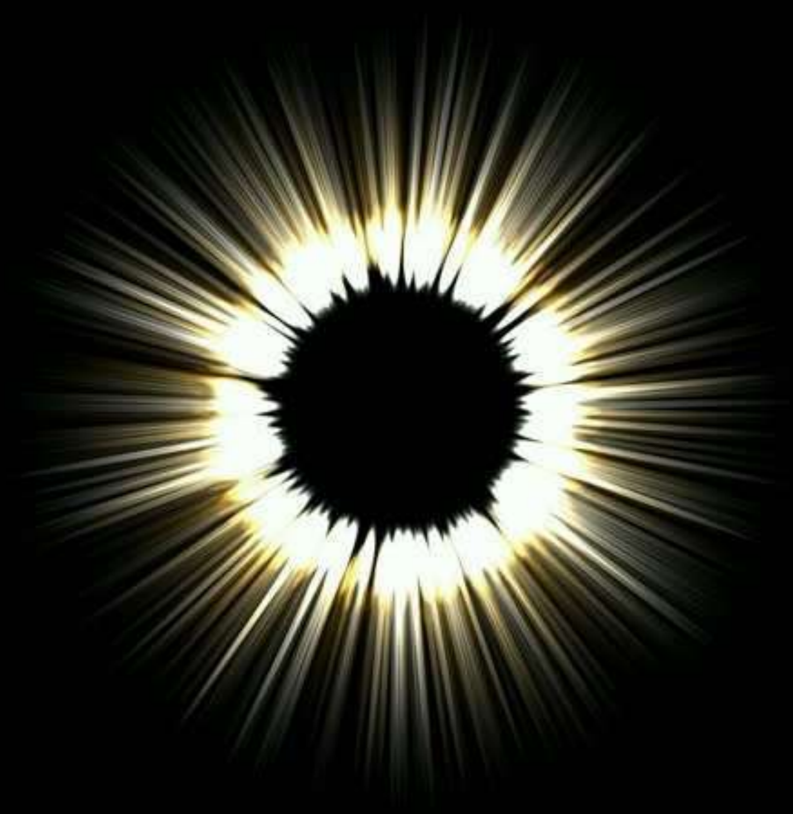
Classical and Quantum Black Holes

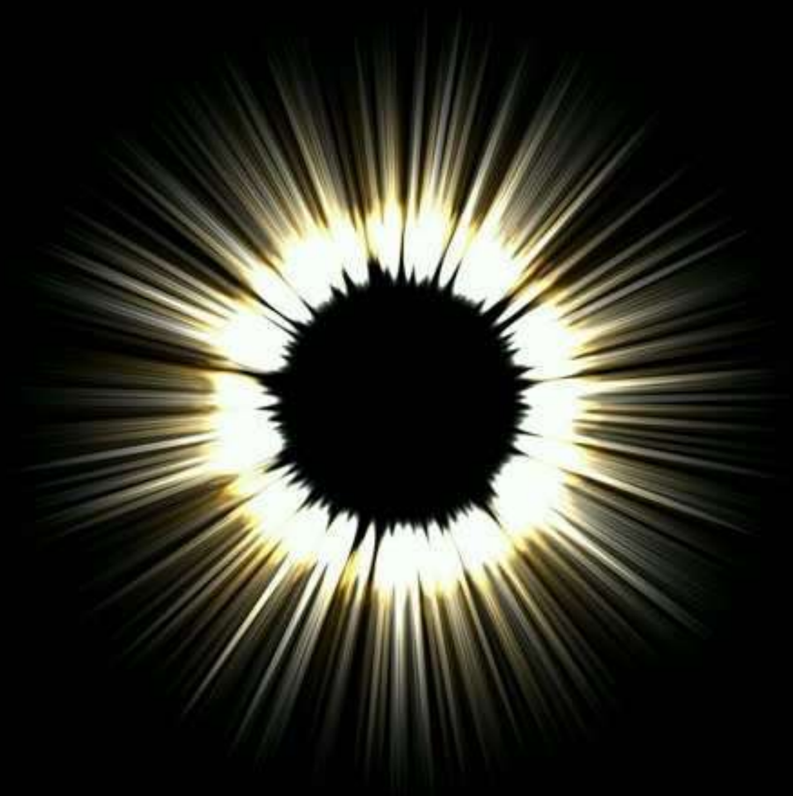
John March-Russell



Morning of Theoretical Physics, 9th Jan 2021

BHs are among the most awe-inspiring objects in the Universe

They imply the end of space-time as we know it, and *appear* to imply deep paradoxes when we try to meld with quantum mechanics

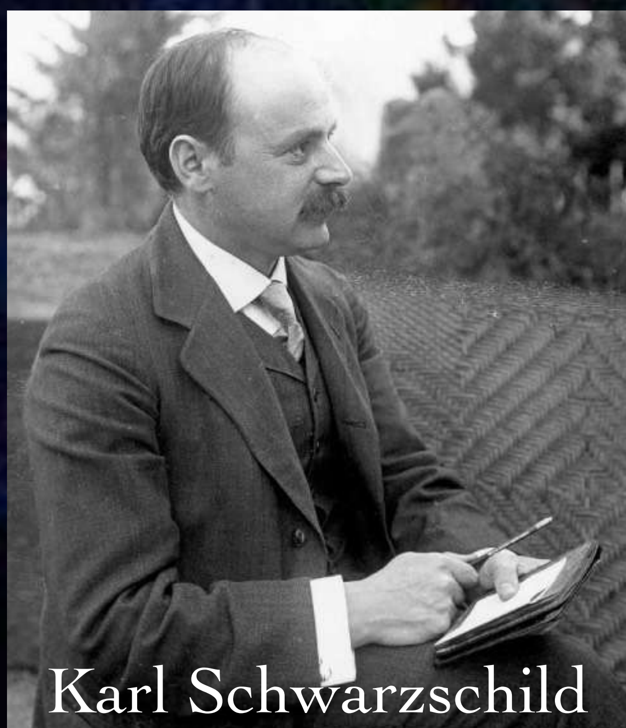


There has been significant progress, both observational *and* theoretical in BH physics in the past few years!

"Hydrogen Atom of the 21st Century"

Classical BHs

We now know that astrophysical BHs exist and behave, to a very good approximation, the way that Einstein's classical General Relativity says they should (at least the exterior)


Karl Schwarzschild

Schwarzschild wrote down first & simplest BH solution while fighting & dying in WW1

Schwarzschild radius of BH event horizon

$$r_s = \frac{2GM}{c^2}$$

BH mass

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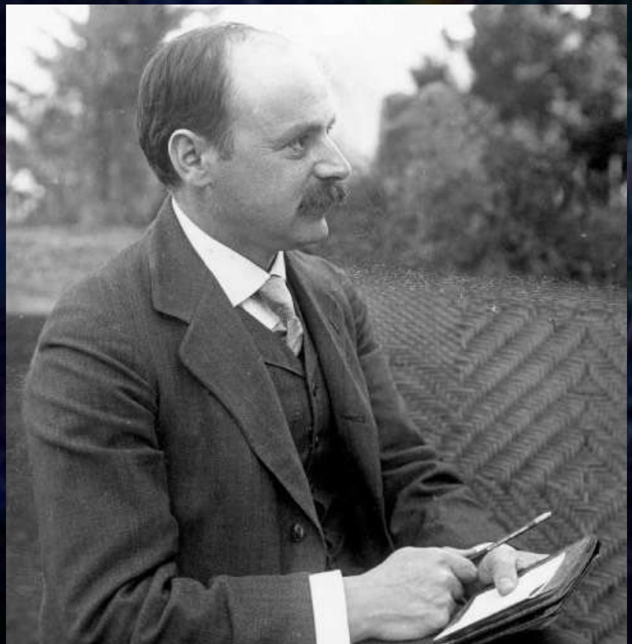

Karl Schwarzschild

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Schwarzschild radius of BH event horizon

$$r_s = \frac{2GM}{c^2}$$

Solar-mass BH: $r_s = 2.93$ km

Galactic centre BHs of $10^{6\text{-}9}$ solar masses: $r_s \sim 10^{6\text{-}9}$ km

Hypothetical BH of Earth mass: $r_s = 0.84$ cm

Classical BHs

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

according to static external observer, time dilation as horizon at r_s approached

