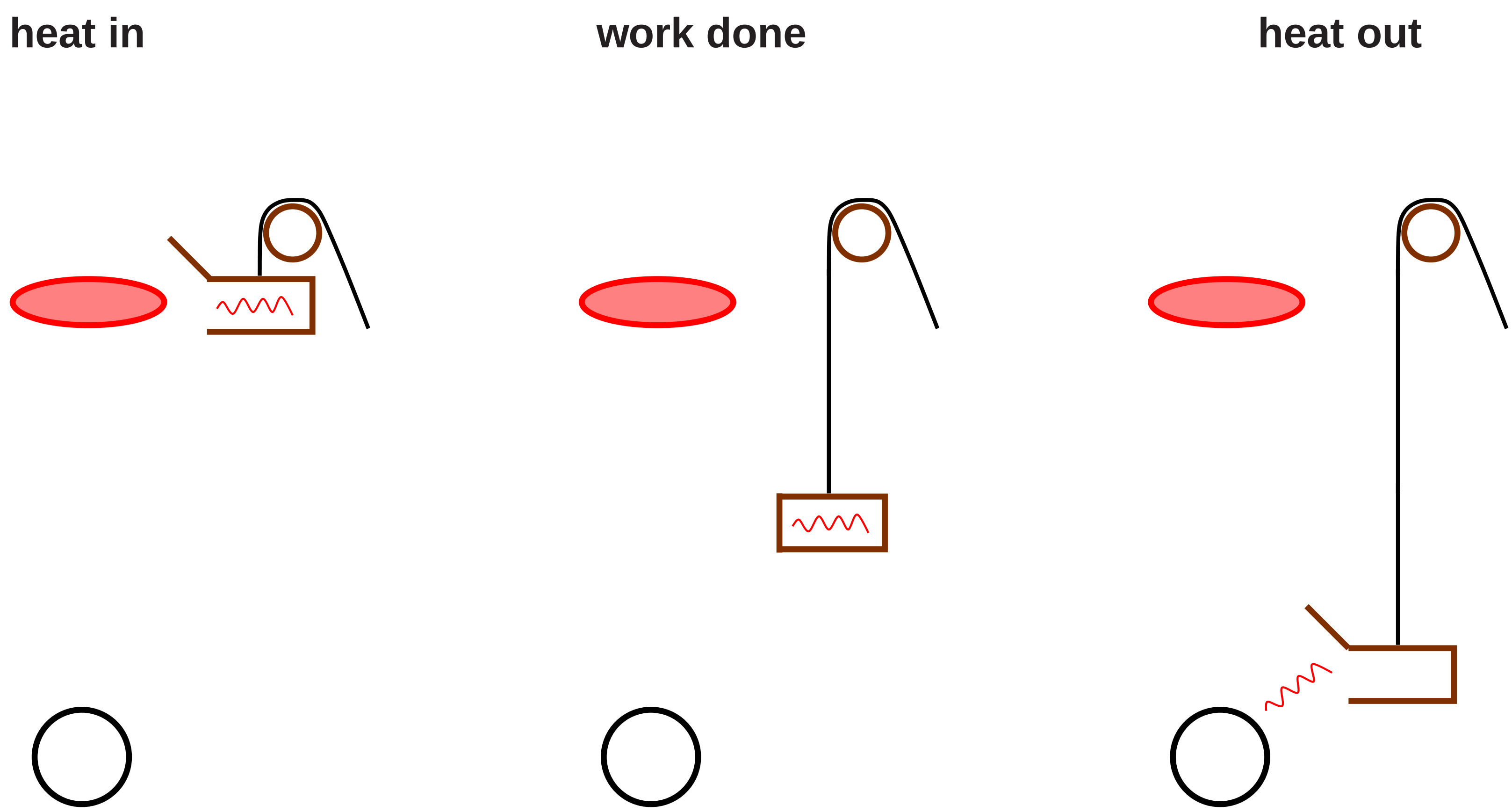
Radius of event horizon

$$R = \frac{2GM}{c^2}$$

Gravitational acceleration at event horizon

$$g = \frac{GM}{R^2} \sim \frac{c^4}{GM}$$

Operation cycle



Efficiency?

Would be 100% if heat from source lowered

fully to event horizon

$\eta = 1 -$ **fraction of potential energy not extracted**

$$= 1 - \frac{mgd}{Q_{\mathrm{in}}} = 1 - \frac{gd}{c^2}$$

How small can we make d **?**

Radiation from source must fit in box $d \gtrsim \lambda$

$$k_{\mathrm{B}} T_{\mathrm{source}} \sim \hbar\omega \sim \hbar c/\lambda \qquad \Rightarrow \qquad \boxed{d \gtrsim \hbar c / k_{\mathrm{B}} T_{\mathrm{source}}}$$

Implications?

