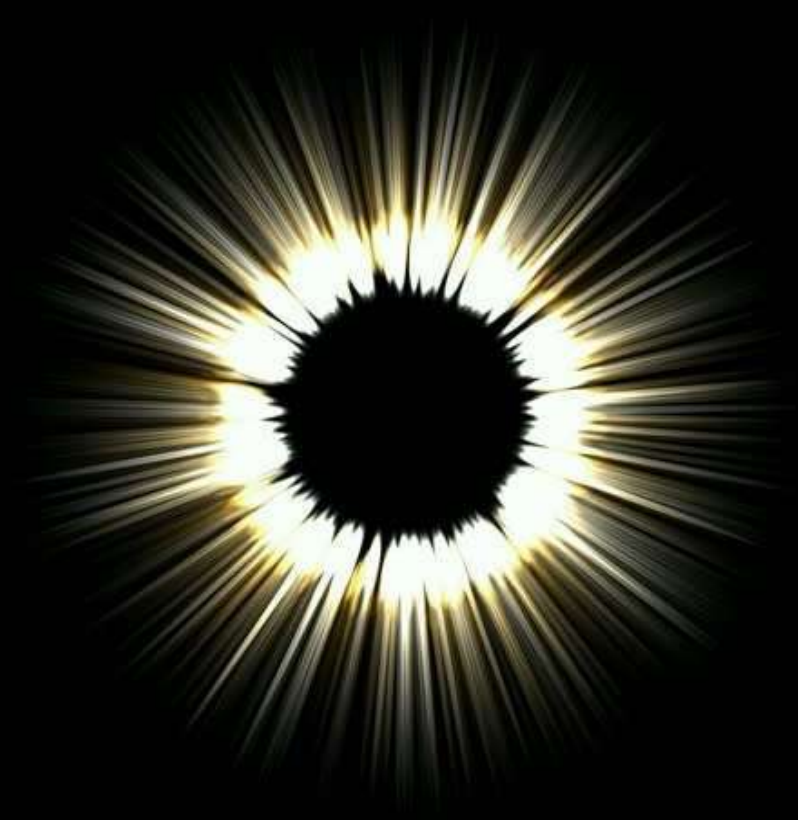


# Classical and Quantum Black Holes

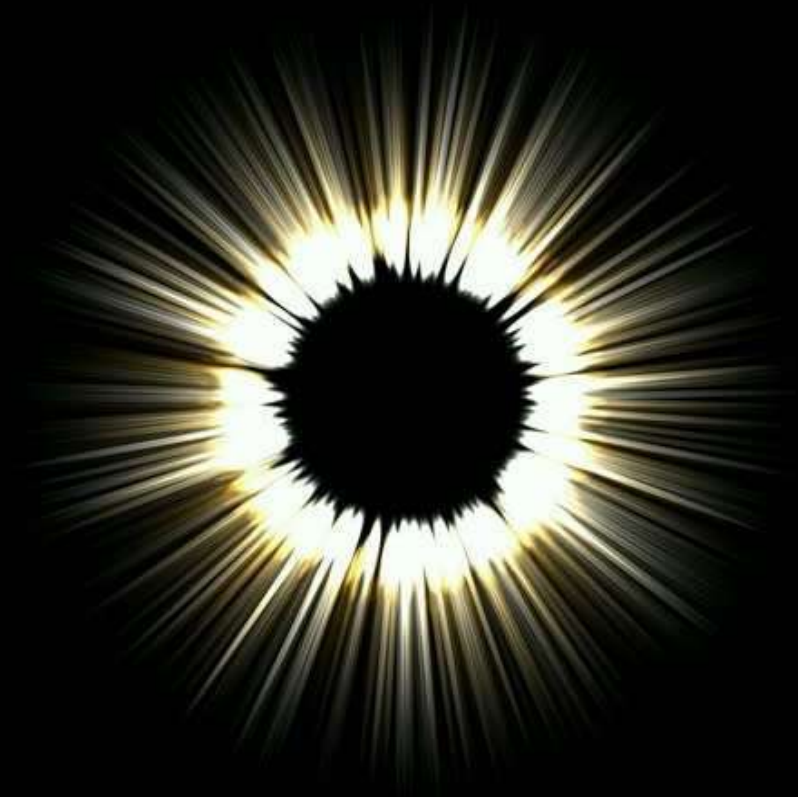
John March-Russell



Morning of Theoretical Physics, 9th Jan 2021

BHs are among the most awe-inspiring objects in the Universe

They imply the end of space-time as we know it, and *appear* to imply deep paradoxes when we try to meld with quantum mechanics

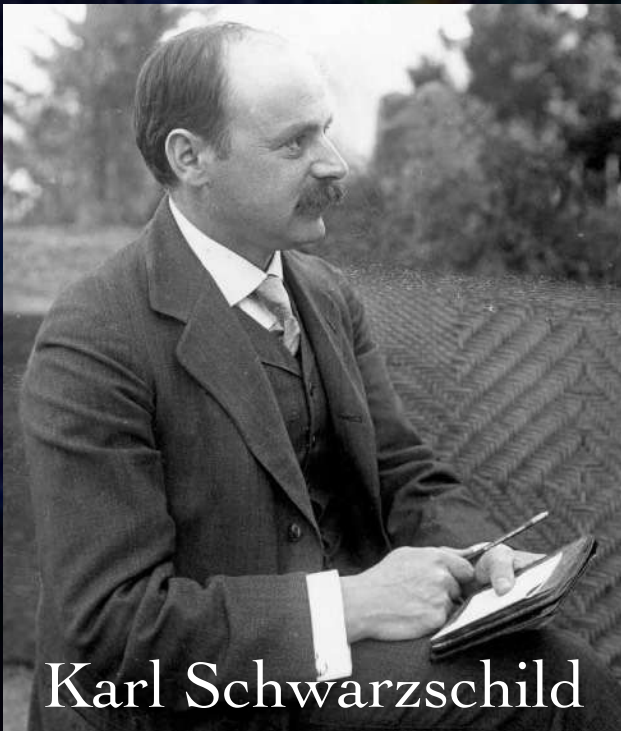


There has been significant progress, both observational *and* theoretical in BH physics in the past few years!

"Hydrogen Atom of the 21st Century"

# Classical BHs

We **now know** that astrophysical BHs exist and behave, to a very good approximation, the way that Einstein's classical General Relativity says they should (at least the exterior)



Karl Schwarzschild

Schwarzschild wrote down first & simplest BH solution while fighting & dying in WW1

Schwarzschild radius  
of BH event horizon

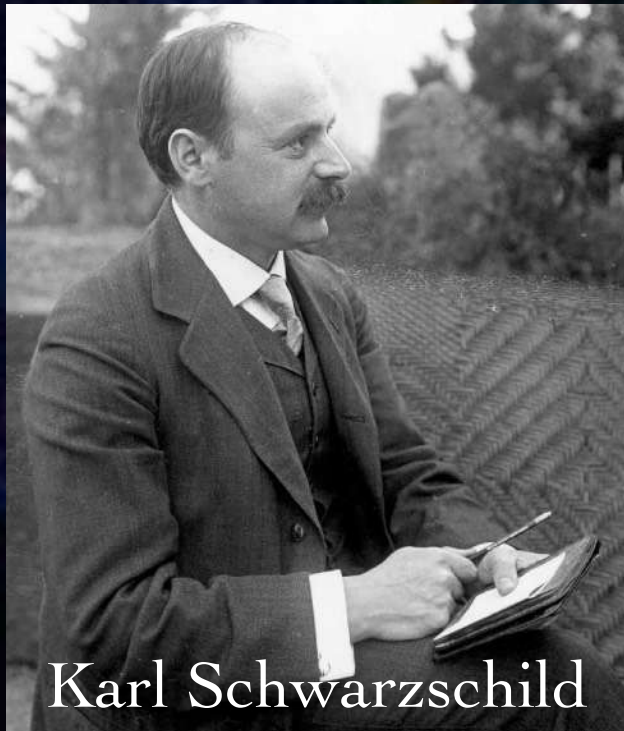
$$r_s = \frac{2GM}{c^2}$$

BH  
mass



# Classical BHs

We **now know** that astrophysical BHs exist and behave, to a very good approximation, the way that Einstein's classical General Relativity says they should (at least the exterior)



Karl Schwarzschild

Schwarzschild wrote down first & simplest BH solution while fighting & dying in WW1

Schwarzschild radius  
of BH event horizon

$$r_s = \frac{2GM}{c^2}$$

Solar-mass BH:  $r_s = 2.93$  km

Galactic centre BHs of  $10^6$ - $10^9$  solar masses:  $r_s \sim 10^6$ - $10^9$  km

Hypothetical BH of Earth mass:  $r_s = 0.84$  cm



# Classical BHs

$$ds^2 = -c^2 d\tau^2 = - \left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

according to static external  
observer, time dilation as  
horizon at  $r_s$  approached

radial length contraction  
as horizon approached

space-time distance  
between events

Modern understanding of classical  
BHs due to Wheeler (who coined the  
name), Kruskal, Kerr, Thorne,  
Hawking, and especially Penrose