radial length contraction as horizon approached

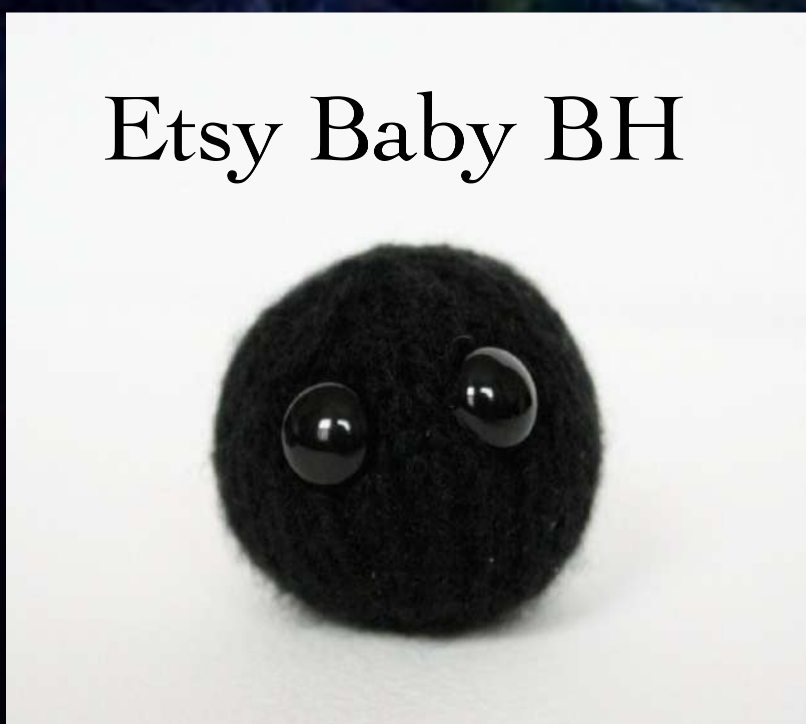
space-time distance between events

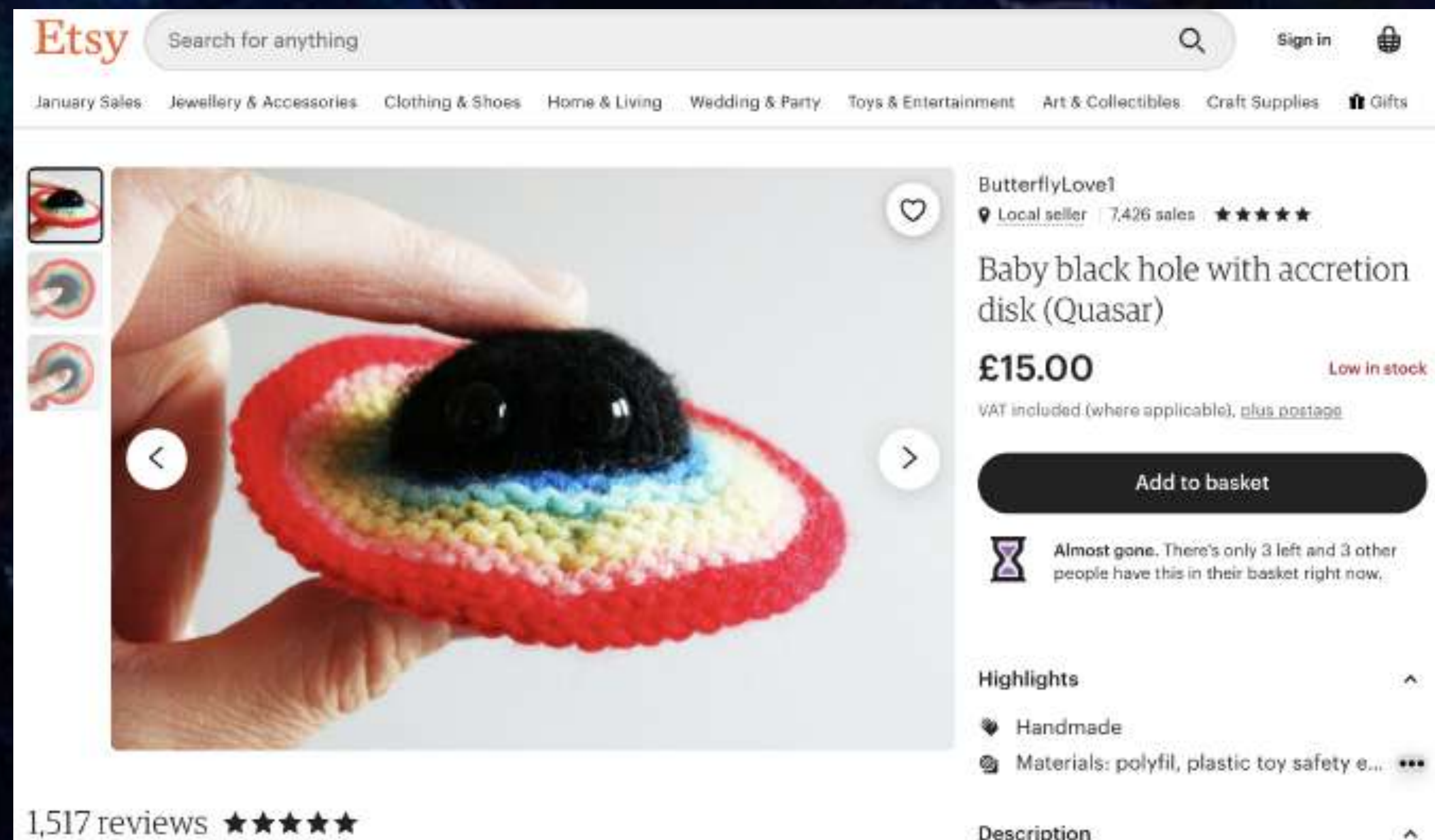
Modern understanding of classical BHs due to Wheeler (who coined the name), Kruskal, Kerr, Thorne, Hawking, and especially Penrose

Roger Penrose

Classical BHs

What do these awe-inspiring objects look like?





Classical BHs

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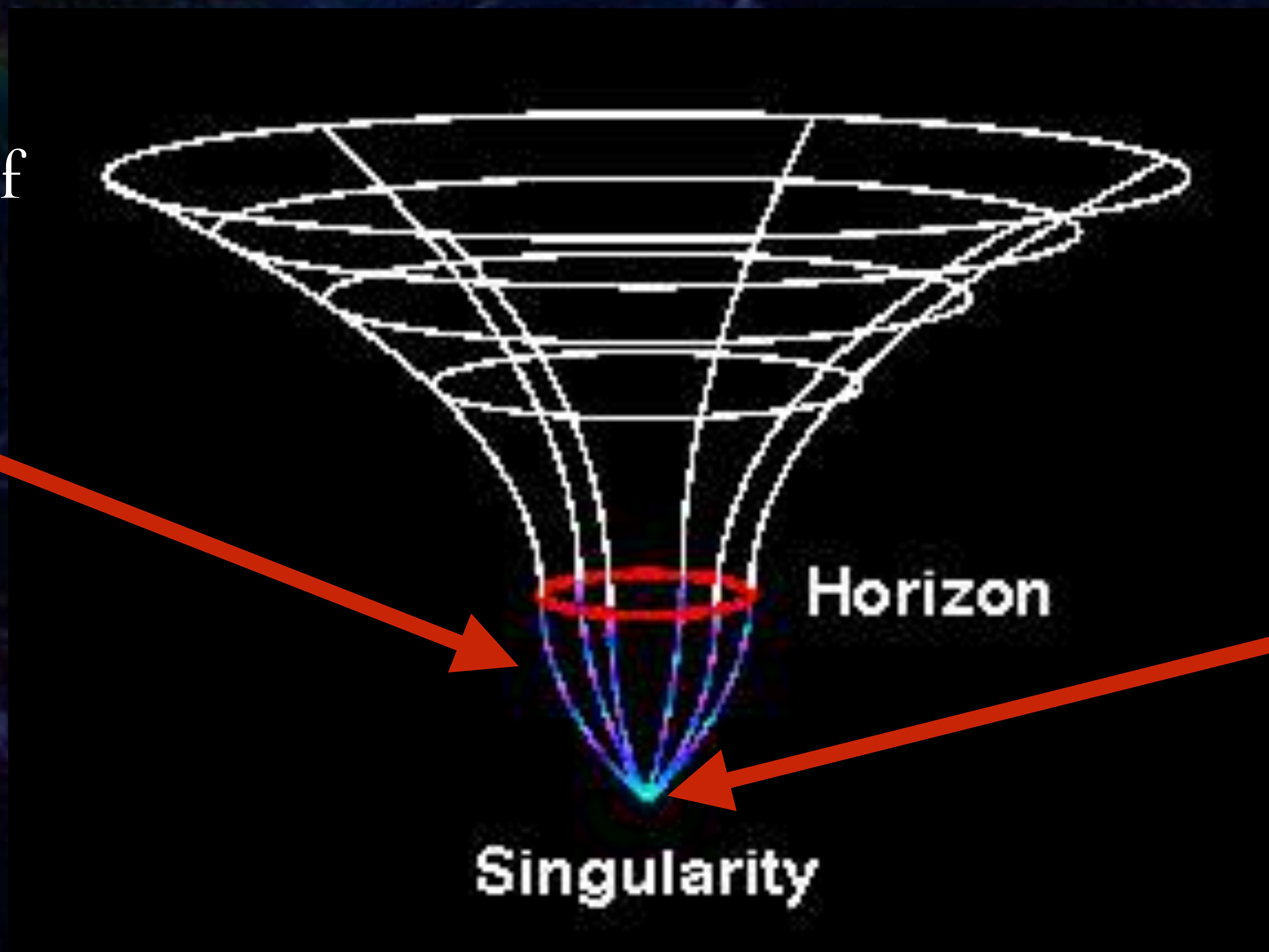
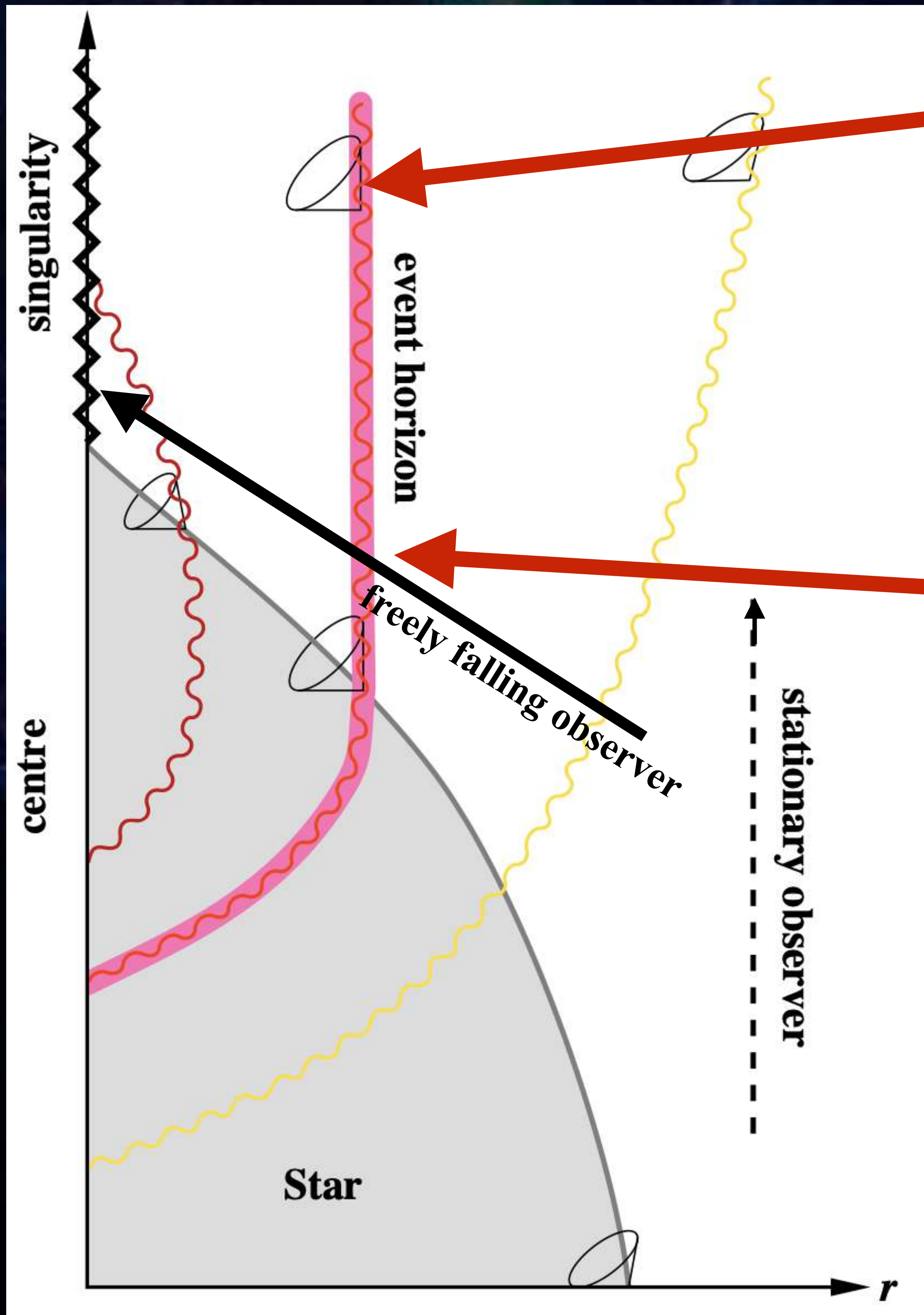
space-time itself is not static inside horizon ("r becomes a temporal coordinate")