Thermodynamics

$$\eta \leq 1 - \frac{T_{\mathrm{sink}}}{T_{\mathrm{source}}}$$

Black hole engine

$$\eta = 1 - \frac{gd}{c^2} \leq 1 - \frac{g\hbar}{ck_{\mathrm{B}}T_{\mathrm{source}}}$$

Black hole temperature

$$\boxed{T_{\mathrm{sink}} \sim \frac{g\hbar}{ck_{\mathrm{B}}} \sim \frac{\hbar c^3}{k_{\mathrm{B}}GM}}$$

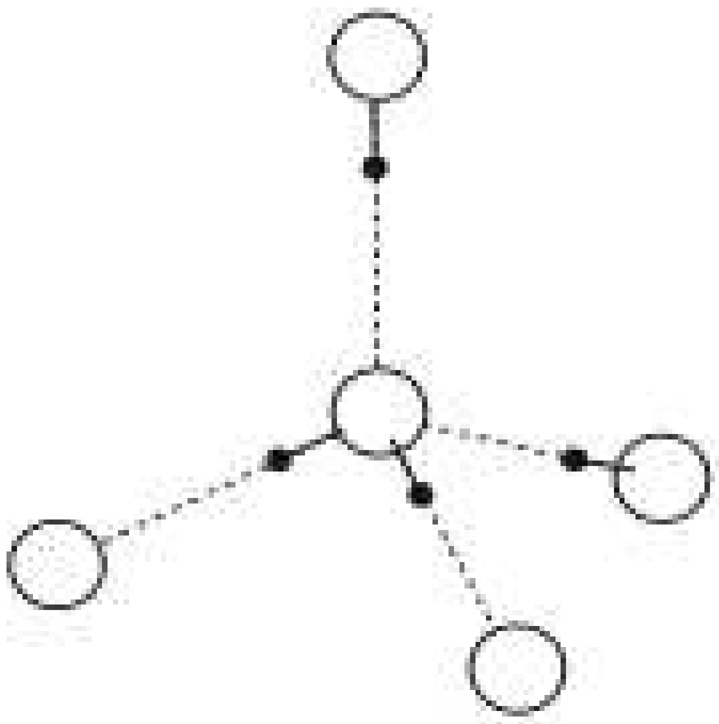
Beckenstein (1973)

Hawking (1974)