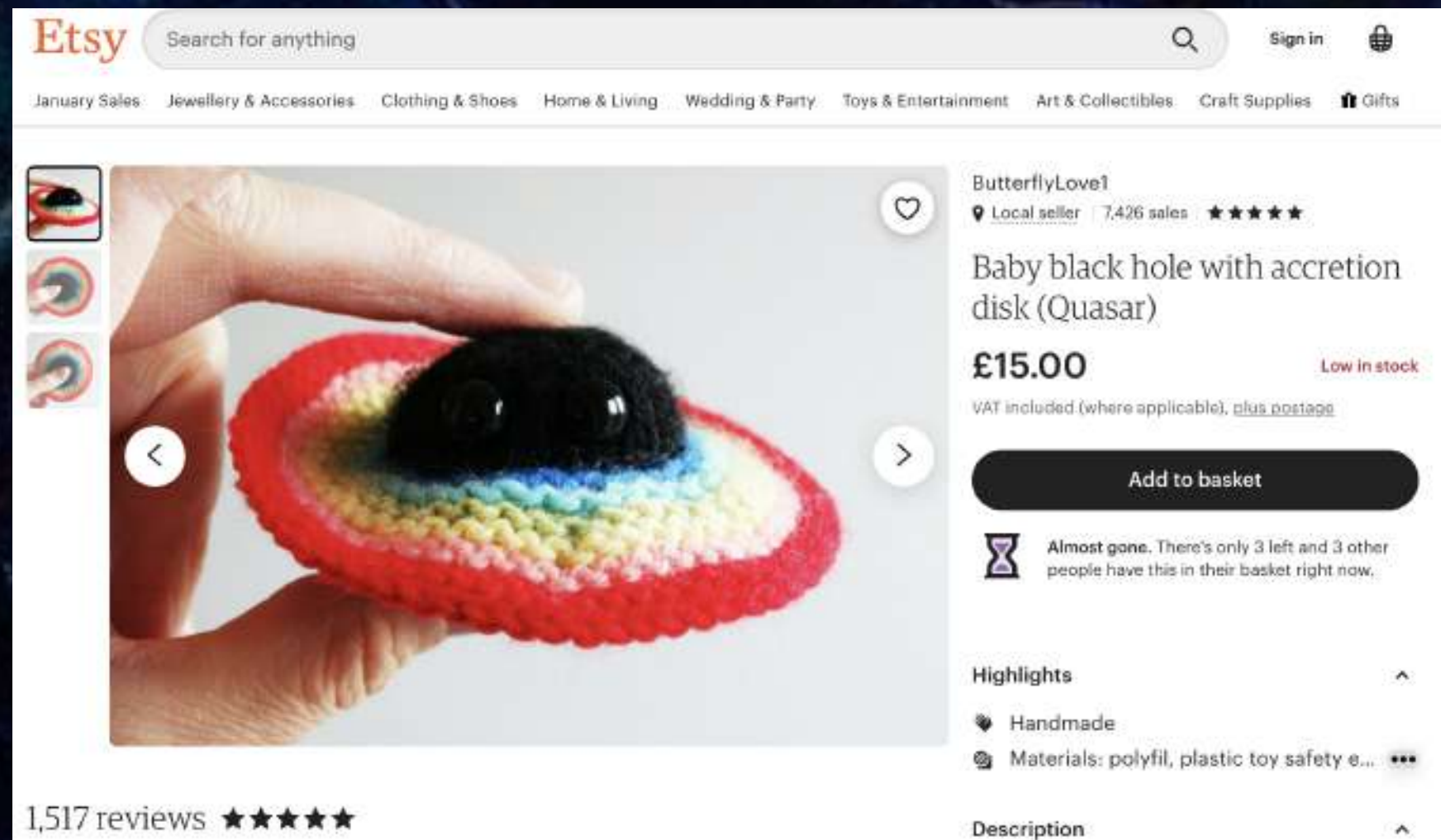
Roger Penrose



# Classical BHs

What do these awe-inspiring objects look like?

## Etsy Baby BH

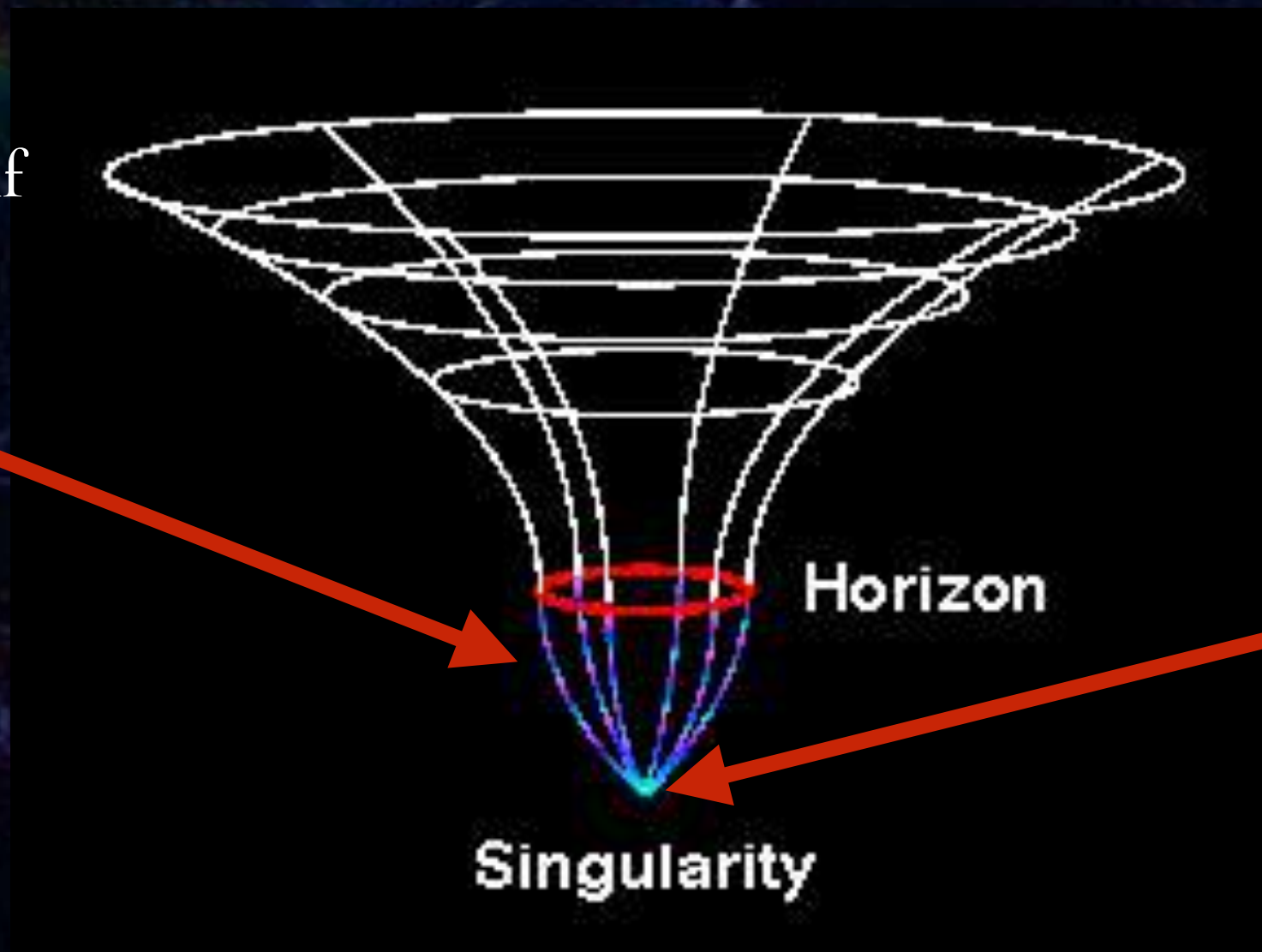




# Classical BHs

$$ds^2 = -c^2 d\tau^2 = - \left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

space-time itself  
is not static  
inside horizon  
("r becomes a  
temporal  
coordinate")

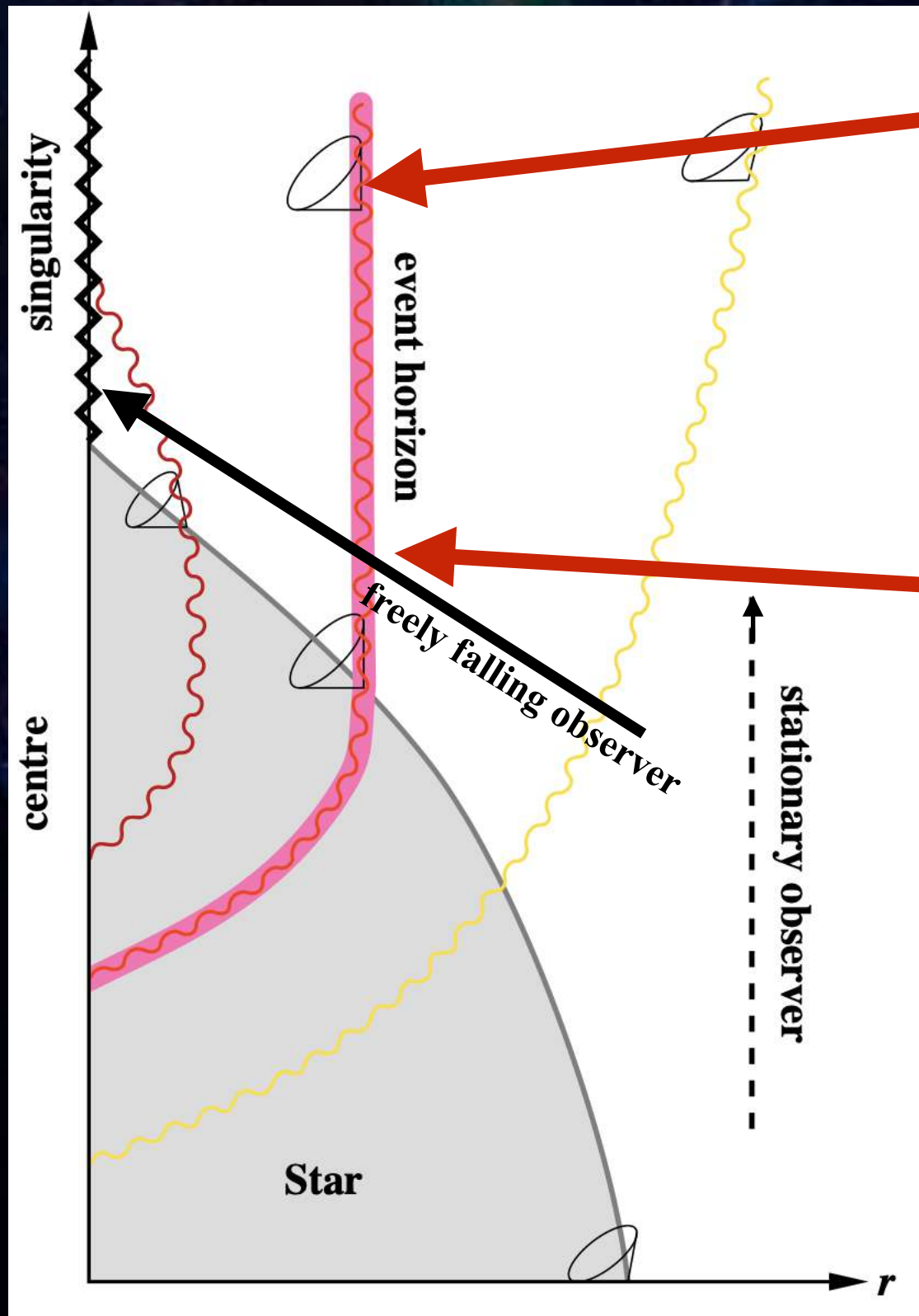


space-time "ends"  
at singularity, but  
classical solution  
(and GR itself)  
not reliable here

Credit: Andrew Hamilton (JILA)



# Classical BHs



At  $r=r_s$  horizon the "light cones" tip so far over that avoidance of  $r=0$  singularity is impossible ("forward in time" becomes "radial inward")

BUT, a freely falling observer feels **nothing special** as she falls through the horizon (just usual, finite, tidal forces). To such an observer the region around the horizon is just like normal, empty Minkowski space!