Credit: Andrew Hamilton (JILA)

space-time "ends" at singularity, but classical solution (and GR itself) not reliable here

Classical BHs



Adapted from: Droz, Israel, Morsink

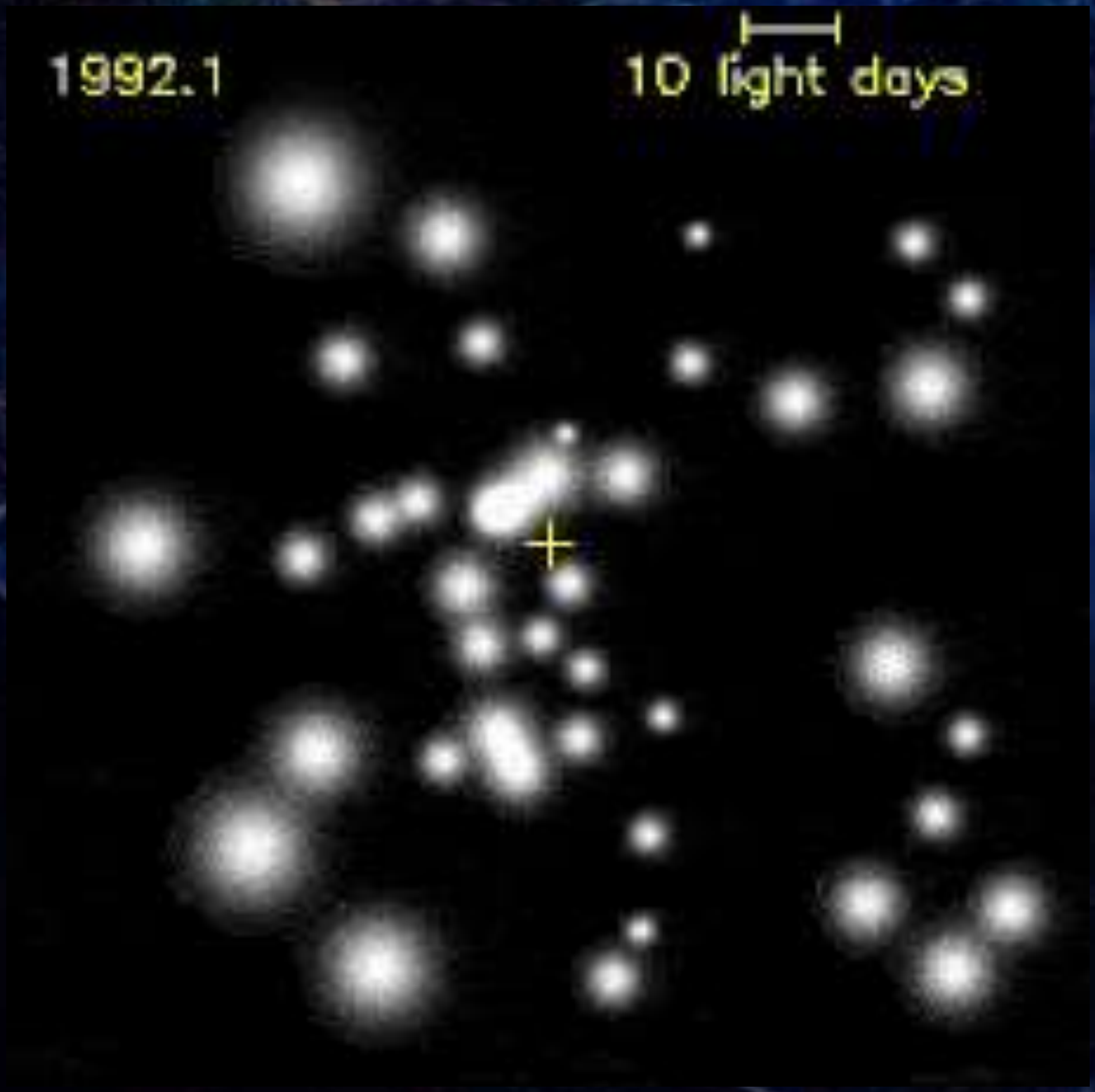
At $r=r_s$ horizon the "light cones" tip so far over that avoidance of r=0 singularity is impossible ("forward in time" becomes "radial inward")

BUT, a freely falling observer feels nothing special as she falls through the horizon (just usual, finite, tidal forces). To such an observer the region around the horizon is just like normal, empty Minkowski space!

The apparent singular behaviour of the Schwarzschild metric at horizon is just a bad choice of coordinates like longitude at N or S poles

Classical BHs

We know to a high degree of confidence that supermassive astrophysical BHs exist in galactic centres