Boltzmann entropy

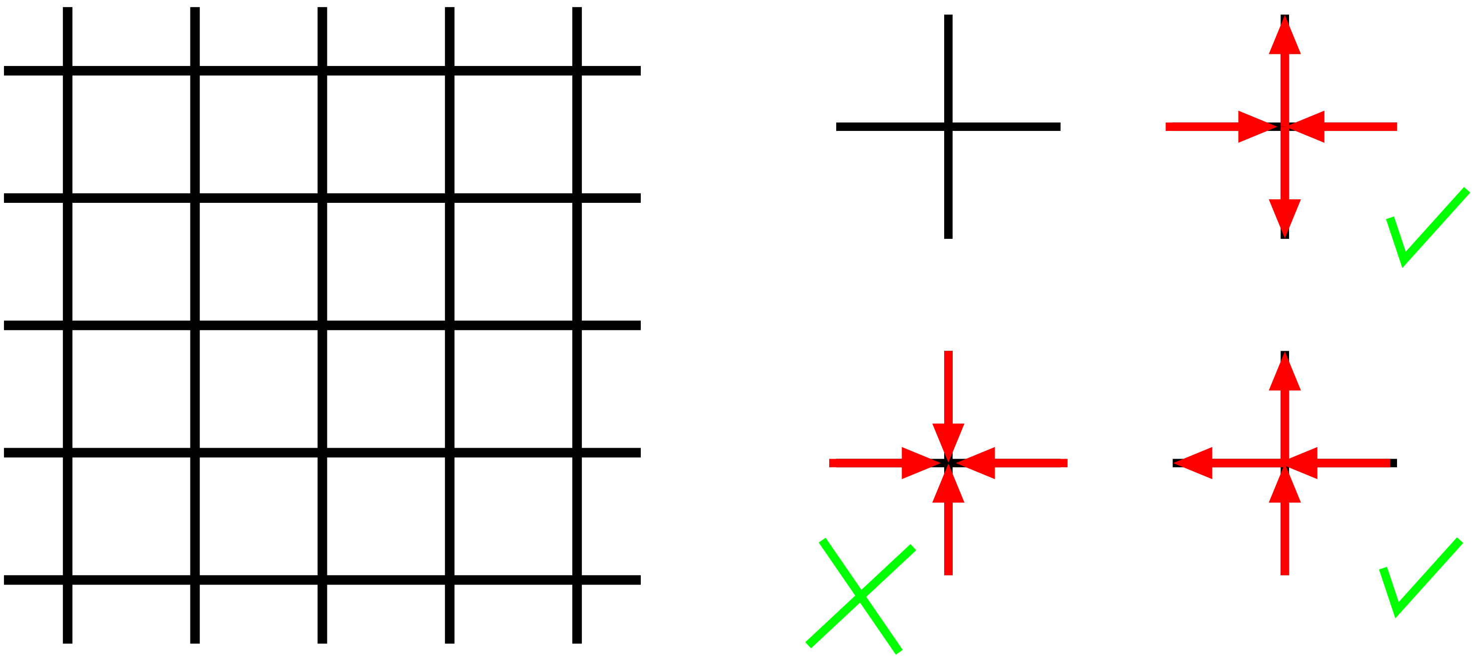
Residual entropy of ice

H_2O

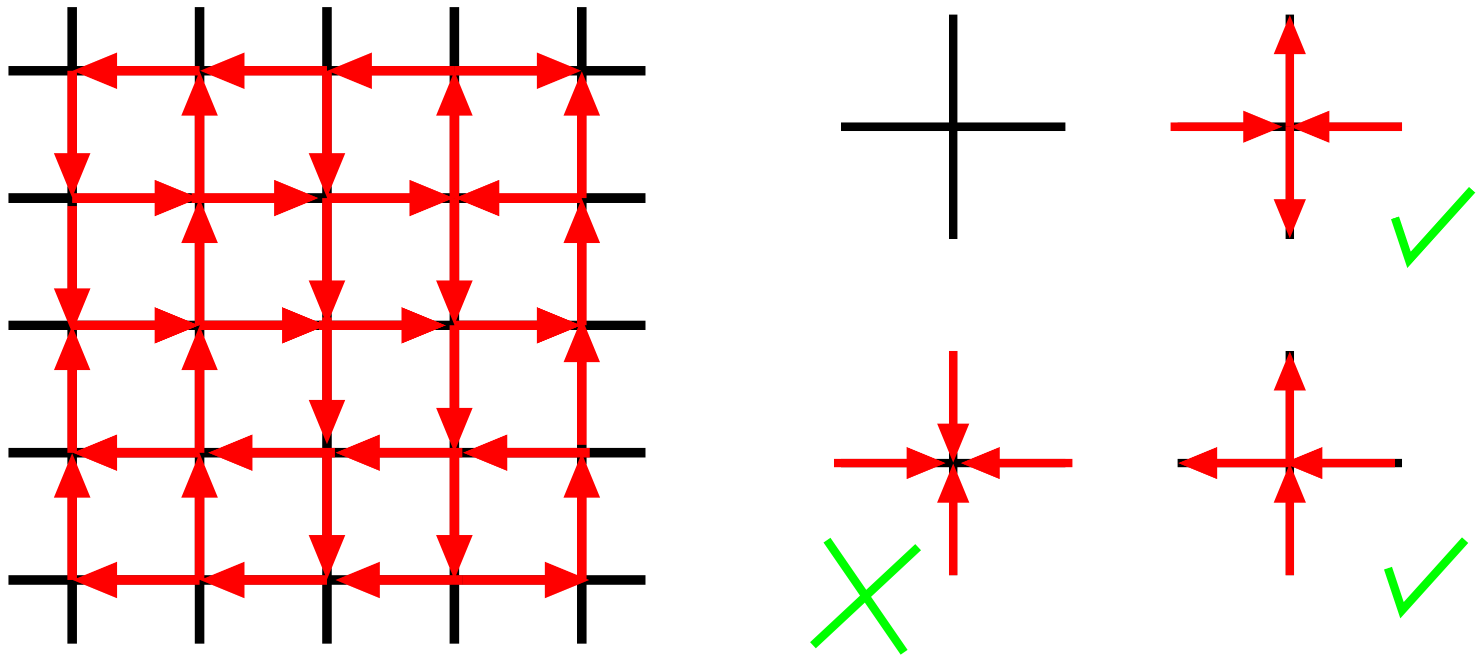


Pauling (1935)