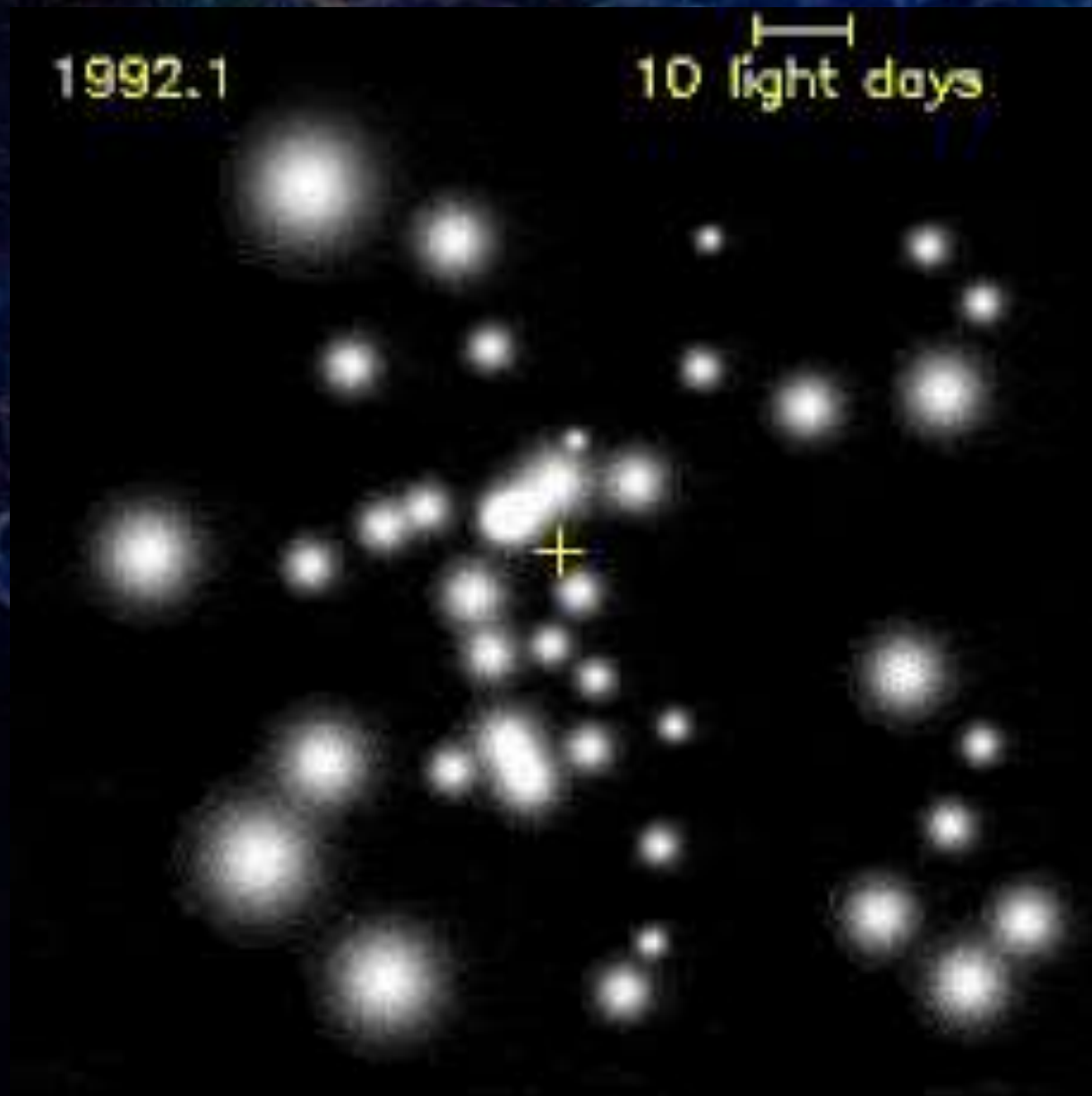
The **apparent** singular behaviour of the Schwarzschild metric at horizon is just a bad choice of coordinates like longitude at N or S poles

Adapted from: Droz, Israel, Morsink



# Classical BHs

We know to a high degree of confidence that supermassive astrophysical BHs exist in galactic centres