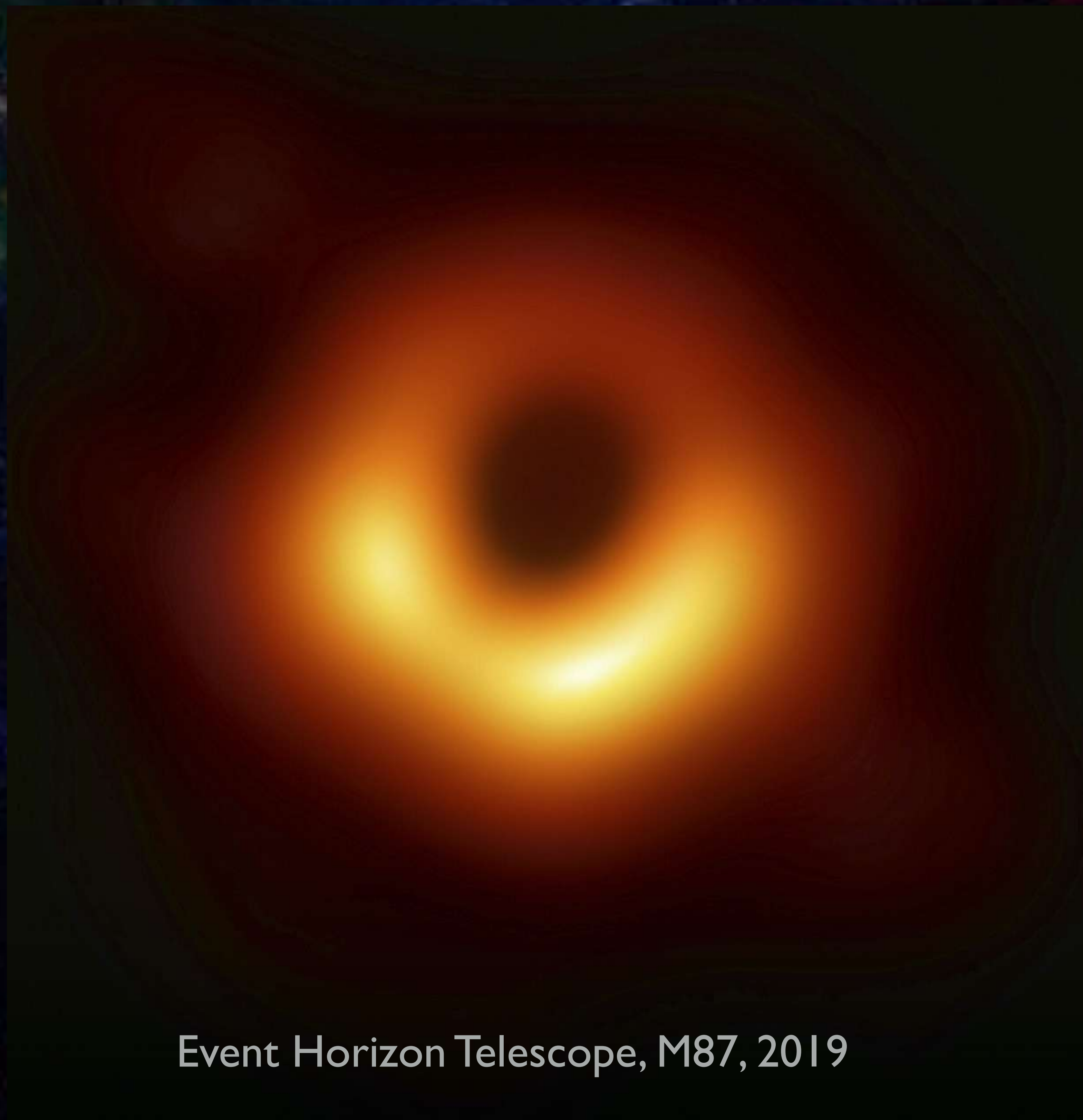


European Southern Observatory

Nobel Prize 2020 shared with Roger Penrose

Classical BHs

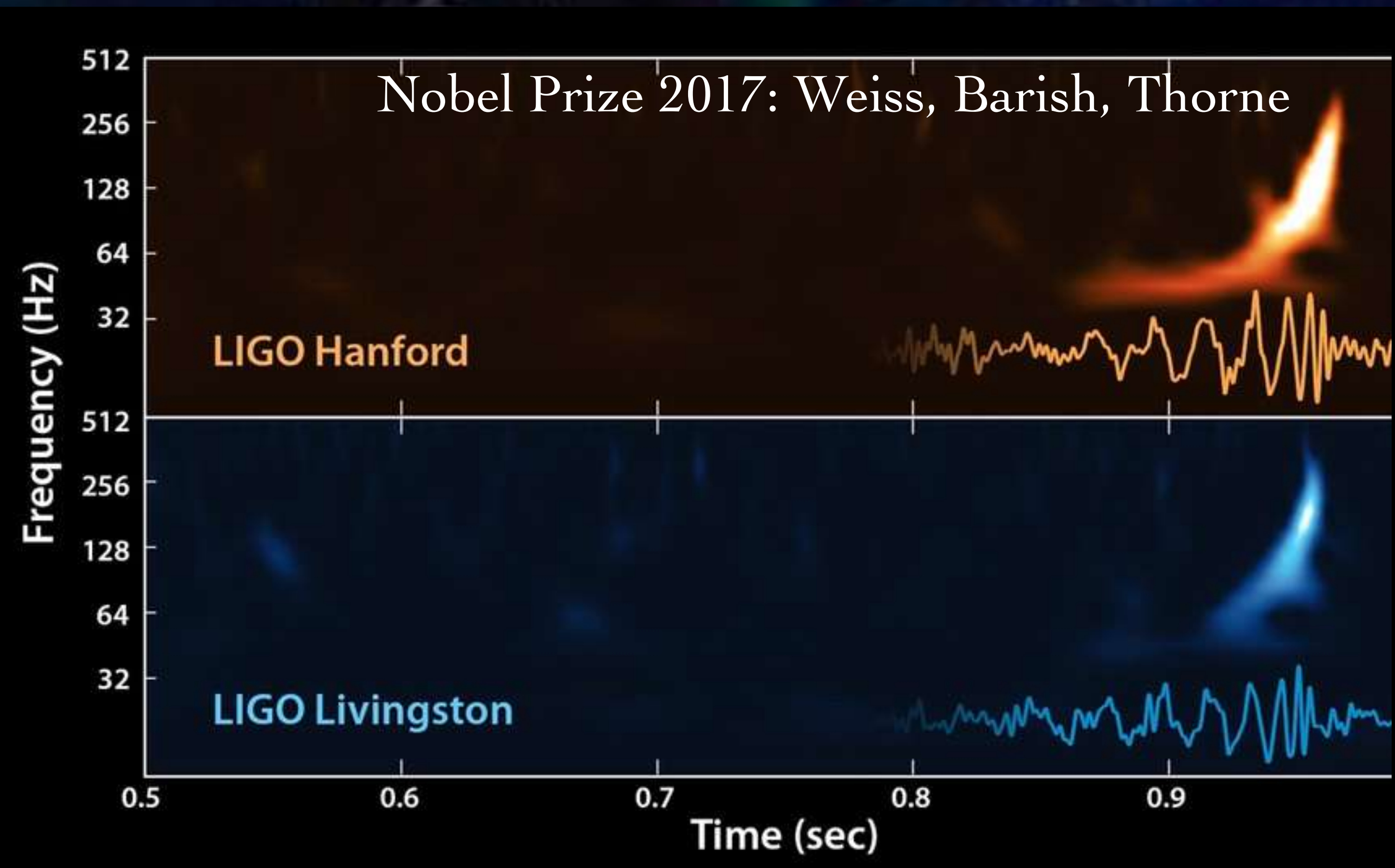
Recently we have a direct image of a supermassive BH