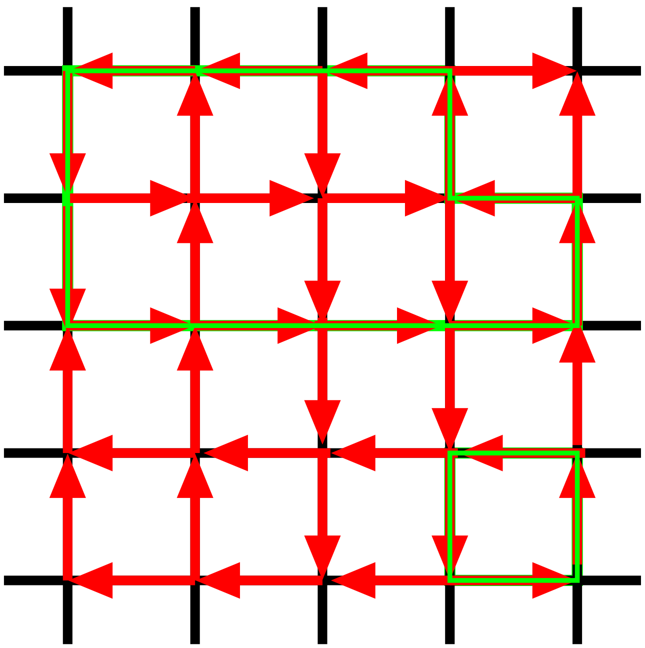
Square ice



Square ice



Square ice



Counting

Consider ice crystal with N oxygen ions and $2N$ protons

Unconstrained no of proton positions: 2^{2N}

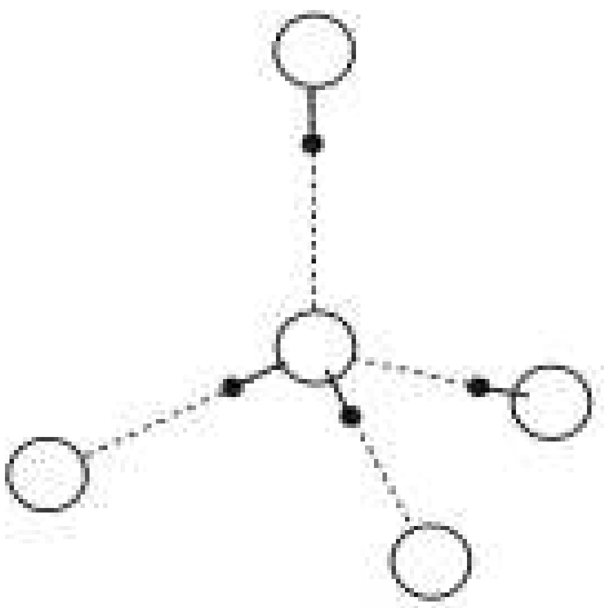
Around one oxygen ion: $\frac{6}{16}$ **of positions permitted**

For whole crystal: $W \approx 2^{2N} \times \left(\frac{6}{16}\right)^N = \left(\frac{24}{16}\right)^N = \left(\frac{3}{2}\right)^N$

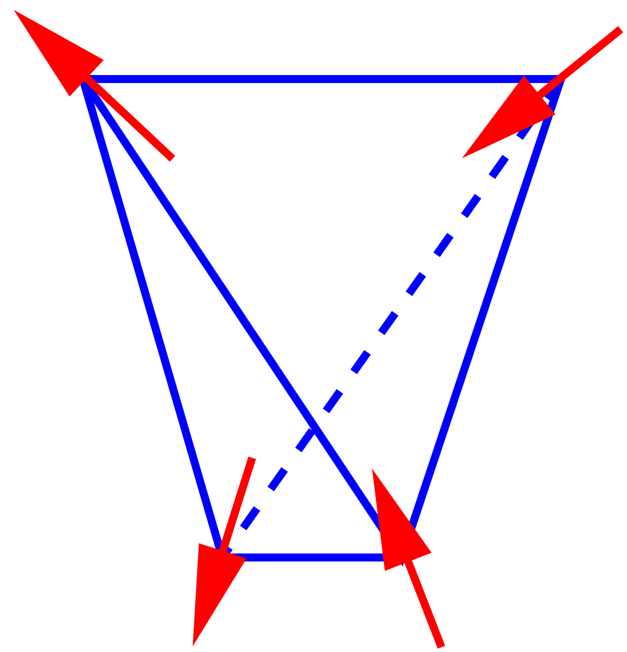
Residual entropy of ice: $\boxed{k_\mathrm{B} \ln W \approx N k_\mathrm{B} \ln(3/2)}$