European Southern Observatory

Nobel Prize 2020 shared with Roger Penrose



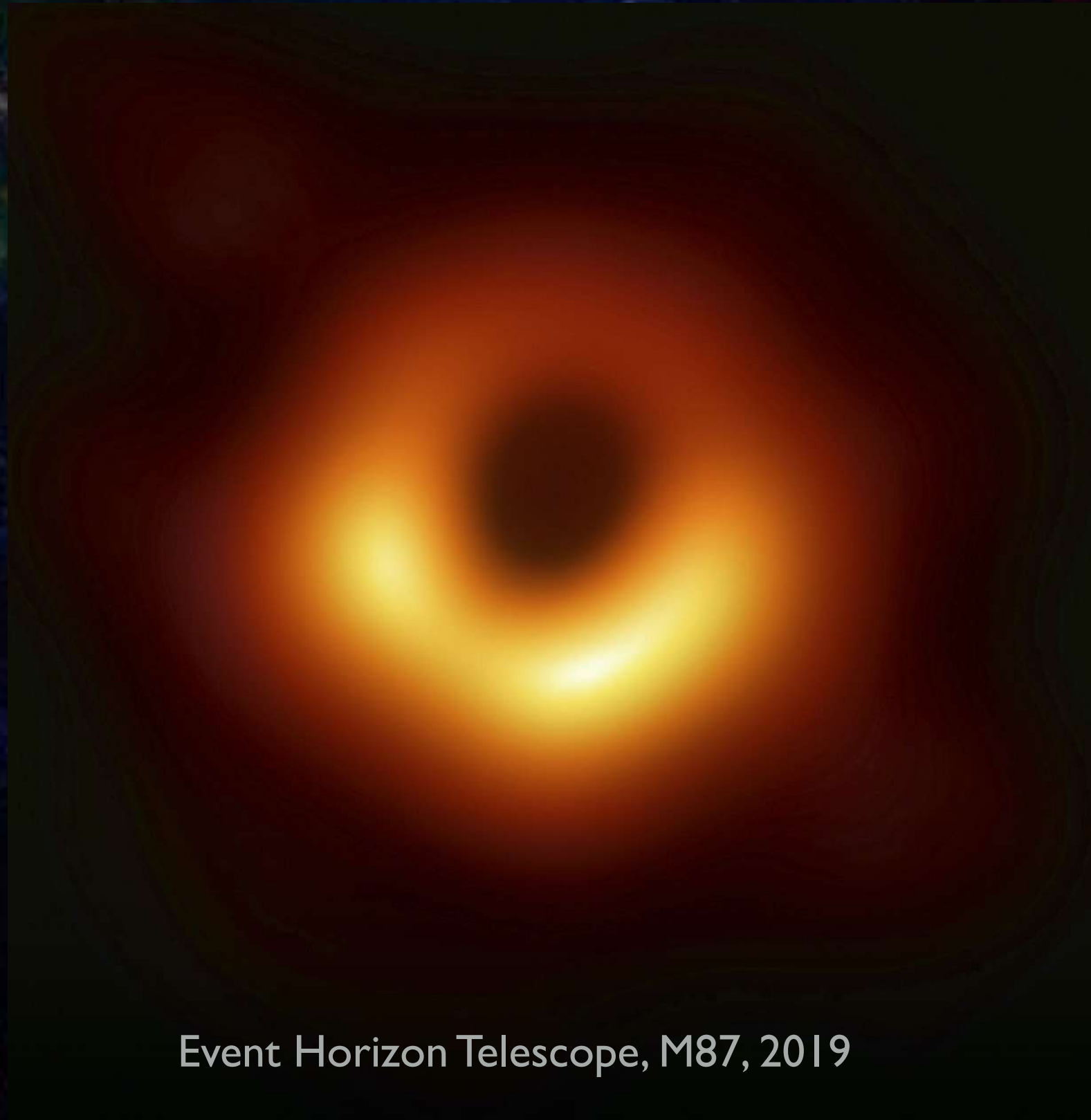
Reinhard Genzel



Andrea Ghez

# Classical BHs

Recently we have a direct image of a supermassive BH

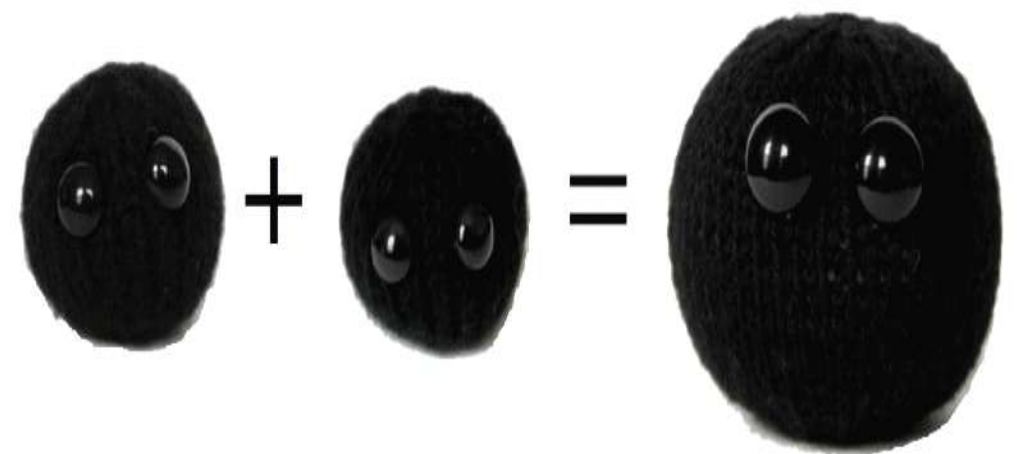
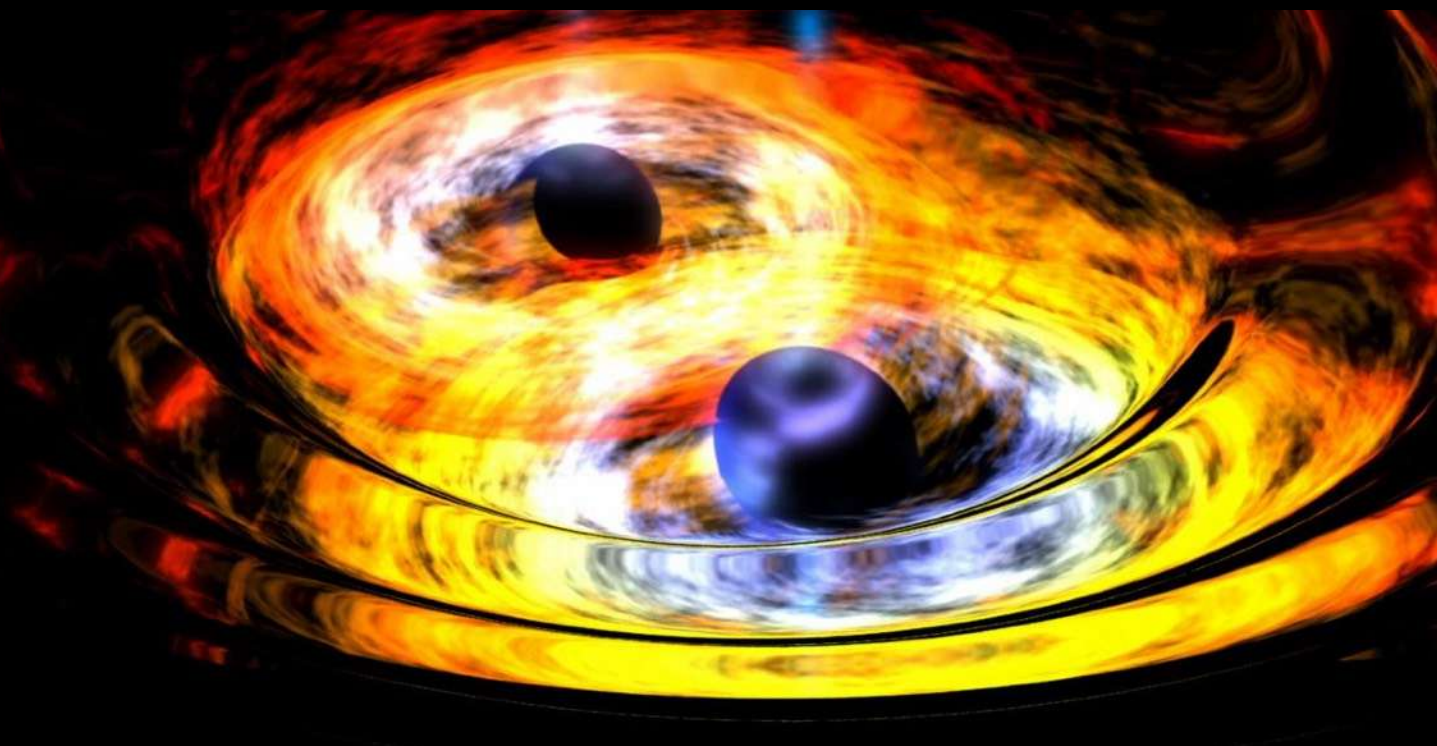
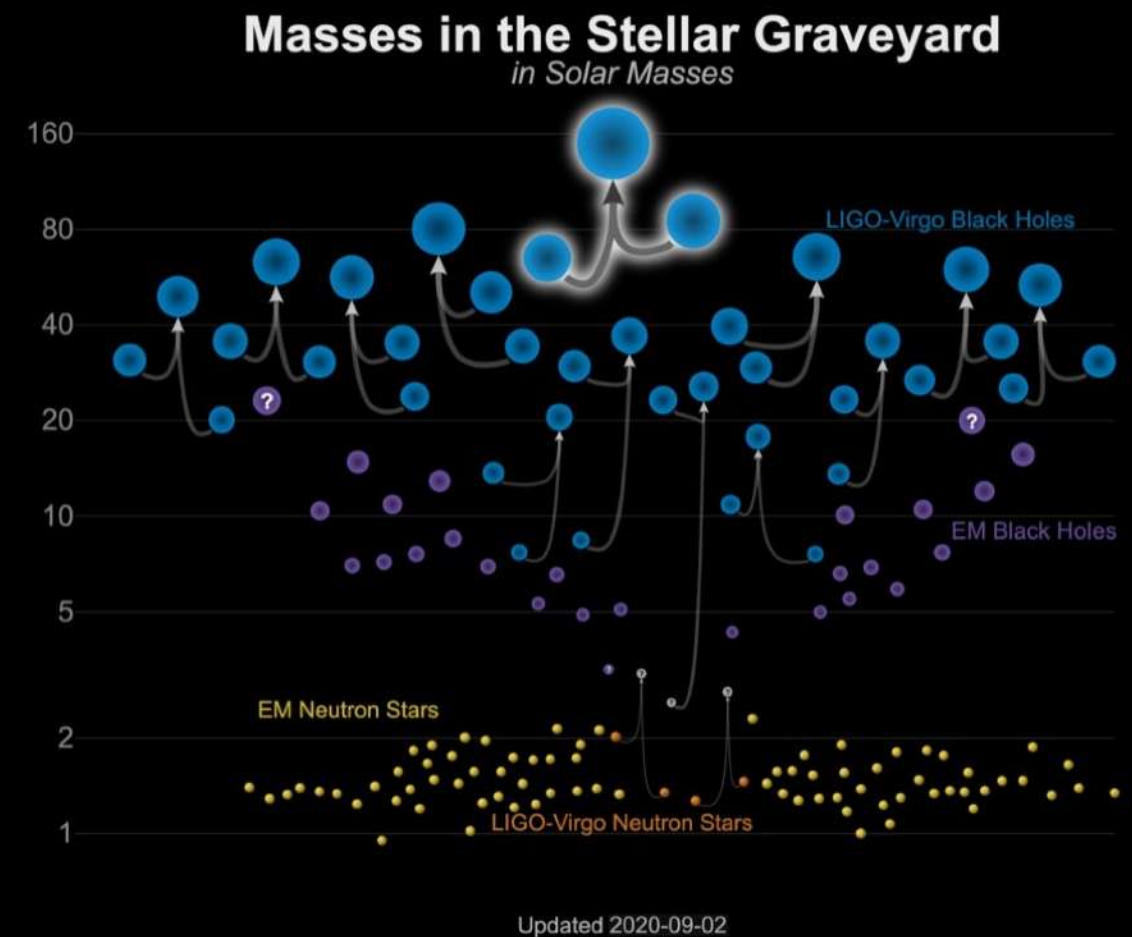
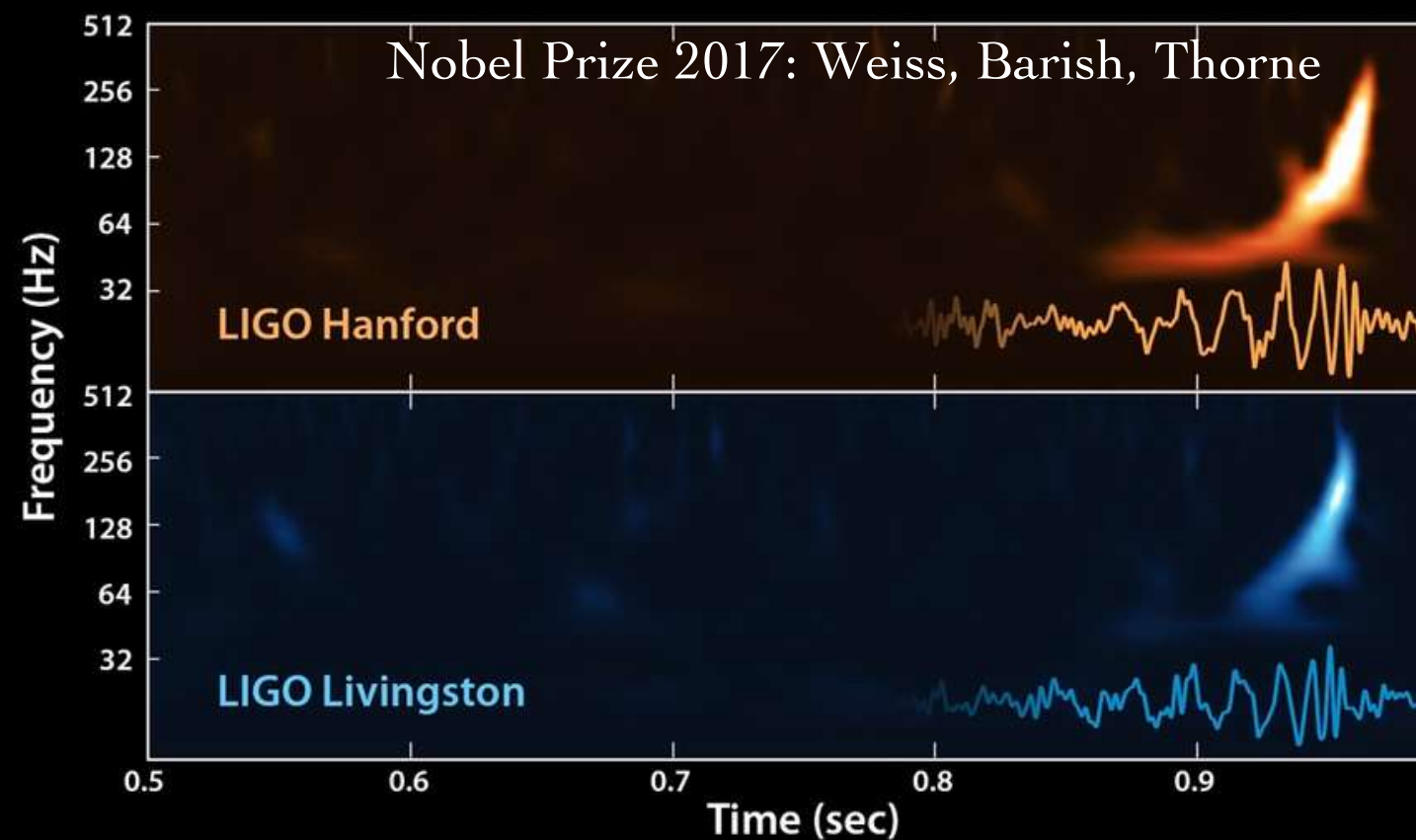


Event Horizon Telescope, M87, 2019



# Classical BHs

Also detected gravitational waves from mergers of smaller BHs



Credit: Chris Berry & Dawn Finney

# Quantum BHs

The classical theory of BHs is extraordinarily rich, but the world is really quantum mechanical