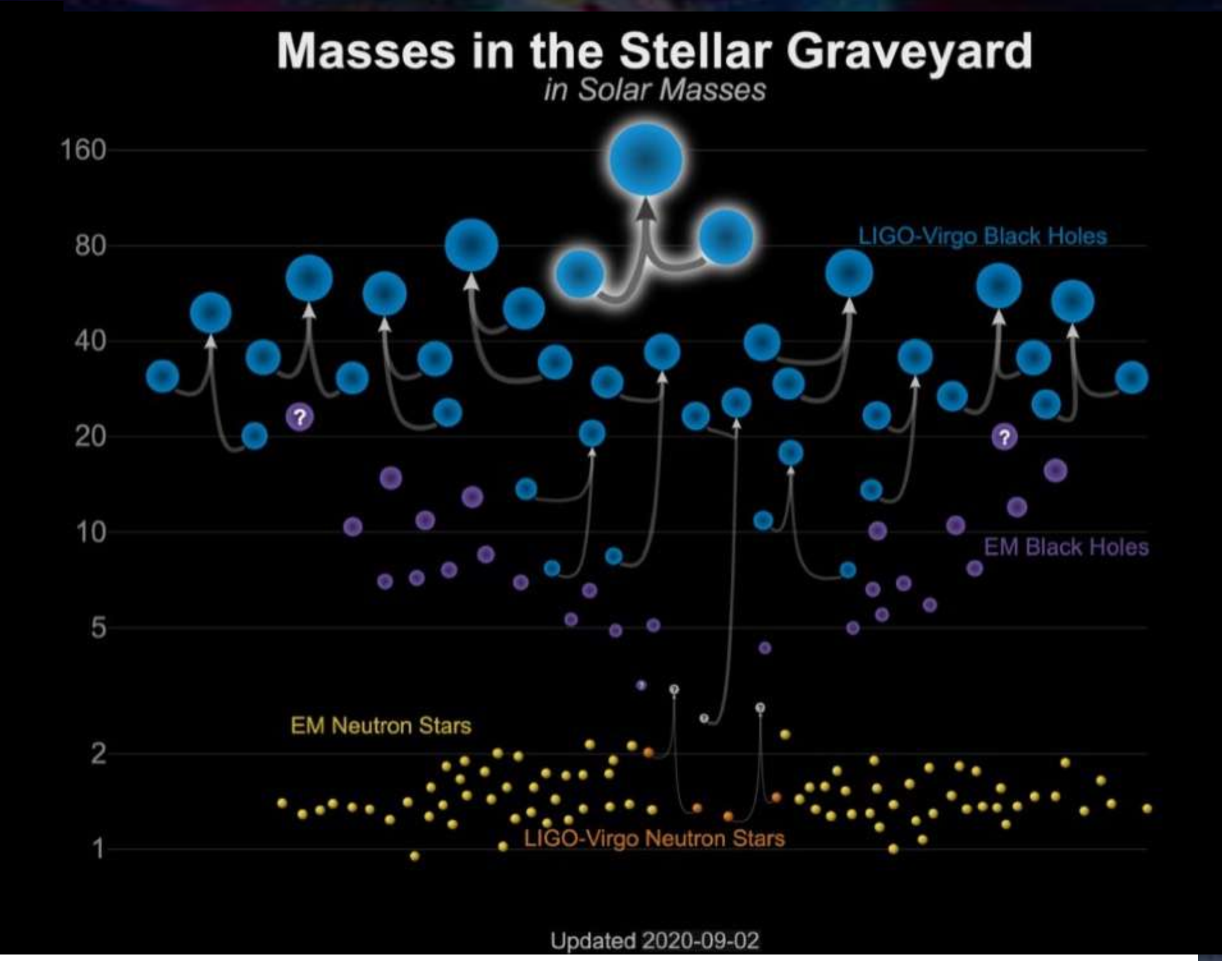


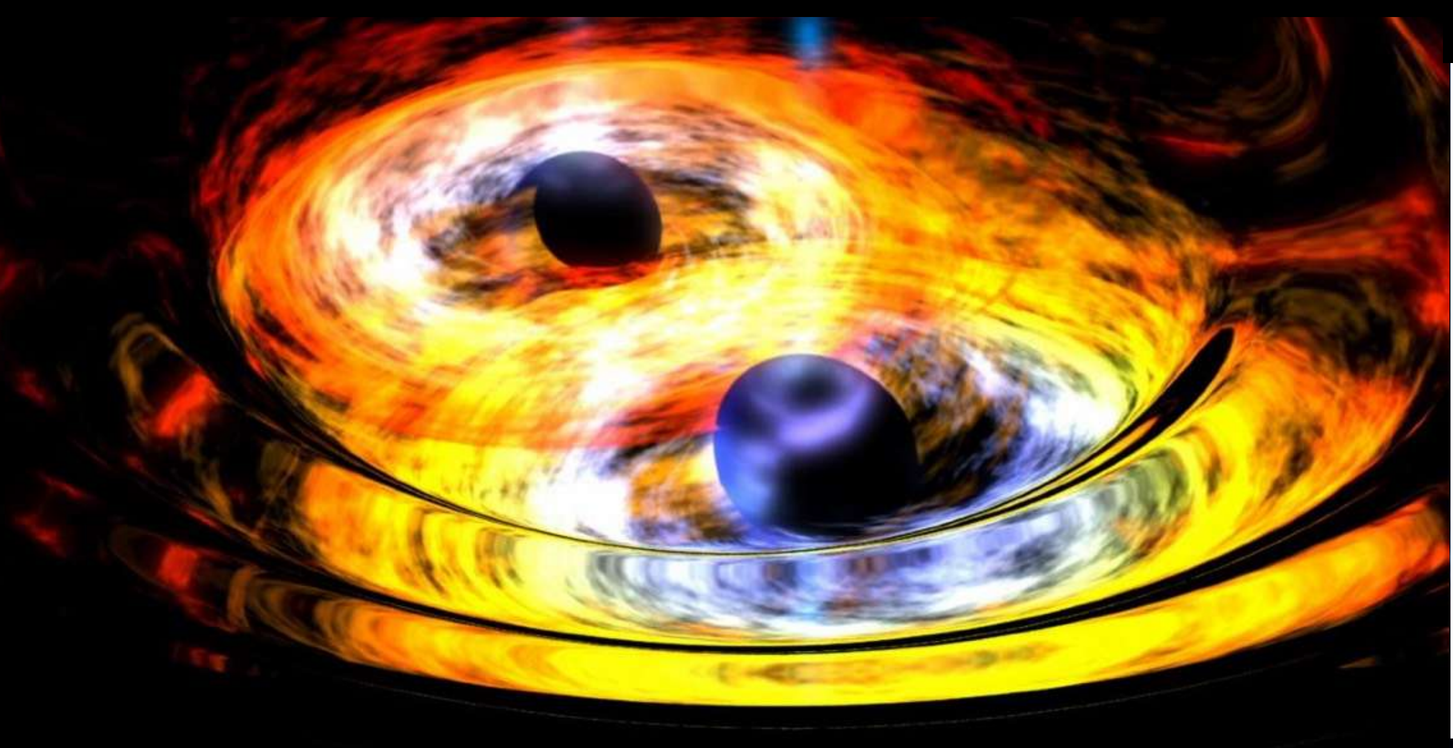
Event Horizon Telescope, M87, 2019

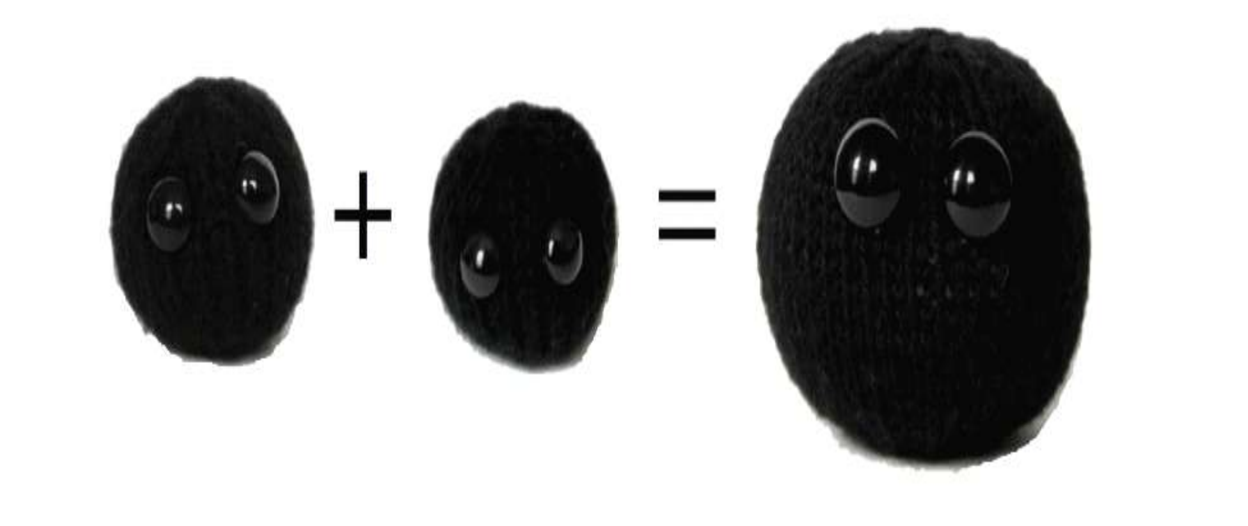
Classical BHs

Also detected gravitational waves from mergers of smaller BHs









Credit: Chris Berry & Dawn Finney

Quantum BHs

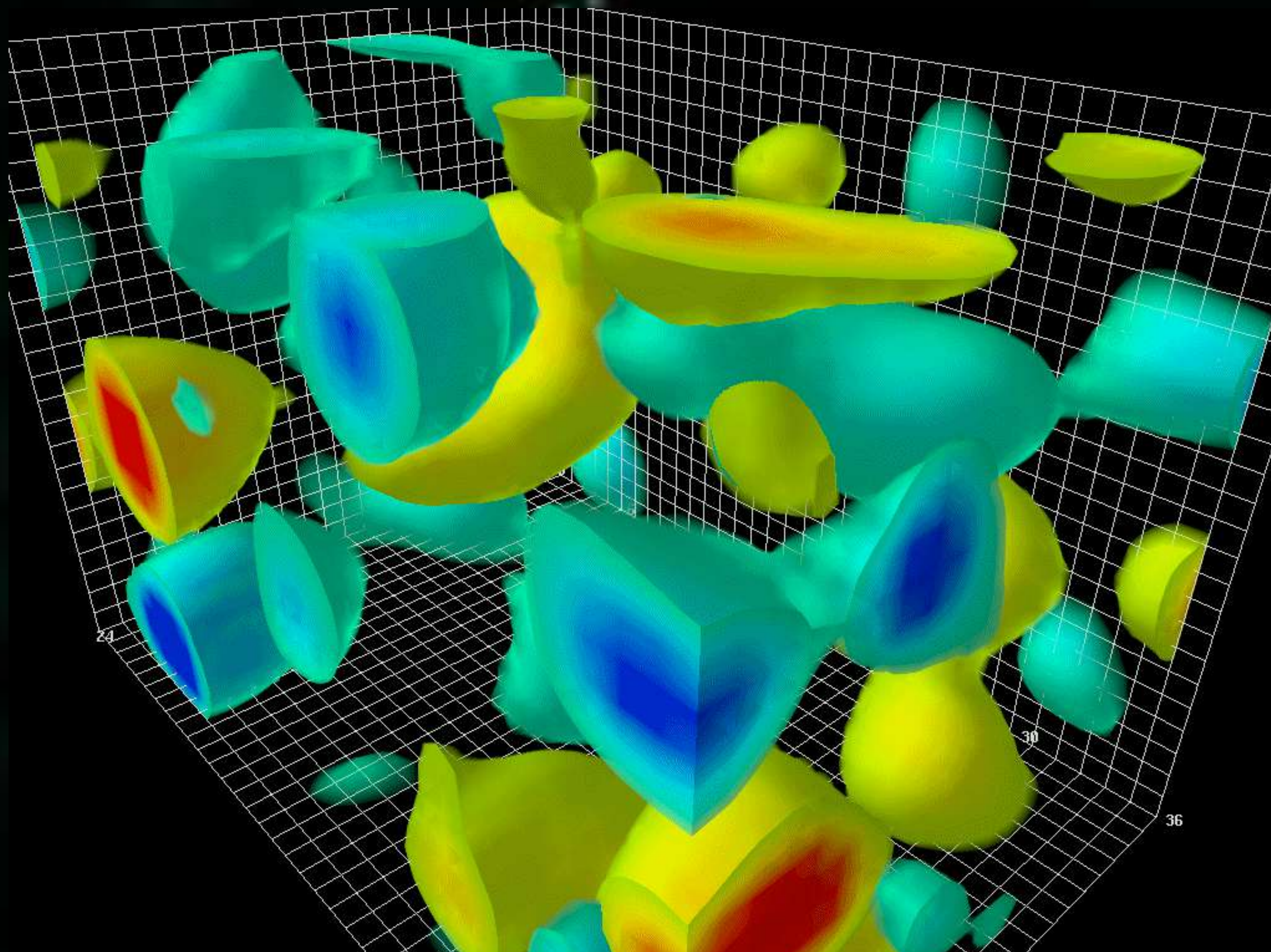
The classical theory of BHs is extraordinarily rich, but the world is really quantum mechanical

The QM of BHs is even more remarkable and leads to apparent deep paradoxes whose resolution is guiding us towards a better understanding of aspects of quantum gravity

Significant progress in last few years....!