Magnetic analogue of ice

Water ice

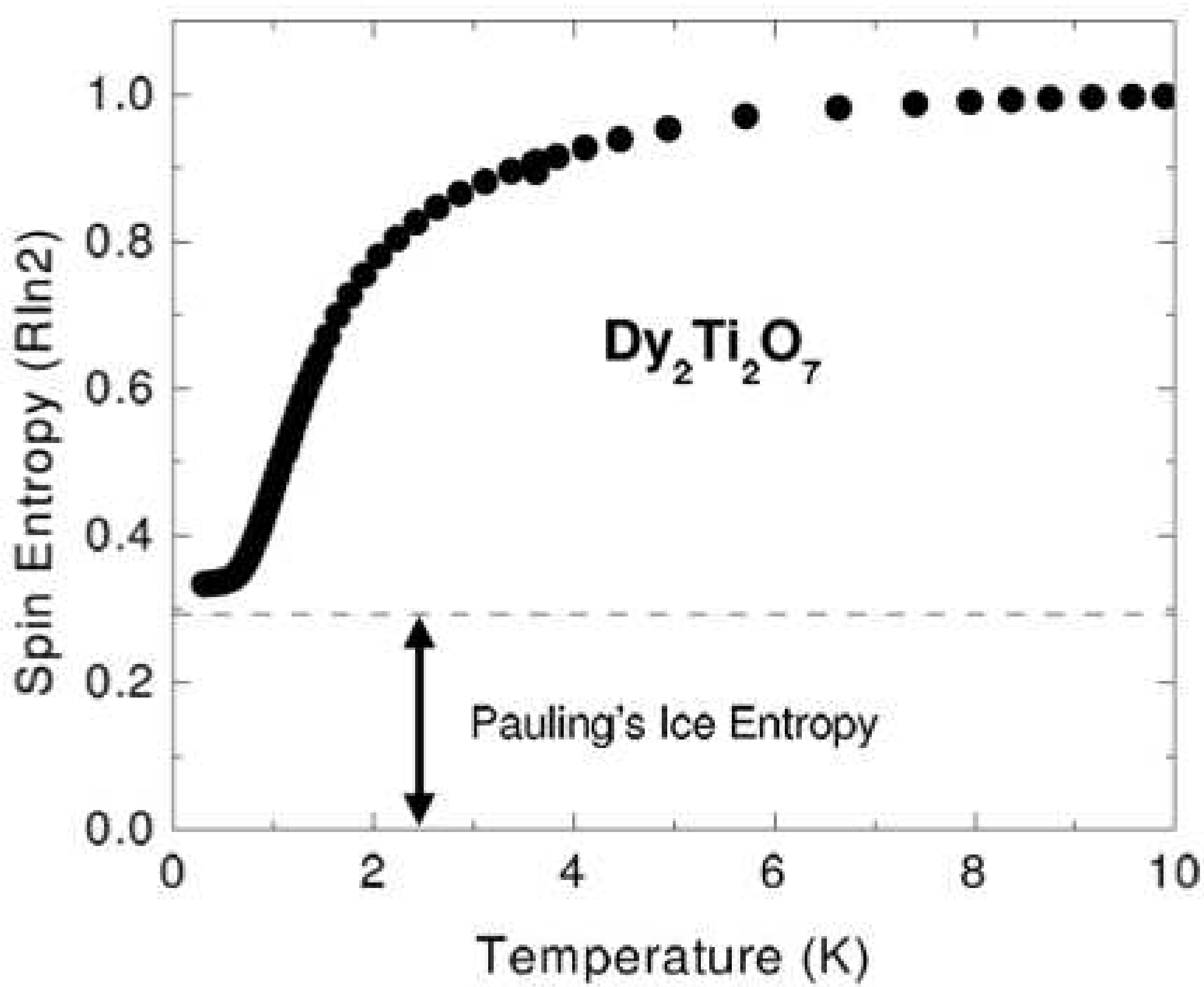


Spin ice



Ground states: ‘two-in, two-out’

Pauling entropy in experiment



$\mathrm{Dy_2Ti_2O_7}$, **Ramirez *et al*, Nature 399, 333 (1999).**

The Nature of the Physical World

Arthur Eddington (1928)

The law that entropy always increases–the second law of thermodynamics – holds, I think, the supreme position among the laws of nature. If someone points out to you that your pet theory of the universe is in disagreement with Maxwell's equations–then so much the worse for Maxwell's equations. If it is found to be contradicted by observation – well, these experimentalists do bungle things sometimes. But if your theory is found to be against the second law of thermodynamics I can give you no hope; there is nothing for it but to collapse in deepest humiliation.