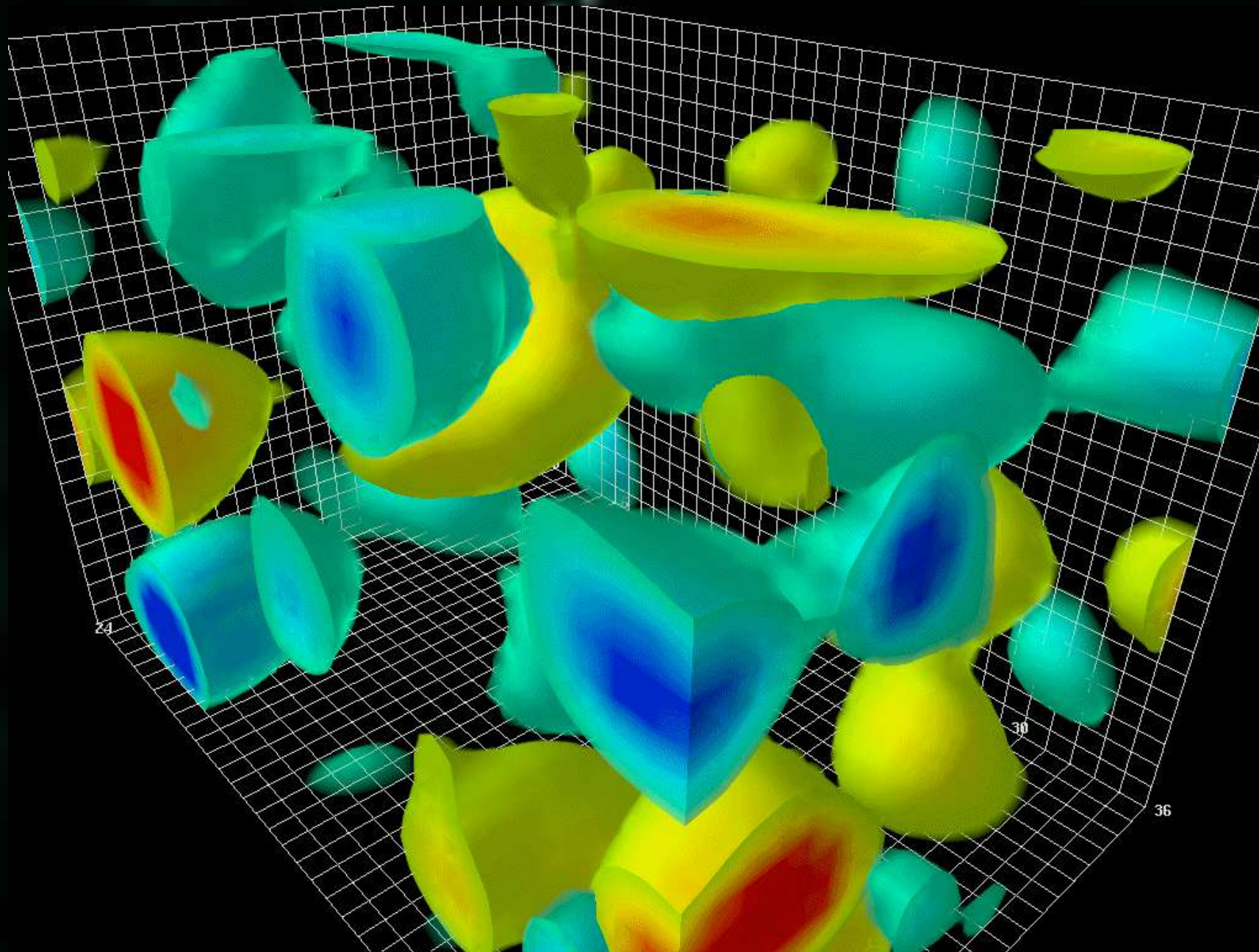
The QM of BHs is even more remarkable and leads to apparent deep paradoxes whose resolution is guiding us towards a better understanding of aspects of quantum gravity

Significant progress in last few years....!



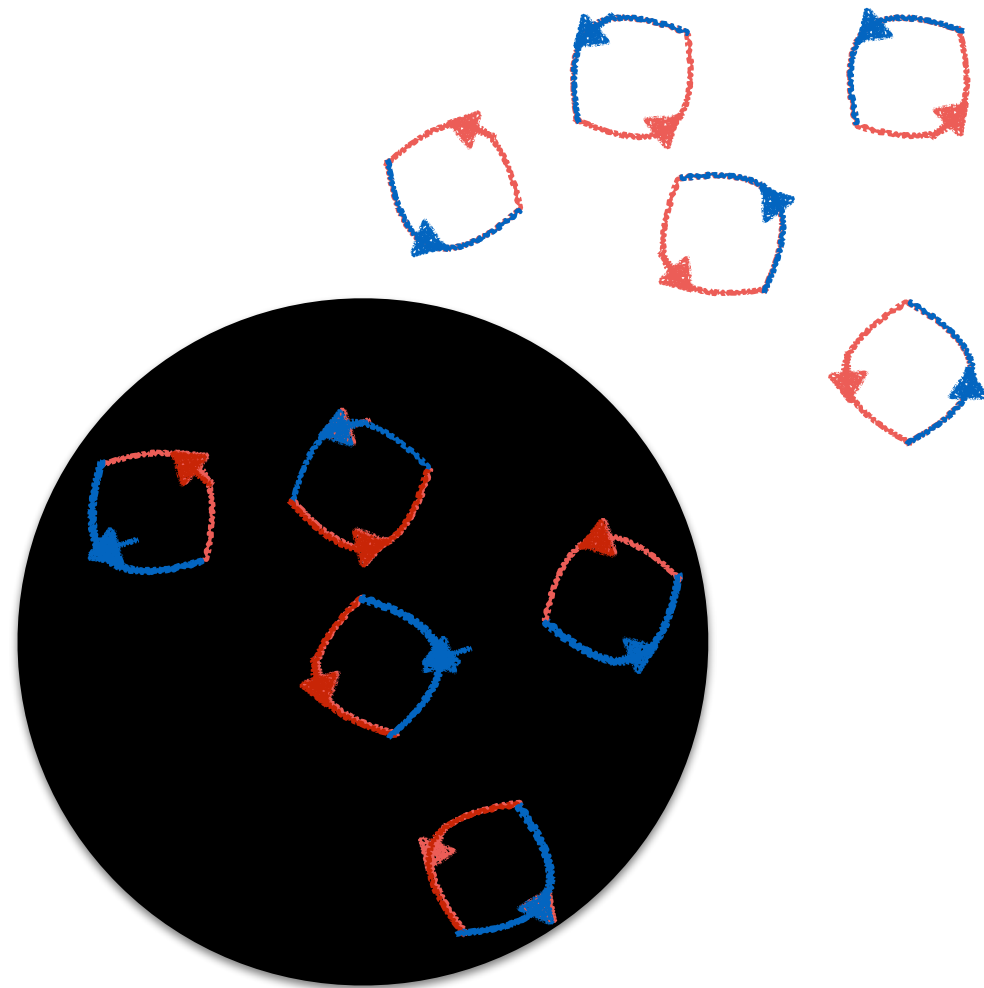
# Hawking Radiation

In relativistic QM empty space (the "vacuum") is full of quantum fluctuations of all fields and particles



Precision numerical evaluation of vacuum of QCD on  $(5\text{fm})^3$  lattice (only one realisation shown!)

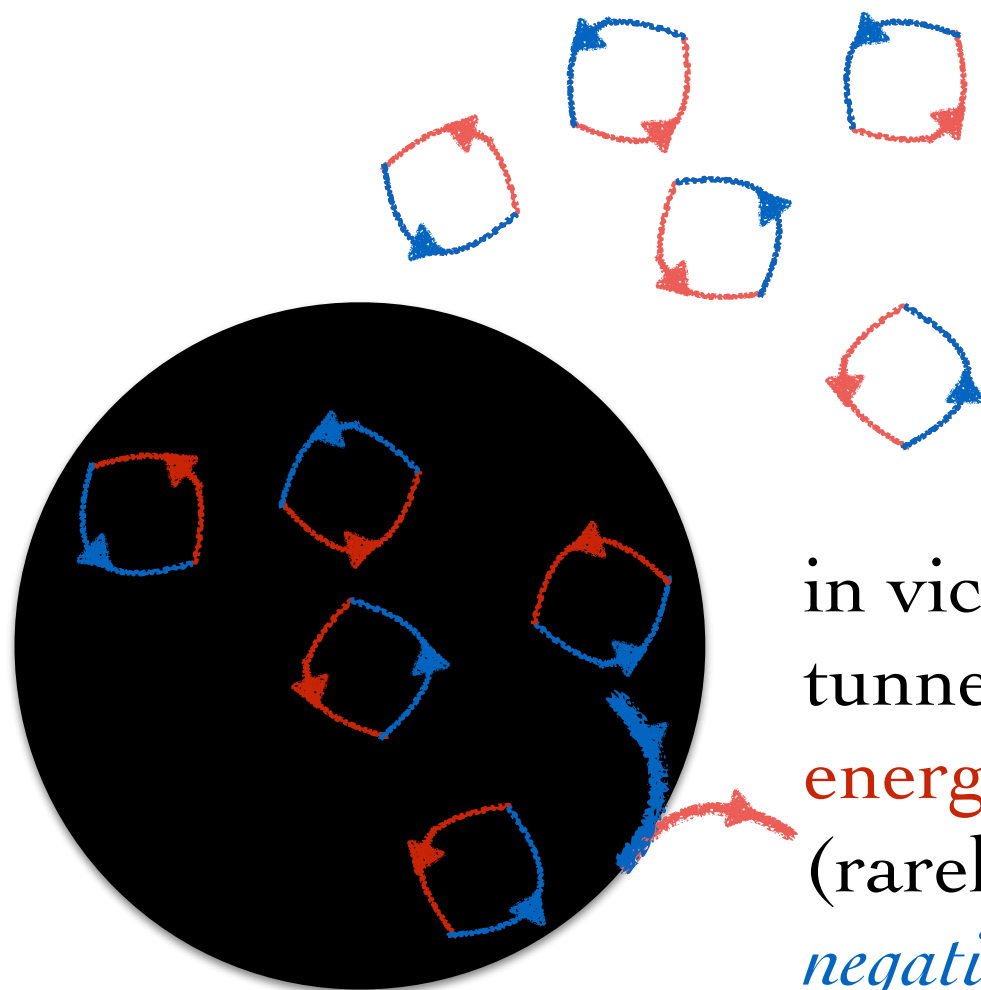
# Hawking Radiation



**positive** & **negative** energy  
'virtual' particle (and anti-  
particle) fluctuations are  
everywhere being created  
and annihilating