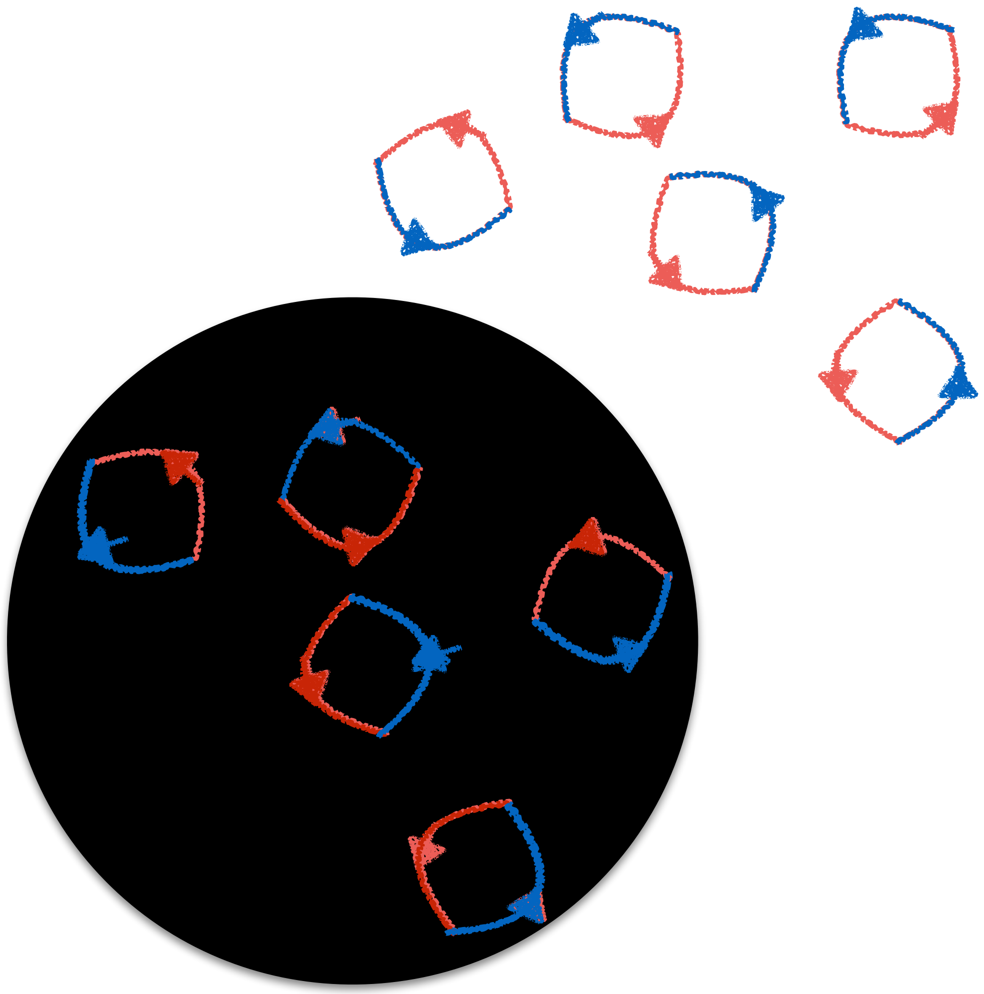
Hawking Radiation

In relativistic QM empty space (the "vacuum") is full of quantum fluctuations of all fields and particles



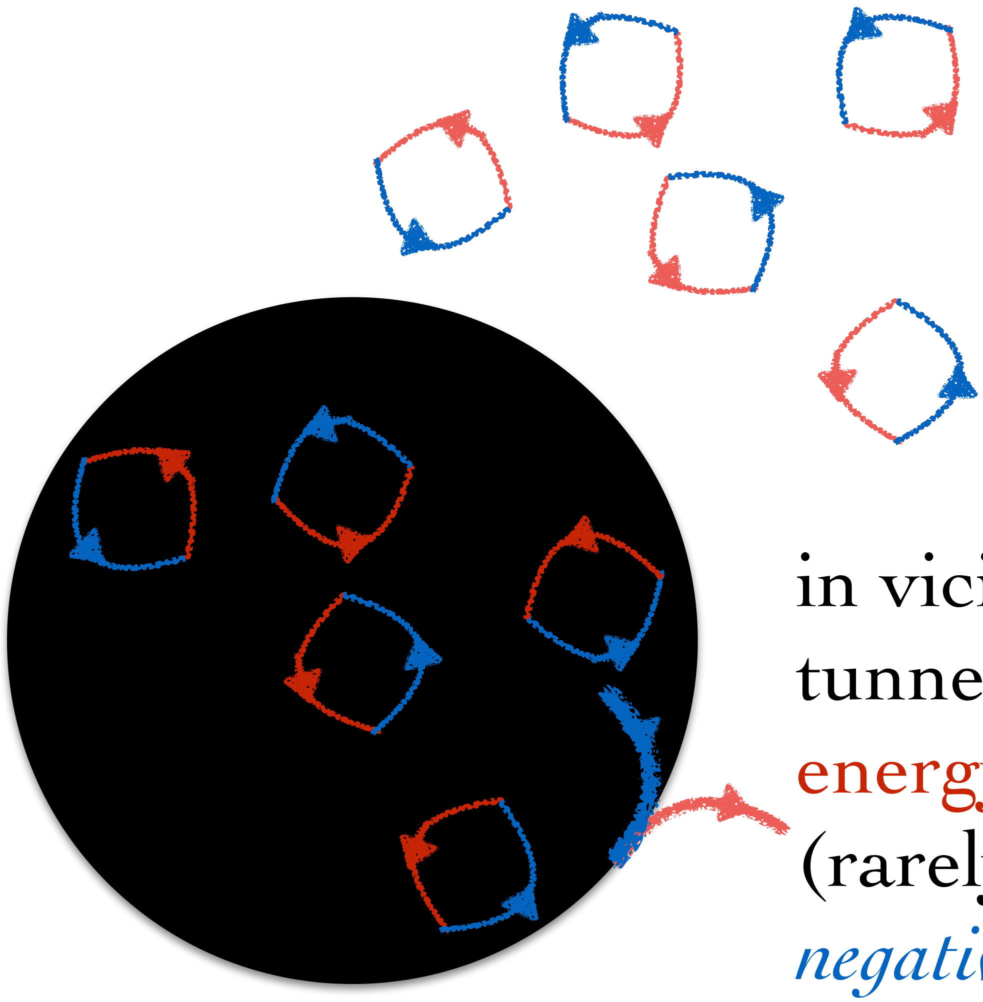
Precision numerical evaluation of vacuum of QCD on $(5\text{fm})^3$ lattice (only one realisation shown!)

Hawking Radiation



positive & negative energy 'virtual' particle (and anti-particle) fluctuations are everywhere being created and annihilating

Hawking Radiation

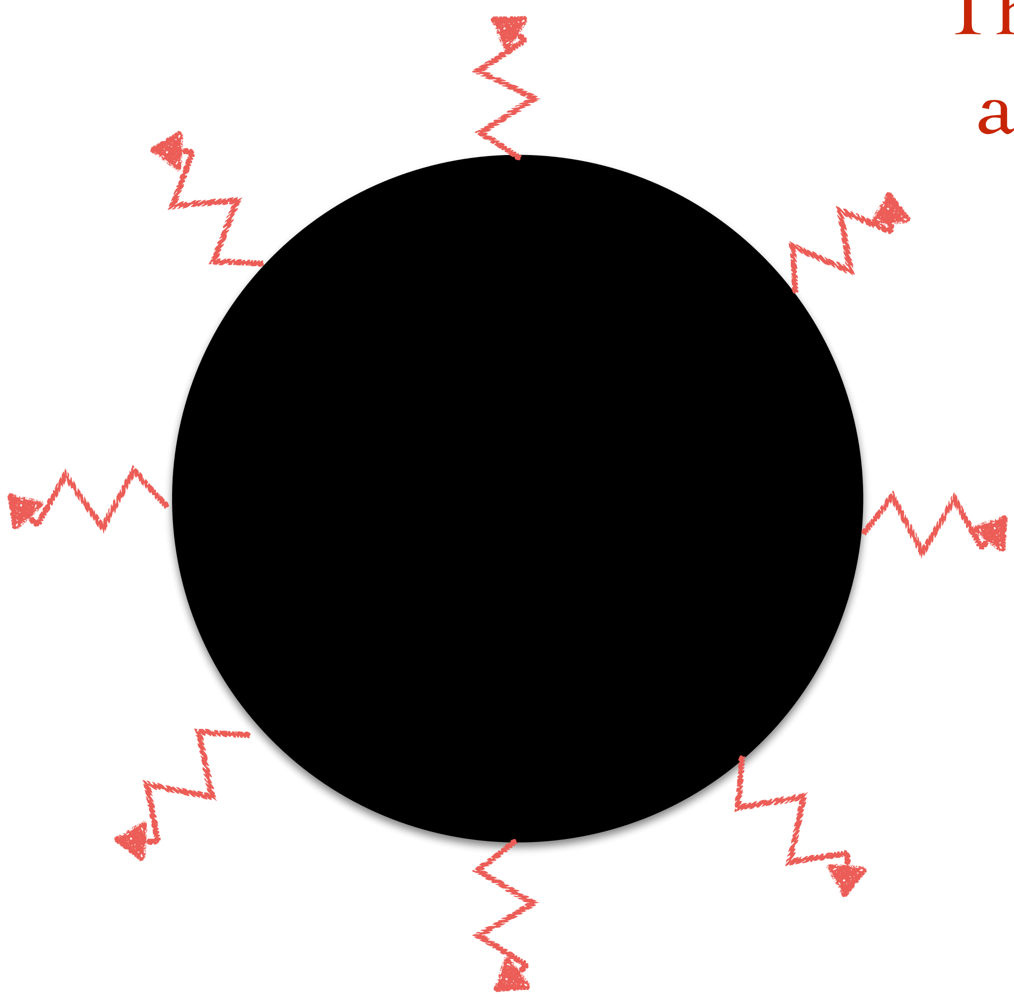


in vicinity of horizon a QM tunnelling process allows a positive energy particle or anti-particle to (rarely) escape to infinity while a *negative energy* (from perspective of distant observer) particle or anti-particle is captured by BH, reducing BH mass

(Note for experts: This tunnelling picture of Hawking radiation can be made *precise* and *gives correct reduction in mass of BH* - see papers of Wilczek etal)

Hawking Radiation

Physical BHs are never truly black!
They produce thermal radiation in
all kinematically allowed modes.



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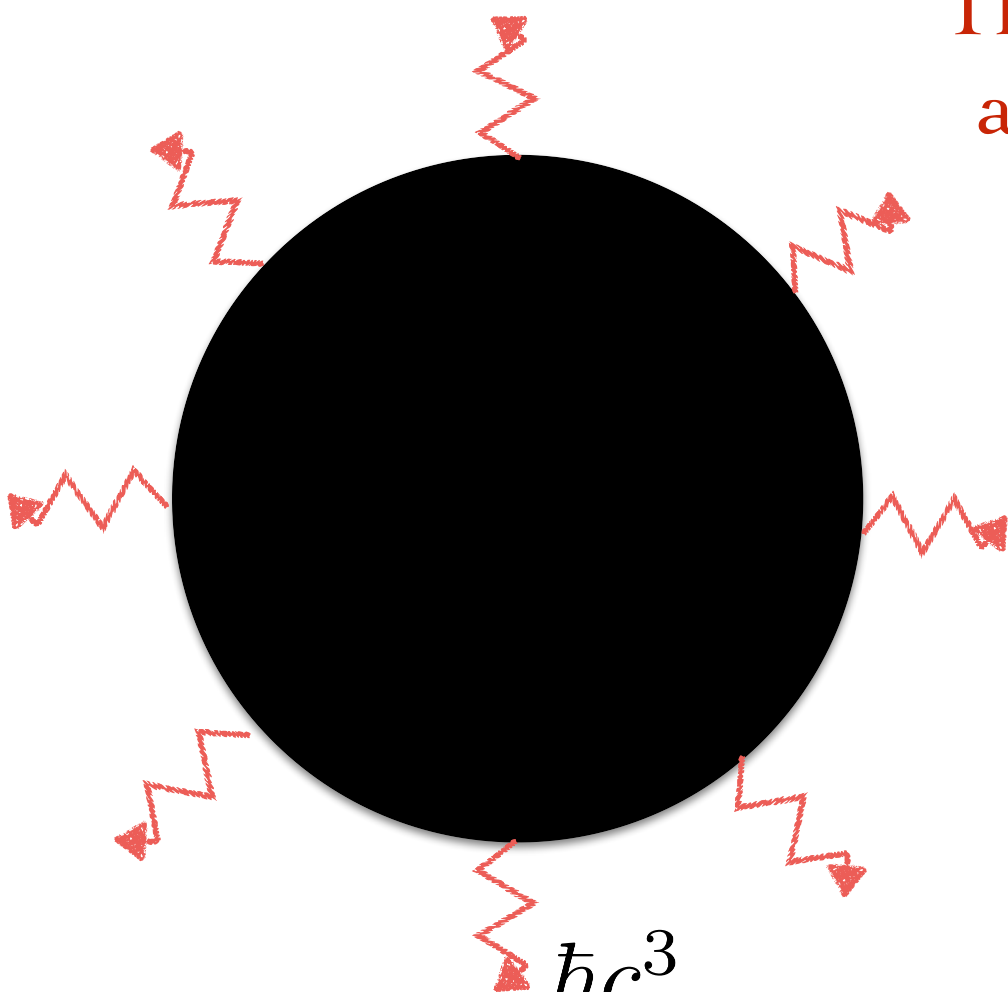

Amazing end result is BH is a thermal system with temperature (as seen by a static observer far away) T_H

$$T_{\text{Hawking}} = \frac{\hbar c^3}{8\pi G_N k_B M} \simeq \frac{1.2\times 10^{23} K}{M(\text{in kg})} \simeq 10^{-7} K \frac{M_\odot}{M}$$

Observed astrophysical BHs are very cold but as mass of BH gets smaller temperature goes up!

Black Hole Entropy

BH Hawking radiation has never been observed, but we now know *many* ways it can be theoretically derived & it solves some otherwise troubling aspects of classical BHs

One of these is associated with the Second Law of Thermodynamics, $\Delta S \geq 0$, and the entropy of BHs

Bekenstein argued that to save 2nd Law one must assign a *finite* entropy to BHs, otherwise one could systematically reduce $S_{universe}$ by throwing in matter with $\Delta S > 0$ which then disappears

Black Hole Entropy

Hawking's calculation confirmed this since 1st Law says $dE = T_H\, dS$ and using T_H and $E = M_{BH}\, c^2$ gives (in $k_B=1$ units)

$$S_{BH} = \frac{c^3}{G\hbar}\frac{A}{4} = \frac{1}{4}\frac{A}{\ell_{Pl}^2}$$

Area of BH event horizon

"Planck length"

$$\ell_{Pl} = (G\hbar/c^3)^{1/2} \simeq 10^{-33}\text{cm}$$

in classical limit $\hbar \to 0$
entropy is infinite

(cf. catastrophe of pre-Planck classical black body spectrum)

Black Hole Entropy

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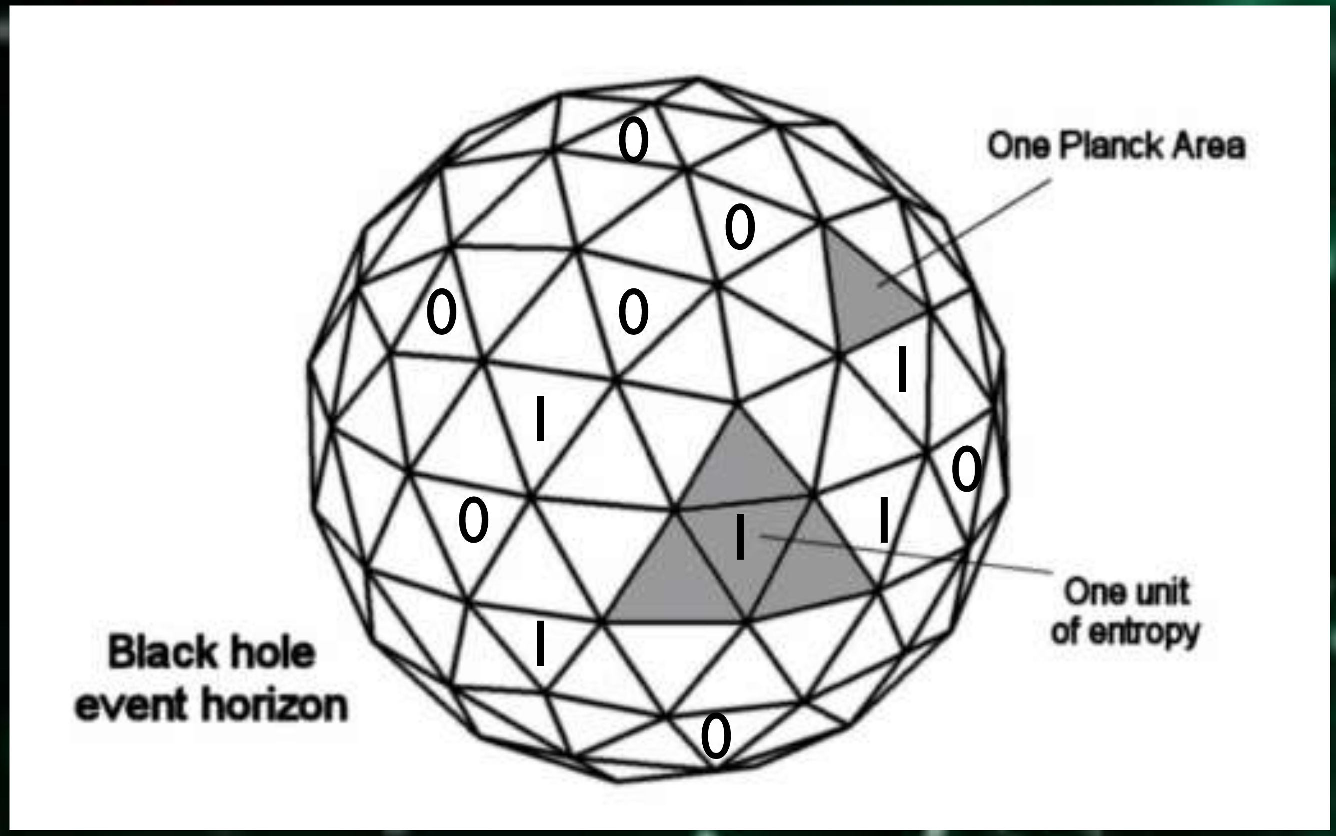
Area of BH event horizon

"One qubit per 4 Planck Areas of horizon"



Black Hole Entropy

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$$S_{BH} = \frac{c^3}{G\hbar}\frac{\mathrm{A}}{4} = \frac{1}{4}\frac{\mathrm{A}}{\ell_{Pl}^2}$$

$$S_{BH} \simeq 10^{77}\left(\frac{M}{M_\odot}\right)^2$$

Entropy of a *single* solar mass BH roughly that of *all* the stars in the Universe!

Quantum BHs are thermal systems with HUGE number of microstates

$$\#\text{states} \simeq \exp(S_{BH})$$

Black Hole Entropy

Some comments:

- Information content of the universe appears to be hugely dominated by BHs!!!
- We don't understand well these "microstates" of BHs - some progress from String Theory, but not complete...
- S_{BH} is not extensive - it goes with area not volume (or mass). Leads to "holographic principle" which motivates Maldacena's amazingly powerful holographic duality for String Theory in AdS (negatively-curved) space-times: AKA "AdS/CFT". We don't understand flat-space holography...

Hawking Evaporation

Now turn to other implications of Hawking Radiation…

Since T_H increases as M decreases, BHs can explosively evaporate by runaway Hawking radiation with $T_H|_{end} \simeq M_{Pl}/8\pi$

Image credit: ortega-pictures / Pixabay.

Hawking Evaporation

Now turn to other implications of Hawking Radiation...

Since T_H increases as M decreases, BHs can explosively evaporate by runaway Hawking radiation with $T_H|_{end} \simeq M_{Pl}/8\pi$

Hawking evaporation is extraordinary: every particle in nature with mass < $M_{Pl}/8\pi$ can be produced even if it is completely decoupled from Standard Model (nothing hides from gravity)!

Hawking evaporation is a scan over creation

Hawking Evaporation

One can produce the dark matter this way, and even start the hot phase of the Big Bang...