# Hawking Radiation

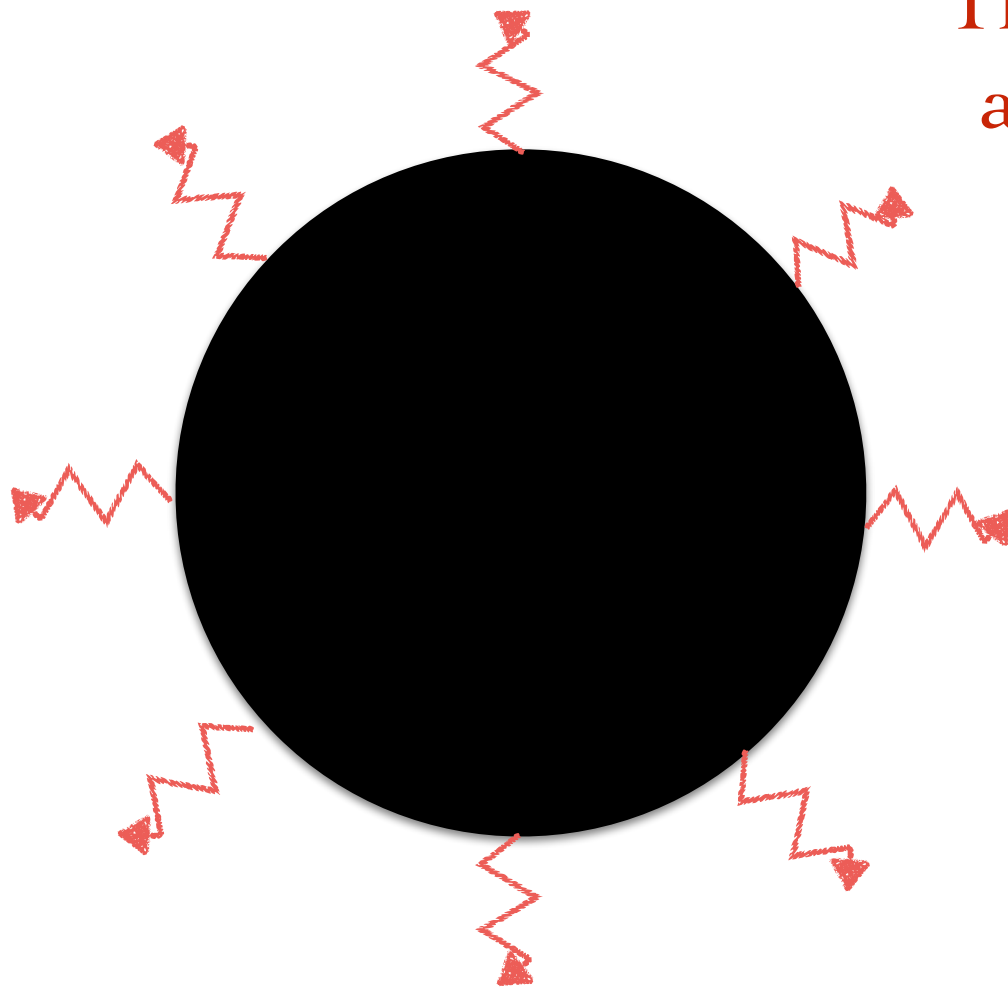


in vicinity of horizon a QM tunnelling process allows a **positive energy** particle or anti-particle to (rarely) escape to infinity while a *negative energy* (from perspective of distant observer) particle or anti-particle is captured by BH, reducing BH mass

(Note for experts: This tunnelling picture of Hawking radiation can be made *precise* and *gives correct reduction in mass of BH* - see papers of Wilczek et al)

# Hawking Radiation

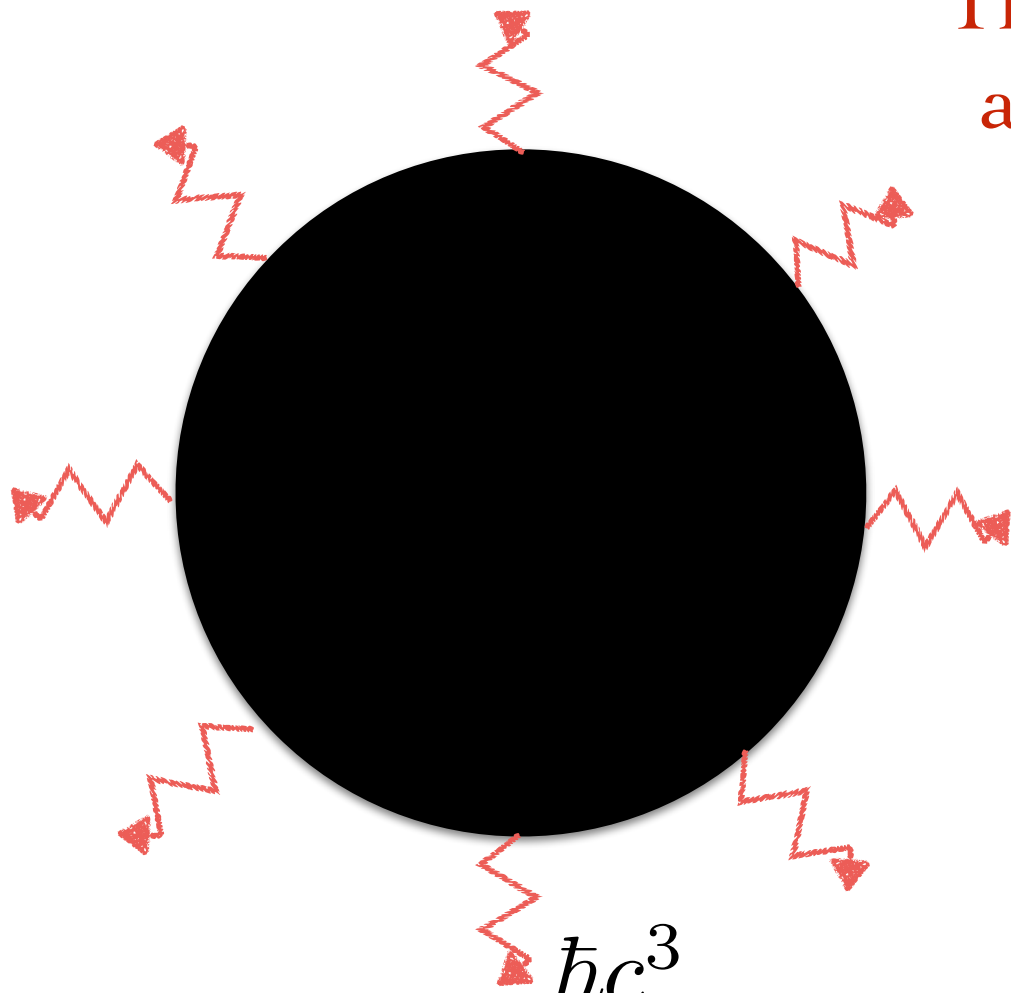
Physical BHs are never truly black!  
They produce thermal radiation in  
all kinematically allowed modes.





# Hawking Radiation

Physical BHs are never truly black!  
They produce thermal radiation in  
all kinematically allowed modes.



Amazing end result is BH  
is a thermal system with  
temperature (as seen by a  
static observer far away)  $T_H$

$$T_{\text{Hawking}} = \frac{\hbar c^3}{8\pi G_N k_B M} \simeq \frac{1.2 \times 10^{23} K}{M(\text{in kg})} \simeq 10^{-7} K \frac{M_\odot}{M}$$

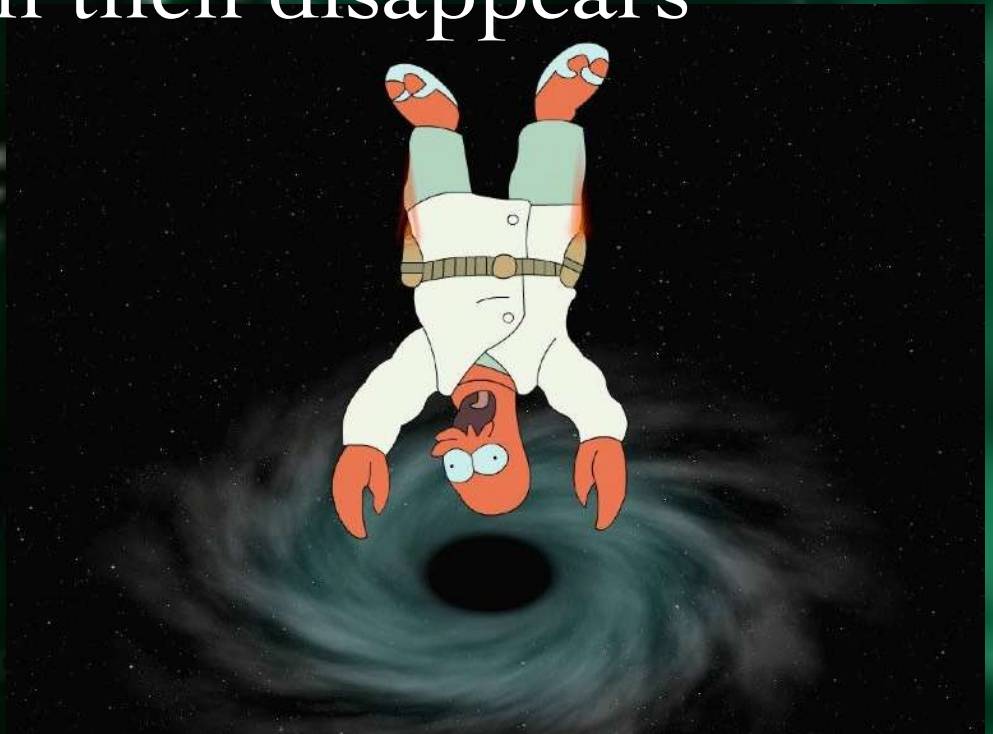
Observed astrophysical BHs are very cold but as mass of BH gets smaller temperature goes up!

# Black Hole Entropy

BH Hawking radiation has never been observed, but we now know *many* ways it can be theoretically derived & it solves some otherwise troubling aspects of classical BHs

One of these is associated with the Second Law of Thermodynamics,  $\Delta S \geq 0$ , and the entropy of BHs

Bekenstein argued that to save 2nd Law one must assign a *finite entropy* to BHs, otherwise one could systematically reduce  $S_{\text{universe}}$  by throwing in matter with  $\Delta S > 0$  which then disappears





# Black Hole Entropy

Hawking's calculation confirmed this since 1st Law says  $dE = T_H dS$  and using  $T_H$  and  $E = M_{BH} c^2$  gives (in  $k_B=1$  units)

$$S_{BH} = \frac{c^3}{G\hbar} \frac{A}{4} = \frac{1}{4} \frac{A}{\ell_{Pl}^2}$$

Area of BH  
event horizon

"Planck length"

$$\ell_{Pl} = (G\hbar/c^3)^{1/2} \simeq 10^{-33} \text{cm}$$

in classical limit  $\hbar \rightarrow 0$   
entropy is infinite

(cf. catastrophe of pre-Planck classical black body spectrum)