Black Hole Genesis of Dark Matter

Olivier Lennon, John March-Russell, Rudin Petrossian-Byrne, Hannah Tillim

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rudin.petrossian-byrne@physics.ox.ac.uk,

2 [hep-ph] 7 Feb 2018

Abstract. We present a purely gravitational infra-red-calculable product dark matter (DM). The source of both the DM relic abundance and the hot (SM) plasma is a primordial density of micro black holes (BHs), which evaporate emission into both the dark and SM sectors. The mechanism has four qualitative regimes depending upon whether the BH evaporation is 'fast' or 'slow' relative to th

Apr 2020

FERMILAB-PUB-20-12

Hot Gravitons and Gravitational Waves From Kerr Black Holes in the Early Univers

Dan Hooper[a,b,c,*] Gordan Krnjaic[a,†] John March-Russell[d,‡] Samuel D. McDermott[a,§] and Rudin Petrossian-Byrne[d,¶]

[a]Fermi National Accelerator Laboratory, Theoretical Astrophysics Group
[b]University of Chicago, Kavli Institute for Cosmological Physics
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[d]University of Oxford, Rudolf Peierls Centre for Theoretical Physics

(Dated: April 2, 2020)

Any abundance of black holes that was present in the early universe will evolve as matter, making up an increasingly large fraction of the total energy density as space expands. This motivates us to consider scenarios in which the early universe included an era that was dominated by low-mass ($M \lesssim 5 \times 10^8$ g) black holes which evaporate prior to primordial nucleosynthesis. In significant regions of parameter space, these black holes will become gravitationally bound within binary systems, and undergo mergers before evaporating. Such mergers result in three potentially observable signatures. First, any black holes that have undergone one or more mergers will possess substantial angular momentum, causing their Hawking evaporation to produce significant quantities of high-energy gravitons. These products of Hawking evaporation are predicted to constitute a background of hot (∼eV-keV) gravitons today, with an energy density corresponding to $\Delta N_{\rm eff} \sim 0.01 - 0.03$. Second, these mergers will produce a stochastic background of high-frequency gravitational waves. And third, the energy density of these gravitational waves can be as large as $\Delta N_{\rm eff} \sim 0.3$, depending on

could be observable signatures of Hawking radiation hiding in DM and gravitational waves (would have won Stephen Hawking his deserved Nobel....)

Hawking Evaporation

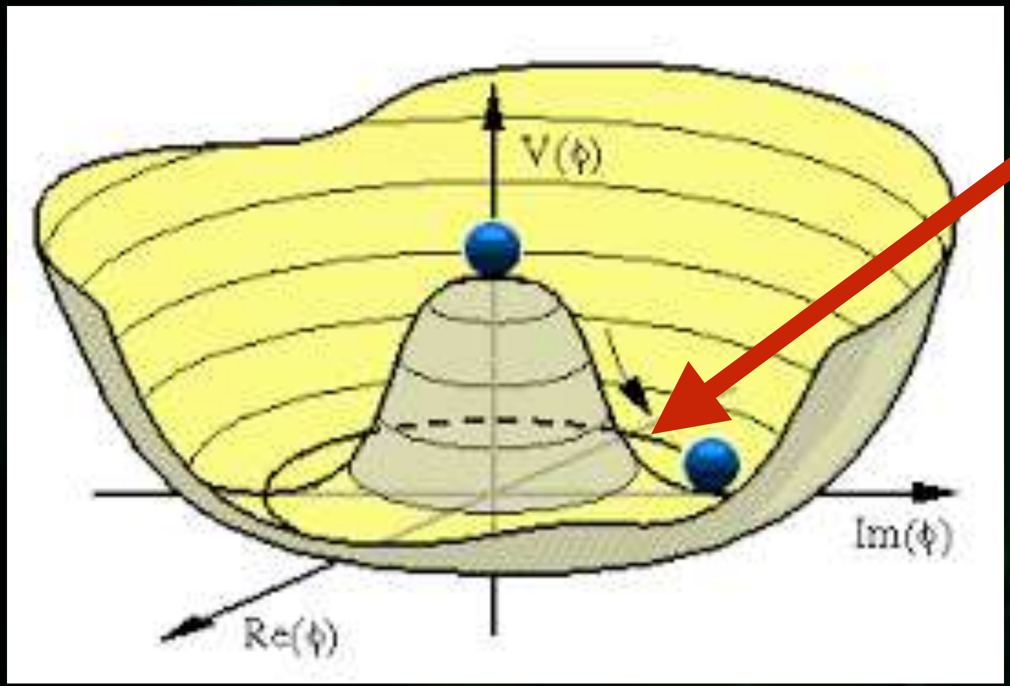
But there's more...!

Impossibility of Exact Global Symmetries

Hawking evaporation of BHs implies remarkable consequence that, in any 4d theory including gravity, non-space-time GSs are always violated (thus approximate at best)

So, eg, impossible to have *exactly* degenerate, physically distinct states resulting from such internal symmetries

can't be exactly equi-potential



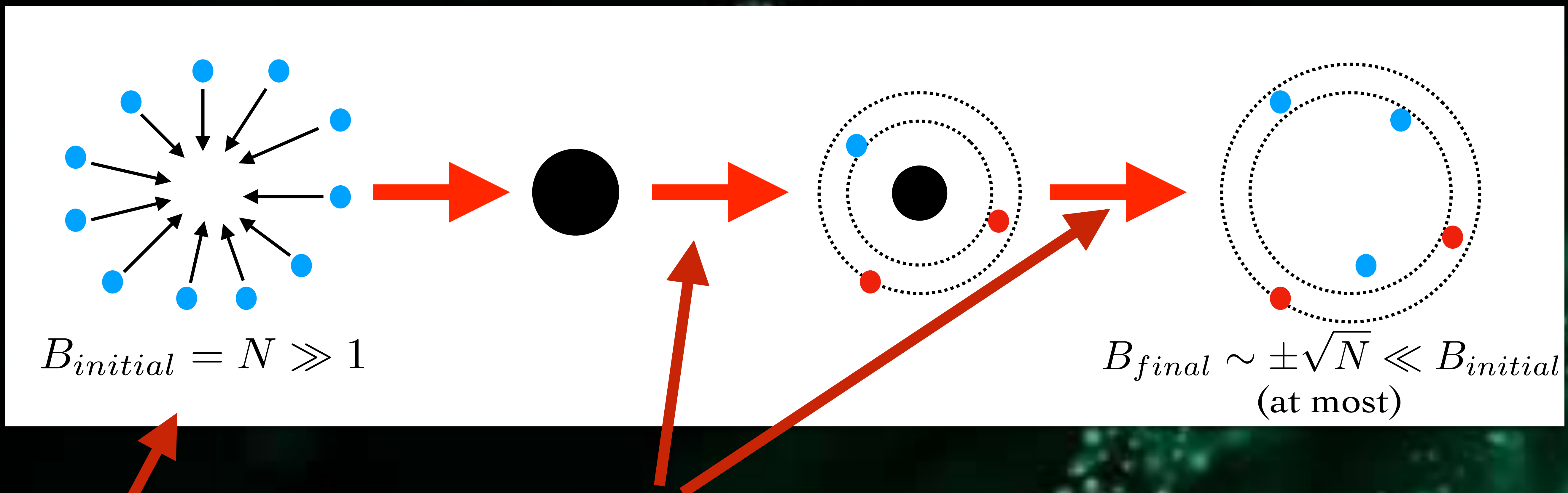
Impossibility of Exact Global Symmetries

By Emmy Noether's great theorem this means in turn that all the associated would-be-conserved "global charges" are not conserved

For example conservation of baryon number associated with internal $U(1)_B$ global symmetry must be violated

Impossibility of Exact Global Symmetries

Why is this true? The basic idea (goes back to Zeldovich):



BH formed by large initial number, N, of neutrons

Hawking radiation is independent of whether baryon or anti-baryon emitted (*only* cares about mass, spin, & charge)

(Note for experts: here, for simplicity, I am taking the BH to fully evaporate - no remnants! - but argument can be sharpened to exclude this case)

Impossibility of Exact Global Symmetries

We don't yet fully understand exactly what is the form and size of the global-symmetry-violating interactions that must be induced by quantum gravitational effects, but progress is being made!

Towards a Swampland Global Symmetry Conjecture using weak gravity

Tristan Daus [a], Arthur Hebecker [a], Sascha Leonhardt [a], John March-Russell [b]

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https://doi.org/10.1016/j.nuclphysb.2020.115167

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Abstract

It is widely believed and in part established that exact global symmetries are inconsistent with quantum gravity. One then expects that approximate global symmetries can be *quantitatively* constrained by quantum gravity or swa
arguments. We provide such a bound for an important class of global
Those arising from a gauged $U(1)$ with the vector made massive via
an axion. The latter necessarily couples to instantons, and their acti
constrained, using both the electric and magnetic version of the ax
gravity conjecture, in terms of the cutoff of the theory. As a result

20 Oct 2020 [hep-th]

Global symmetry, Euclidean gravity, and the black hole information problem

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Abstract: In this paper we argue for a close connection between the non-existence of global symmetries in quantum gravity and a unitary resolution
information problem. In particular we show how
calculations of the Page curve of an
a recent argument

9 Jan 2020 [hep-th]

Approximate Symmetries and Gravity

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[b] ICTP South American Institute for Fundamental Research & IFT-UNESP, R. Dr. Bento Teobaldo Ferraz 271, São Paulo, Brazil 2019-032

Abstract

There are strong reasons to believe that global symmetries of quantum theories cannot be exact in the presence of gravity. While this has been argued at the qualitative level, establishing a quantitative statement is more challenging. In this work we take new steps

Final Words

One issue I haven't spoken about is if QM & BHs are in fact inconsistent? This was view of Stephen Hawking for many years (though not last 5), and might still be Roger's

Hawking evaporation of BHs appears to allow QM "pure states" to evolve to "mixed states" - if so this would violate a fundamental axiom of QM and imply information loss

But in last 2 years there has been major progress and a consistent calculational framework is emerging

Quantum BHs live..!

Solving BH Information Paradox

Basic idea: Hawking's calculation of thermal density matrix was never exact

$$\rho_R = \rho^{thermal}\,(1 + \text{loop effects}) + \mathcal{O}\left(e^{-A/\ell_{Pl}^2}\right)$$

All these usual local QFT terms don't help

New tiny, non-perturbative and non-local, effects

$$S_{rad} = -\sum_{\text{states } i} \lambda_i \log \lambda_i$$

$(e^{A/\ell_{Pl}^2}$ terms $\times\, e^{-A/\ell_{Pl}^2}$ corrections$) \to \mathcal{O}(1)$ modification

Quantum BHs

QM of BHs is even more remarkable and is guiding us towards a better understanding of quantum gravity (& information!)

"Hydrogen Atom of the 21st Century"