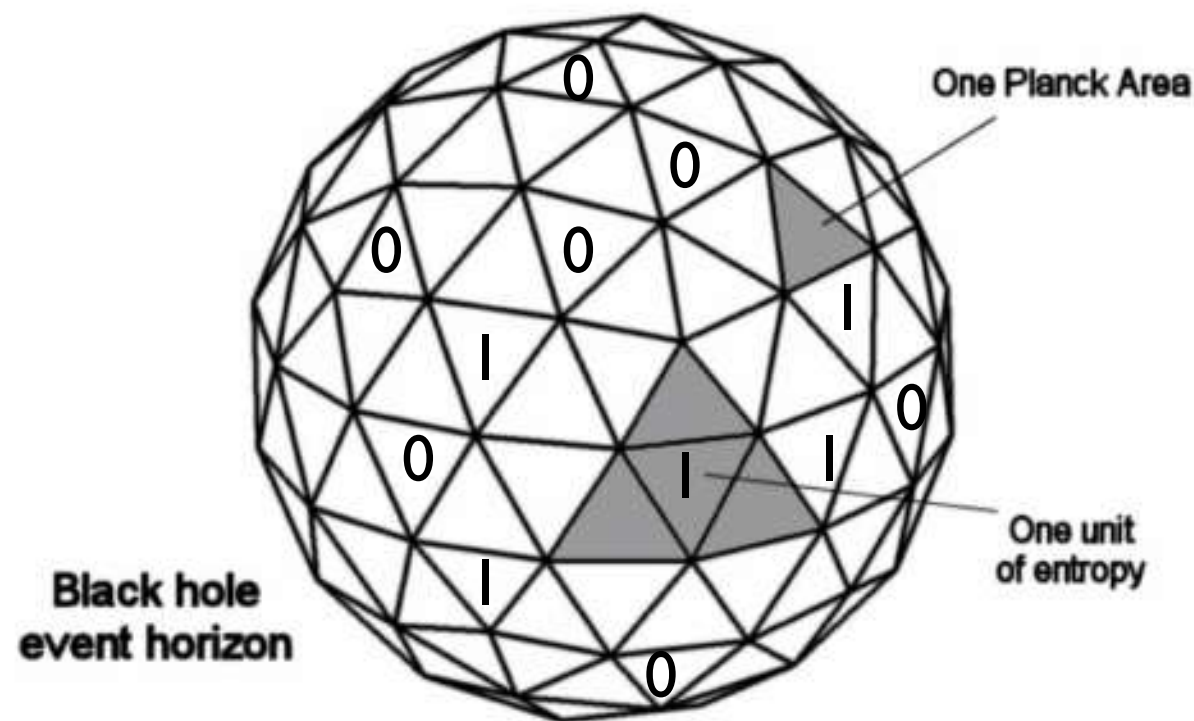
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$$S_{BH} = \frac{c^3}{G\hbar} \frac{A}{4} = \frac{1}{4} \frac{A}{\ell_{Pl}^2}$$

Area of BH  
event horizon

"One qubit per  
4 Planck Areas  
of horizon"





# Black Hole Entropy

Hawking's calculation confirmed this since 1st Law says  $dE = T_H dS$  and using  $T_H$  and  $E = M_{BH} c^2$  gives (in  $k_B=1$  units)

$$S_{BH} = \frac{c^3}{G\hbar} \frac{A}{4} = \frac{1}{4} \frac{A}{\ell_{Pl}^2}$$

$$S_{BH} \simeq 10^{77} \left( \frac{M}{M_\odot} \right)^2$$

Entropy of a *single* solar mass BH roughly that of *all* the stars in the Universe!

Quantum BHs are thermal systems with HUGE number of microstates

$$\#states \simeq \exp(S_{BH})$$

# Black Hole Entropy

Some comments:

- Information content of the universe appears to be hugely dominated by BHs!!!
- We don't understand well these "microstates" of BHs - some progress from String Theory, but not complete...
- $S_{\text{BH}}$  is **not extensive** - it goes with **area** not volume (or mass). Leads to "**holographic principle**" which motivates Maldacena's amazingly powerful holographic duality for String Theory in AdS (negatively-curved) space-times: AKA "AdS/CFT". We don't understand flat-space holography...



# Hawking Evaporation

Now turn to other implications of Hawking Radiation...

Since  $T_H$  increases as  $M$  decreases, BHs can explosively evaporate by runaway Hawking radiation with  $T_H|_{end} \simeq M_{Pl}/8\pi$



Image credit: ortega-pictures / Pixabay.

# Hawking Evaporation

Now turn to other implications of Hawking Radiation...

Since  $T_H$  increases as  $M$  decreases, BHs can explosively evaporate by runaway Hawking radiation with  $T_H|_{end} \simeq M_{Pl}/8\pi$

Hawking evaporation is extraordinary: every particle in nature with mass  $< M_{Pl}/8\pi$  can be produced even if it is completely decoupled from Standard Model (nothing hides from gravity)!

Hawking evaporation is a scan over creation



# Hawking Evaporation

One can produce the dark matter this way, and even start the hot phase of the Big Bang...

## Black Hole Genesis of Dark Matter

Olivier Lennon, John March-Russell, Rudin Petrossian-Byrne  
Hannah Tillim

Rudolf Peierls Centre for Theoretical Physics, University of Oxford, 1 Keble Rd, Oxford OX1 3NP, United Kingdom  
E-mail: [olivier.lennon@physics.ox.ac.uk](mailto:olivier.lennon@physics.ox.ac.uk), [jmr@thphys.ox.ac.uk](mailto:jmr@thphys.ox.ac.uk), [rudin.petrossian-byrne@physics.ox.ac.uk](mailto:rudin.petrossian-byrne@physics.ox.ac.uk), [hannah.tillim@physics.ox.ac.uk](mailto:hannah.tillim@physics.ox.ac.uk)

**Abstract.** We present a purely gravitational infra-red-calculable production mechanism for dark matter (DM). The source of both the DM relic abundance and the hot (SM) plasma is a primordial density of micro black holes (BHs), which evaporate into both the dark and SM sectors. The mechanism has four qualitative regimes depending upon whether the BH evaporation is 'fast' or 'slow' relative to the expansion of the universe.

## Hot Gravitons and Gravitational Waves From Kerr Black Holes in the Early Universe

Dan Hooper<sup>a,b,c,\*</sup>, Gordan Krnjaic<sup>a,†</sup>, John March-Russell<sup>d,‡</sup>, Samuel D. McDermott<sup>a,§</sup>, and Rudin Petrossian-Byrne<sup>d,¶</sup>  
<sup>a</sup>Fermi National Accelerator Laboratory, Theoretical Astrophysics Group  
<sup>b</sup>University of Chicago, Kavli Institute for Cosmological Physics and Department of Astronomy and Astrophysics  
<sup>c</sup>University of Chicago, Rudolf Peierls Centre for Theoretical Physics  
<sup>d</sup>University of Oxford, Rudolf Peierls Centre for Theoretical Physics  
(Dated: April 2, 2020)

Any abundance of black holes that was present in the early universe will evolve as matter, making up an increasingly large fraction of the total energy density as space expands. This motivates us to consider scenarios in which the early universe included an era that was dominated by low-mass ( $M \lesssim 5 \times 10^8$  g) black holes which evaporate prior to primordial nucleosynthesis. In significant regions of parameter space, these black holes will become gravitationally bound within binary systems, and undergo mergers before evaporating. Such mergers result in three potentially observable signatures. First, any black holes that have undergone one or more mergers will possess substantial angular momentum, causing their Hawking evaporation to produce quantities of high-energy gravitons. These products of Hawking evaporation are predicted to constitute a background of hot ( $\sim$  eV-keV) gravitons today, with an energy density corresponding to  $\Delta N_{\text{eff}} \sim 0.01 - 0.03$ . Second, these mergers will produce a stochastic background of high-frequency gravitational waves. And third, the energy density of these gravitational waves can be as large as  $\Delta N_{\text{eff}} \sim 0.3$ , depending on

*could be observable signatures of Hawking radiation hiding in DM and gravitational waves (would have won Stephen Hawking his deserved Nobel....)*

# Hawking Evaporation

But there's more...!

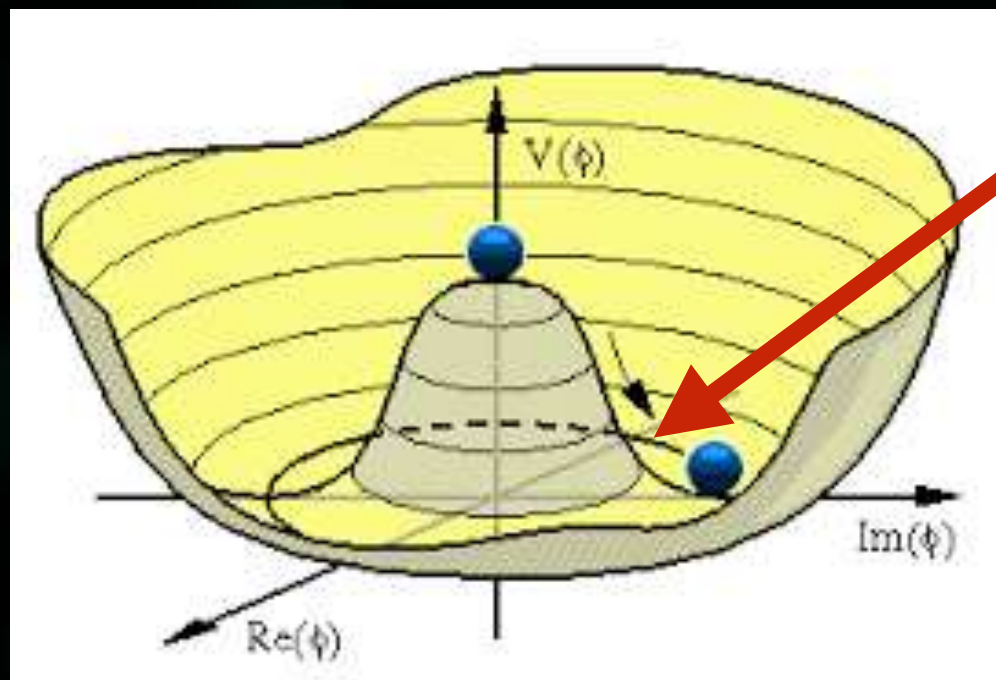




# Impossibility of Exact Global Symmetries

Hawking evaporation of BHs implies remarkable consequence that, **in any 4d theory including gravity, non-space-time GSs are always violated (thus approximate at best)**

So, eg, impossible to have *exactly* degenerate, physically distinct states resulting from such internal symmetries



can't be **exactly** equi-potential

# Impossibility of Exact Global Symmetries

By Emmy Noether's great theorem this means in turn that all the associated would-be-conserved "global charges" are not conserved

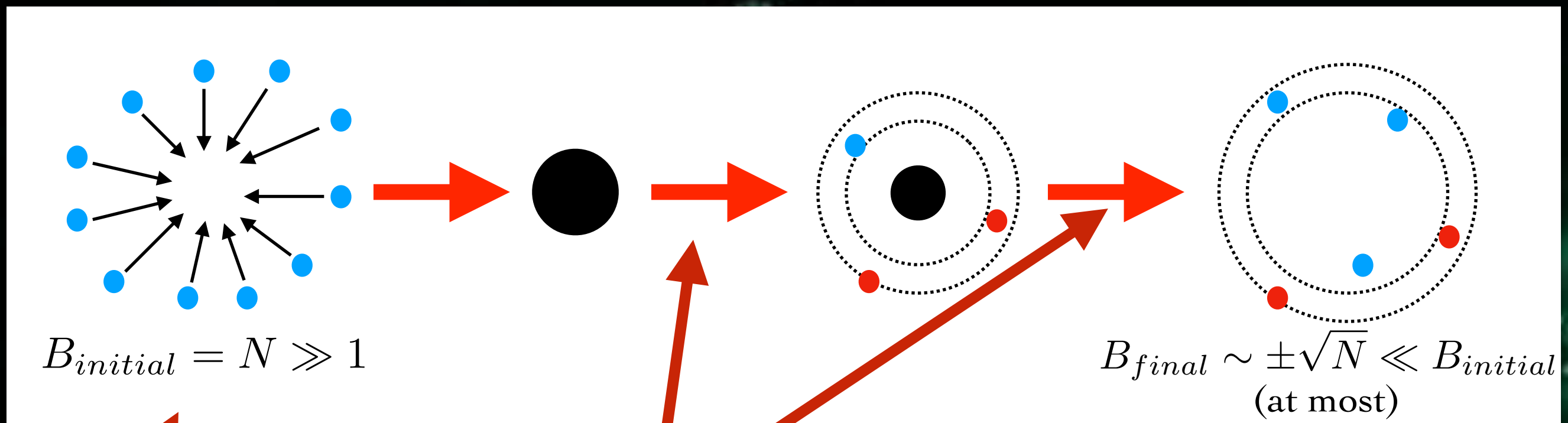
For example conservation of baryon number associated with internal  $U(1)_B$  global symmetry **must** be violated





# Impossibility of Exact Global Symmetries

Why is this true? The basic idea (goes back to Zeldovich):



BH formed by large initial number,  $N$ , of neutrons

Hawking radiation is **independent** of whether baryon or anti-baryon emitted (*only* cares about mass, spin, & charge)

(Note for experts: here, for simplicity, I am taking the BH to fully evaporate - no remnants! - but argument can be sharpened to exclude this case)

# Impossibility of Exact Global Symmetries

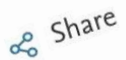
We don't yet fully understand exactly what is the **form and size of the global-symmetry-violating interactions** that must be induced by quantum gravitational effects, but progress is being made!

## Towards a Swampland Global Symmetry Conjecture using weak gravity

Tristan Daus<sup>a</sup>, Arthur Hebecker<sup>a</sup>, Sascha Leonhardt<sup>a</sup>, John March-Russell<sup>b</sup>

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### Abstract

It is widely believed and in part established that exact global symmetries are inconsistent with quantum gravity. One then expects that approximate global symmetries can be quantitatively constrained by quantum gravity or swampland arguments. We provide such a bound for an important class of global symmetries arising from a gauged  $U(1)$  with the vector made massive via a Higgs mechanism. Those arising from a gauged  $U(1)$  with the vector made massive via a Higgs mechanism. The latter necessarily couples to instantons, and their action is constrained, using both the electric and magnetic version of the axion-gravity conjecture, in terms of the cutoff of the theory. As a result,

## Global symmetry, Euclidean gravity, and the black hole information problem

Daniel Harlow<sup>a</sup> and Edgar Shaghoulian<sup>b</sup>

<sup>a</sup>Center for Theoretical Physics  
Massachusetts Institute of Technology, Cambridge, MA 02139, USA  
<sup>b</sup>David Rittenhouse Laboratory, University of Pennsylvania  
209 S.33rd Street, Philadelphia PA, 19104, USA  
E-mail: [harlow@mit.edu](mailto:harlow@mit.edu), [eshag@sas.upenn.edu](mailto:eshag@sas.upenn.edu)

ABSTRACT: In this paper we argue for a close connection between the non-existence of global symmetries in quantum gravity and a unitary resolution of the black hole information problem. In particular we show how a recent argument based on the Page curve of an eternal black hole can be used to generalize the no-hair conjecture to a more general context.

### Approximate Symmetries and Gravity

Sylvain Fichet<sup>a,\*</sup>, Prashant Saraswat<sup>a,†</sup>

<sup>a</sup> Walter Burke Institute for Theoretical Physics, California Institute of Technology,  
Pasadena, CA 91125, California, USA

<sup>†</sup> ICTP South American Institute for Fundamental Research & IFT-UNESP,  
R. Dr. Bento Teobaldo Ferraz 271, São Paulo, Brazil 2019-032

### Abstract

There are strong reasons to believe that global symmetries of quantum theories cannot be exact in the presence of gravity. While this has been argued at the qualitative level, establishing a quantitative statement is more challenging. In this work we take new steps



# Final Words

One issue I haven't spoken about is **if QM & BHs are in fact inconsistent?** This was view of Stephen Hawking for many years (though not last 5), and might still be Roger's

Hawking evaporation of BHs appears to allow QM "pure states" to evolve to "mixed states" - if so this would violate a fundamental axiom of QM and imply information loss

**But in last 2 years there has been major progress and a consistent calculational framework is emerging**

# Quantum BHs live..!

## Entanglement wedge reconstruction and the information paradox

Geoffrey Penington<sup>1</sup>

<sup>1</sup>Stanford Institute for Theoretical Physics, Stanford University, Str

### Simple holographic models of black hole evaporation

Chris Akers, Netta Engelhardt, and Daniel Harlow  
Center for Theoretical Physics  
Massachusetts Institute of Technology, Cambridge, MA 02139, USA  
E-mail: [cakers@mit.edu](mailto:cakers@mit.edu), [engeln@mit.edu](mailto:engeln@mit.edu), [harlow@mit.edu](mailto:harlow@mit.edu)

ABSTRACT: Several recent papers have shown a close relationship between entanglement wedge reconstruction and the unitarity of black hole evaporation. The analysis of these papers however has a rather puzzling feature: the entanglement wedge is not essentially those regions which are causally connected to the boundary.

### The entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole

Ahmed Almheiri,<sup>a</sup> Netta Engelhardt,<sup>b,c</sup> Donald Marolf,<sup>d</sup> Henry Maxfield,<sup>e</sup>  
<sup>a</sup>Institute for Advanced Study, Princeton, NJ 08540, USA  
<sup>b</sup>Department of Physics, Princeton University, Princeton, NJ 08544, USA  
<sup>c</sup>Gravity Initiative, Princeton University, Princeton NJ 08544, USA  
<sup>d</sup>Physics Department, University of California, Santa Barbara, CA 93106  
E-mail: [almheiri@ias.edu](mailto:almheiri@ias.edu), [engelhardt@princeton.edu](mailto:engelhardt@princeton.edu),  
[marolf@ucsb.edu](mailto:marolf@ucsb.edu), [hmaxfield@physics.ucsb.edu](mailto:hmaxfield@physics.ucsb.edu)

ABSTRACT: Bulk quantum fields are often said to contribute to the entropy  $\frac{A}{4G_N} + S_{\text{bulk}}$  only at  $O(1)$ . Nonetheless, in the context of evaporating black holes,  $O(1/G_N)$  gradients in  $S_{\text{bulk}}$  can arise due to large boosts, introducing a new extremal surface far from any classical extremal surface. We examine the bulk quantum effects on quantum extremal surfaces (QESs) and the resulting entanglement wedge in a simple two-boundary 2d bulk system defined by Jackiw-Teitelboim gravity.

#### Abstract

When absorbing boundary conditions are used to evaporate black holes, we show that there is a phase transition in the location of the entanglement surface, at precisely the Page time. The new RT surface is located at an inflection point of the Page curve.

### The Page curve of Hawking radiation from semiclassical geometry

Ahmed Almheiri,<sup>a</sup> Raghu Mahajan,<sup>a,b</sup> Juan Maldacena<sup>a</sup> and Ying Zhao<sup>a</sup>  
<sup>a</sup>School of Natural Sciences, Institute for Advanced Study,  
1 Einstein Dr, Princeton, NJ 08540, U.S.A.  
<sup>b</sup>Department of Physics, Princeton University,  
Jadwin Hall, Washington Road, Princeton, NJ 08544, U.S.A.  
E-mail: [almheiri@ias.edu](mailto:almheiri@ias.edu), [raghu.m@princeton.edu](mailto:raghu.m@princeton.edu), [malda@ias.edu](mailto:malda@ias.edu),  
[zhaoying@ias.edu](mailto:zhaoying@ias.edu)

ABSTRACT: We consider a gravity theory coupled to matter, where the matter has a higher-dimensional holographic dual. In such a theory, finding quantum extremal surfaces becomes equivalent to finding the RT/HRT surfaces in the higher-dimensional theory. Using this, we compute the entropy of Hawking radiation and argue that it follows the Page curve, as suggested by recent computations of the entropy and entanglement wedges for old black holes. The higher-dimensional geometry connects the radiation to the black hole interior in the spirit of ER=EPR. The black hole interior then becomes part of the entanglement wedge.

### Replica wormholes and the black hole interior

Geoff Penington, Stephen H. Shenker, Douglas Stanford, and Zhenbin Yang  
Stanford Institute for Theoretical Physics,  
Stanford University, Stanford, CA 94305

#### Abstract

Recent work has shown how to obtain the Page curve for black hole evaporation using the replica trick. This has been justified using the replica trick, but the different replicas are not connected in the bulk geometry.

### Cool horizons for entangled black holes

Juan Maldacena<sup>a</sup> and Leonard Susskind<sup>b</sup>  
<sup>a</sup>Institute for Advanced Study,  
Princeton University, Princeton, NJ 08540, U.S.A.  
<sup>b</sup>Department of Physics, Princeton University,  
Jadwin Hall, Washington Road, Princeton, NJ 08544, U.S.A.

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JHEP03(2020)149

172v2 [hep-th] 5 Dec 2019

162v3 [hep-th] 4 Nov 2019

Aug 2020

177v2 [hep-th] 15 Apr 2020



# Solving BH Information Paradox

Basic idea: Hawking's calculation of thermal density matrix was never exact

$$\rho_R = \rho^{thermal} (1 + \text{loop effects}) + \mathcal{O}\left(e^{-A/\ell_{Pl}^2}\right)$$

All these usual local QFT terms **don't** help

New tiny, non-perturbative and **non-local**, effects

$$S_{rad} = - \sum_{\text{states } i} \lambda_i \log \lambda_i$$

$(e^{A/\ell_{Pl}^2} \text{ terms} \times e^{-A/\ell_{Pl}^2} \text{ corrections}) \rightarrow \mathcal{O}(1) \text{ modification}$

# Quantum BHs

QM of BHs is even more remarkable and is guiding us towards a better understanding of quantum gravity (& information!)

"Hydrogen Atom of the 21st Century"